Year 2003

VCE

Mathematical Methods
Trial Examination 1

Suggested Solutions

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These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.

Question 1 D

Amplitude of $4\cos(3x-\frac{\pi}{2})$ is 4

This graph has a maximum of 4 and a minimum of - 4.

When $y = 4\cos(3x-\frac{\pi}{2})-1$ the graph has a maximum of 4 - 1 = 3 and a minimum of -4 - 1 = -5

$\therefore$ Range is $[-5,3]$

Question 2 C

When $t=2,\ \ y=2$ which is true for A,C and E

Looking only at A,C and E,

When $t=6,\ \ y=1.5$ which is not true for A or E

Question 3 E

$\sqrt{2}\cos^2 x+\cos x-\sqrt{2}=0 \qquad 0\le x\le 2\pi$

$(\sqrt{2}\cos x-1)(\cos x+\sqrt{2})=0$

$\Rightarrow \sqrt{2}\cos x=1$ or $\cos x=-\sqrt{2}$

$-1\le \cos x\le 1$

$\therefore \cos x\ne -\sqrt{2}$

$\therefore \cos x=\frac{1}{\sqrt{2}} \qquad 0\le x\le 2\pi$

$\therefore x=\frac{\pi}{4},\frac{7\pi}{4}$

Sum of solutions $=\frac{\pi}{4}+\frac{7\pi}{4}=2\pi$

Question 4 A

This is a sin or cos graph.

Maximum = 4

Minimum = –2

$\therefore$ Amplitude = 3 (midway between –2 and 4)

Graph has been translated up 1 so +1 on end.

Period = π = $\frac{2\pi}{n}$

$\therefore$ n = 2

Phase shift is $\frac{\pi}{4}$ to the right, $\therefore$ $(x-\frac{\pi}{4})$

The graph without the phase shift would have had a maximum when x = 0, $\therefore$ cos graph.

Question 5 E

$$(2x-3)^7=(2x)^7-\binom{7}{1}(2x)^6(3)^1+\binom{7}{2}(2x)^5(3)^2$$
$$-\binom{7}{3}(2x)^4(3)^3+\binom{7}{4}(2x)^3(3)^4$$
$$-\binom{7}{5}(2x)^2(3)^5+.....$$

Coefficient of x^2 is $-\binom{7}{5}(2)^2(3)^5=-20412$

Question 6 B

$\log_e(x^2)-\log_e(2x)=q$

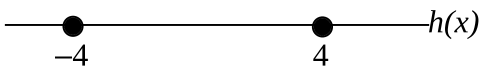
$\log_e\frac{x^2}{2x}=q$

$\log_e\frac{x}{2}=q$

$e^q=\frac{x}{2}$

$x=2e^q$

Question 7 E



If $g(x)$ is $[-4,0)$ then $f(x)$ is $[-4,0)$
If $g(x)$ is $[-4,4]$ then $f(x)$ is $[-4,4]$
If $g(x)$ is $(0,4)$ then $f(x)$ is $(0,4)$
If $g(x)$ is $(-4,0]$ then $f(x)$ is $(-4,0]$
If $g(x)$ is $(0,6]$ then $f(x)$ is $(0,4]$

Question 8 C

The graph of $y = \frac{1}{x}$ is reflected in the x axis to give $y = -\frac{1}{x}$. It is translated 2 units to the right to give $y = -\frac{1}{x-2}$. It is dilated by a factor k to give $y = -\frac{k}{x-2}$

When $x = 0$, $y = 1$

$y = -\frac{k}{-2} = 1$

$\therefore k = 2$

$y = -\frac{2}{x-2}$ which means the original graph has been reflected in the x axis, translated 2 units to the right parallel to the x axis and dilated by a factor of 2

Question 9 C

For this many-one function to have an inverse that is also a function, it must have its domain restricted so that it is a one-one function. This can be done by restricting the domain from the axis of symmetry, which in this case is $x = 0$.

$\therefore [0,3]$ which means $a = 0$

Question 10 C

Asymptote for $y = be^x$ is $y = 0$
$\therefore$ asymptote for $y = be^x + a$ is $y = a$
Asymptote on the given graph is $y = -6$
$\therefore a = -6$
When $x = 0$, $y = a + b = 1$ from graph
$\therefore b = 7$

Question 11 C

Let $y = e^x$

$y = 1 + \frac{4}{y}$

$y^2 = y + 4$

$y^2 - y - 4 = 0$

$y = \frac{1 \pm \sqrt{1+16}}{2}$

$y = \frac{1 \pm \sqrt{17}}{2}$

$\therefore e^x = \frac{1+\sqrt{17}}{2}$ or $\frac{1-\sqrt{17}}{2}$

But $e^x > 0$

$\therefore e^x = \frac{1+\sqrt{17}}{2} = 2.56155$

$\therefore x = \log_e 2.56155 = 0.9406$

Question 12 E

Equation of graph is of the form

$y = k(x - a)(x - b)(x - c)^2$

When $x = 0$, $y < 0$

$\therefore$ k is negative and could be -1

$\therefore y = -(x - a)(x - b)(x - c)^2$

$\therefore y = -(x - b)(x - a)(x - c)^2$

$\therefore y = (b - x)(x - a)(x - c)^2$

$\therefore y = (x - a)(b - x)(x - c)^2$

Question 13 D

Let $u = 3x^2 + 5$

$\frac{du}{dx} = 6x$

$y = \log_e u$

$\frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$\frac{dy}{dx} = \frac{1}{u} \times 6x$

$\frac{dy}{dx} = \frac{6x}{u} = \frac{6x}{3x^2+5}$

Question 14 B

Gradient of curve at point of tangency = gradient of tangent line = 1

$\therefore \frac{dy}{dx} = 1$ at point of tangency

$\therefore 2x = 1$

$\Rightarrow x = \frac{1}{2}$

When $x = \frac{1}{2}$, $y = \frac{1}{2} - 7$

$\Rightarrow y = -6\frac{1}{2}$

$\therefore$ Point of tangency is $\left(\frac{1}{2}, -\frac{13}{2}\right)$

On curve when $x = \frac{1}{2}$, $y = -\frac{13}{2}$

$\therefore -\frac{13}{2} = \frac{1}{4} + c$

$\Rightarrow c = -\frac{26}{4} - \frac{1}{4} = -\frac{27}{4}$

Question 15 B

$\frac{dy}{dx} = x^2 \times \frac{d}{dx}\cos 2x + \cos 2x \times \frac{d}{dx}x^2$

$= x^2 \times (-2\sin 2x) + \cos 2x \times 2x$

$= -2x^2 \sin 2x + 2x\cos 2x$

When $x = \pi$

$\frac{dy}{dx} = -2\pi^2 \sin 2\pi + 2\pi\cos 2\pi$

$\frac{dy}{dx} = -0 + 2\pi = 2\pi$

Question 16 B

Graph of $f(x)$ has gradient =0 when $x = 2$ and when $x = 4$

For $0 < x < 4$ except when $x = 2$, $y = f(x)$ is an increasing graph, hence the gradient, i.e. $f'(x)$ is greater than 0 in this region.

Question 17 B

The derivative does not exist at $x = 2$ or at $x = -1$, or at $x = 4$

$\therefore$ domain of $f^1(x) = R \setminus \{-1, 2, 4\}$

Question 18 B

$\frac{dy}{dx} = -3x^2 + 4x + 7 = 0$ for T.P.

$(3x-7)(-x-1) = 0$

$\therefore 3x = 7$ or $x = -1$ $\therefore x = \frac{7}{3}$ or $x = -1$

$\therefore$ turning points exist at $x = \frac{7}{3}$ and $x = -1$

When $x < -1$ $\frac{dy}{dx} < 0$

When $-1 < x < \frac{7}{3}$ $\frac{dy}{dx} > 0$

When $x > \frac{7}{3}$ $\frac{dy}{dx} < 0$

Hence, local minimum when $x = -1$ and gradient is positive for $-1 < x < \frac{7}{3}$

Question 19 C

$y = \int \frac{dx}{2x+1}$

$y = \frac{1}{2}\int \frac{2dx}{2x+1}$

$y = \frac{1}{2}\log_e(2x+1) + c$

Question 20 D

$f(x) = \int (3e^{\pi} \sin\frac{x}{4})dx$

$f(x) = 3e^{\pi}\int (\sin\frac{1}{4}x)dx$

$f(x) = 3e^{\pi}(-\cos\frac{x}{4}) \div \frac{1}{4} + c$

$f(x) = -12e^{\pi}\cos\frac{x}{4} + c$

Question 21 D

Area of trapezium $= \frac{1}{2}(a+b)h$

$\therefore$ Area under graph $= \frac{1}{2}(f(1) + f(1.5))0.5 + \frac{1}{2}(f(1.5) + f(2))0.5$

$f(1) = 1 + 3 = 4$

$f(1.5) = 2.25 + 3 = 5.25$

$f(2) = 4 + 3 = 7$

$\therefore$ Area under graph $= \frac{1}{4}(f(1) + 2f(1.5) + f(2))$

$\therefore$ Area under graph $= \frac{1}{4}(4 + 10.5 + 7) = 5.375$

Question 22 B

$\int_3^a e^{2x}dx = 21623.037$

$\frac{1}{2}e^{2x}]_3^a = 21623.037$

$\frac{1}{2}[e^{2a} - e^6] = 21623.037$

$e^{2a} - e^6] = 43246.074$

$e^{2a} = 43246.074 + e^6$

$e^{2a} = 43649.50279$

$2a = \log_e 43649.50279$

$2a = 10.684$

$a = 5.3$

Question 23 D

$\sum \Pr = 1$

$\therefore b = 1 - (0.2 + 0.3 + 0.1)$

$\therefore b = 1 - 0.6$

$\therefore b = 0.4$

$\mu = \sum x \Pr(X = x) = 1.2$

$\therefore -0.4 - 0.3 + 0.4a + 0.1a + 0.4 = 1.2$

$\therefore 0.5a = 1.5$

$\therefore a = 3$

Question 24 C

$Z = \frac{x - \mu}{\sigma}$

$1.5 = \frac{a - 10}{4}$

$6 = a - 10$

$a = 16$

Question 25 C

Without replacement, hypergeometric

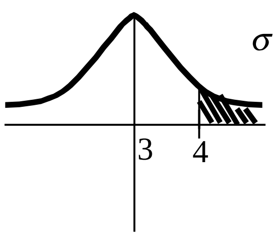
Pr at least one green = Pr 1 is green or Pr 2 are green

$\Pr(x = 1) + \Pr(x = 2)$

$= \frac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}} + \frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}}$

$= 0.6$

Question 26 A



$\sigma = \frac{1}{2}$

$\Pr(X > 4) = \Pr(Z > 2)$ $\quad Z = \frac{x - \mu}{\sigma}$

$\Pr(X > 4) = 1 - \Pr(Z < 2)$ $\quad Z = \frac{4 - 3}{\frac{1}{2}} = 2$

$\Pr(X > 4) = 1 - 0.9772$

$\Pr(X > 4) = 0.0228$

$\Pr(X > 4) = 2.28\%$

Question 27 D

Binomial

$p = 0.4$

$q = 0.6$

$n = 3$

$x = 1$

$\Pr(X = 1) = \binom{3}{1}(0.4)^1(0.6)^2$

$\Pr(X = 1) = 0.43$

Question 1

$2x - 3 \geq 0$

$\therefore 2x \geq 3$

$\therefore x \geq \frac{3}{2}$

$\therefore$ Domain $[\frac{3}{2}, \infty)$ (1 mark)

When $x = \frac{3}{2}$ $f(x) = -\frac{1}{2} \times 0 + 4 = 4$

When $x \to \infty$ $f(x) \to 4 -$ a very large number

$\therefore f(x) \to -\infty$

Range$(-\infty, 4]$ (1 mark)

Question 2

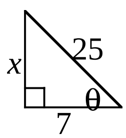
$\frac{\log_a \frac{16}{2}}{\log_a 2} = \frac{\log_a 8}{\log_a 2}$ (1 mark)

$= \frac{\log_a 2^3}{\log_a 2}$

$= \frac{3\log_a 2}{\log_a 2}$

$= 3$ (1 mark)

Question 3



Using Pythagorean triad 7:24:25
$x = 24$
θ is in the 4th quadrant

$\therefore$ tan θ is negative (1 mark)

$\therefore \tan\theta = -\frac{24}{7}$ (1 mark)

Question 4
a.

$f(x) = 3\sin(\frac{1}{3}x)$

$f^1(x) = 3 \times \frac{1}{3}\cos(\frac{1}{3}x)$

$= \cos(\frac{x}{3})$ (1 mark)

b.

Minimum value of $3\sin(\frac{x}{3})$, from the amplitude would be -3 (1 mark)

Question 4
b.(continued)

$3\sin(\frac{x}{3}) = -3$

$\sin(\frac{x}{3}) = -1$ $\quad 0 \leq x \leq 8\pi$

$0 \leq \frac{x}{3} \leq \frac{8\pi}{3}$

$\frac{x}{3} = \frac{3\pi}{2}$

$x = \frac{9\pi}{2}$ (1 mark)

Minimum is -3 when $x = \frac{9\pi}{2}$

Question 5
a.

$\frac{dy}{dx} = -8(3 - 2x) \times (-2)$

$\frac{dy}{dx} = 16(3 - 2x) = 0$ for turning point

$\Rightarrow (3 - 2x) = 0$

$\Rightarrow x = \frac{3}{2}$

When $x = \frac{3}{2}$

$y = 5 - 4(3 - 3)^2$

$\Rightarrow y = 5 - 0 = 5$

Turning point is $(\frac{3}{2}, 5)$ (1 mark)

Question 5
b.

$y = -4(3-2x)^2 + 5$

Graph of $y = x^2$ is translated $\frac{3}{2}$ units to the right parallel to the x axis. (1 mark)

It is translated 5 units up parallel to the y axis (1 mark)

It is reflected in the y axis because of the minus sign in front of the equation (1 mark)

It is dilated by a factor of 16 in the y direction(because of the $-4 \times (-2)^2$ which is the coefficient of x^2 in the expansion (1 mark)

Question 6
a.

$E(x) = \sum x \Pr(X = x)$

$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$

$= \frac{21}{6}$

$= 3.5$ (1 mark)

Question 6
b.

95% confidence limits: $\mu \pm 2\sigma$

$\sigma = \sqrt{x^2 p(x) - \mu^2}$

$x^2 p(x) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}$

$x^2 p(x) = \frac{91}{6} = 15.1667$

$\sigma = \sqrt{15.1667 - 12.25} = 1.708$ (1 mark)

$2\sigma = 3.42$

$\mu \pm 2\sigma = 3.5 \pm 3.42$

$0.08 \le \mu \le 6.92$

$0.1 \le \mu \le 7.0$ (1 mark)

Question 7

Sampling without replacement is hypergeometric

$\Pr(X < 2) = \Pr(x = 0) + \Pr(X = 1)$ (1 mark)

$\Pr(X < 2) = \frac{\binom{6}{0}\binom{24}{10}}{\binom{30}{10}} + \frac{\binom{6}{1}\binom{24}{9}}{\binom{30}{10}}$ (1 mark)

$\Pr(X < 2) = 0.3264$ to four decimal places (1 mark)

Question 8 a.	Question 8 b.
$y = e^{\cos x}$ Let $u = \cos x$ $\frac{du}{dx} = -\sin x$ $y = e^u$ $\frac{dy}{du} = e^u$ $\frac{dy}{dx} = \frac{dy}{du} \quad \frac{du}{dx}$ $\frac{dy}{dx} = e^u(-\sin x)$ $\frac{dy}{dx} = -\sin xe^{\cos x}$ (1 mark)	$\int -\sin xe^{\cos x}dx = e^{\cos x} + c$ where c is a constant $\int \sin xe^{\cos x}dx = -e^{\cos x} - c$ (1 mark) $\therefore \int_0^{\frac{\pi}{2}} \sin xe^{\cos x}dx = -e^{\cos x}]_0^{\frac{\pi}{2}}$ $\therefore \int_0^{\frac{\pi}{2}} \sin xe^{\cos x}dx = (-e^{\cos\frac{\pi}{2}}) - (-e^{\cos 0})$ $\therefore \int_0^{\frac{\pi}{2}} \sin xe^{\cos x}dx = -e^0 + e^1 = e - 1$ (1 mark)

END OF SUGGESTED SOLUTIONS
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