

Year 2003

VCE

Mathematical Methods Trial Examination 1

Suggested Solutions

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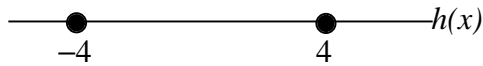
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<p>These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.</p>
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Suggested Solutions Part I

<p>Question 1 D</p> <p>Amplitude of $4 \cos(3x - \frac{\pi}{2})$ is 4</p> <p>This graph has a maximum of 4 and a minimum of -4.</p> <p>When $y = 4 \cos(3x - \frac{\pi}{2}) - 1$ the graph has a maximum of $4 - 1 = 3$ and a minimum of $-4 - 1 = -5$</p> <p>\therefore Range is $[-5, 3]$</p>	<p>Question 2 C</p> <p>When $t = 2$, $y = 2$ which is true for A,C and E</p> <p>Looking only at A,C and E,</p> <p>When $t = 6$, $y = 1.5$ which is not true for A or E</p>
<p>Question 3 E</p> $\sqrt{2} \cos^2 x + \cos x - \sqrt{2} = 0 \quad 0 \leq x \leq 2\pi$ $(\sqrt{2} \cos x - 1)(\cos x + \sqrt{2}) = 0$ $\Rightarrow \sqrt{2} \cos x = 1 \text{ or } \cos x = -\sqrt{2}$ $-1 \leq \cos x \leq 1$ $\therefore \cos x \neq -\sqrt{2}$ $\therefore \cos x = \frac{1}{\sqrt{2}} \quad 0 \leq x \leq 2\pi$ $\therefore x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\text{Sum of solutions} = \frac{\pi}{4} + \frac{7\pi}{4} = 2\pi$	<p>Question 4 A</p> <p>This is a sin or cos graph.</p> <p>Maximum = 4</p> <p>Minimum = -2</p> <p>\therefore Amplitude = 3 (midway between -2 and 4)</p> <p>Graph has been translated up 1 so +1 on end.</p> $\text{Period} = \pi = \frac{2\pi}{n}$ <p>$\therefore n = 2$</p> <p>Phase shift is $\frac{\pi}{4}$ to the right, $\therefore (x - \frac{\pi}{4})$</p> <p>The graph without the phase shift would have had a maximum when $x = 0$, \therefore cos graph.</p>
<p>Question 5 E</p> $(2x - 3)^7 = (2x)^7 - \binom{7}{1}(2x)^6(3)^1 + \binom{7}{2}(2x)^5(3)^2$ $- \binom{7}{3}(2x)^4(3)^3 + \binom{7}{4}(2x)^3(3)^4$ $- \binom{7}{5}(2x)^2(3)^5 + \dots$ <p>Coefficient of x^2 is $-\binom{7}{5}(2)^2(3)^5 = -20412$</p>	<p>Question 6 B</p> $\log_e(x^2) - \log_e(2x) = q$ $\log_e \frac{x^2}{2x} = q$ $\log_e \frac{x}{2} = q$ $e^q = \frac{x}{2}$ $x = 2e^q$

Question 7 E



If $g(x)$ is $[-4,0)$ then $f(x)$ is $[-4,0)$

If $g(x)$ is $[-4,4]$ then $f(x)$ is $[-4,4]$

If $g(x)$ is $(0,4)$ then $f(x)$ is $(0,4)$

If $g(x)$ is $(-4,0]$ then $f(x)$ is $(-4,0]$

If $g(x)$ is $(0,6]$ then $f(x)$ is $(0,4]$

Question 8 C

The graph of $y = \frac{1}{x}$ is reflected in the x axis to

give $y = -\frac{1}{x}$. It is translated 2 units to the right to

give $y = -\frac{1}{x-2}$. It is dilated by a factor k to

give $y = -\frac{k}{x-2}$

When $x = 0$, $y = 1$

$$y = -\frac{k}{-2} = 1$$

$$\therefore k = 2$$

$y = -\frac{2}{x-2}$ which means the original graph has

been reflected in the x axis, translated 2 units to the right parallel to the x axis and dilated by a factor of 2

Question 9 C

For this many-one function to have an inverse that is also a function, it must have its domain restricted so that it is a one-one function. This can be done by restricting the domain from the axis of symmetry, which in this case is $x = 0$.

$$\therefore [0,3] \text{ which means } a = 0$$

Question 10 C

Asymptote for $y = be^x$ is $y = 0$

$$\therefore \text{asymptote for } y = be^x + a \text{ is } y = a$$

Asymptote on the given graph is $y = -6$

$$\therefore a = -6$$

When $x = 0$, $y = a + b = 1$ from graph

$$\therefore b = 7$$

Question 11 C

Let $y = e^x$

$$y = 1 + \frac{4}{y}$$

$$y^2 = y + 4$$

$$y^2 - y - 4 = 0$$

$$y = \frac{1 \pm \sqrt{1+16}}{2}$$

$$y = \frac{1 \pm \sqrt{17}}{2}$$

$$\therefore e^x = \frac{1 + \sqrt{17}}{2} \text{ or } \frac{1 - \sqrt{17}}{2}$$

But $e^x > 0$

$$\therefore e^x = \frac{1 + \sqrt{17}}{2} = 2.56155$$

$$\therefore x = \log_e 2.56155 = 0.9406$$

Question 12 E

Equation of graph is of the form

$$y = k(x - a)(x - b)(x - c)^2$$

When $x = 0$, $y < 0$

$\therefore k$ is negative and could be -1

$$\therefore y = -(x - a)(x - b)(x - c)^2$$

$$\therefore y = -(x - b)(x - a)(x - c)^2$$

$$\therefore y = (b - x)(x - a)(x - c)^2$$

$$\therefore y = (x - a)(b - x)(x - c)^2$$

Question 13 D

Let $u = 3x^2 + 5$

$$\frac{du}{dx} = 6x$$

$$y = \log_e u$$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \times 6x$$

$$\frac{dy}{dx} = \frac{6x}{u} = \frac{6x}{3x^2 + 5}$$

Question 14 B

Gradient of curve at point of tangency = gradient of tangent line = 1

$$\therefore \frac{dy}{dx} = 1 \text{ at point of tangency}$$

$$\therefore 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

When $x = \frac{1}{2}$, $y = \frac{1}{2} - 7$

$$\Rightarrow y = -6\frac{1}{2}$$

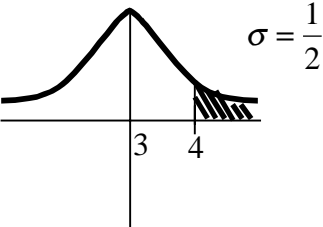
$$\therefore \text{Point of tangency is } \left(\frac{1}{2}, -\frac{13}{2}\right)$$

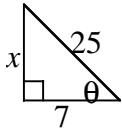
On curve when $x = \frac{1}{2}$, $y = -\frac{13}{2}$

$$\therefore -\frac{13}{2} = \frac{1}{4} + c$$

$$\Rightarrow c = -\frac{26}{4} - \frac{1}{4} = -\frac{27}{4}$$

<p>Question 15 B</p> $\frac{dy}{dx} = x^2 \times \frac{d}{dx} \cos 2x + \cos 2x \times \frac{d}{dx} x^2$ $= x^2 \times (-2 \sin 2x) + \cos 2x \times 2x$ $= -2x^2 \sin 2x + 2x \cos 2x$ <p>When $x = \pi$</p> $\frac{dy}{dx} = -2\pi^2 \sin 2\pi + 2\pi \cos 2\pi$ $\frac{dy}{dx} = -0 + 2\pi = 2\pi$	<p>Question 16 B</p> <p>Graph of $f(x)$ has gradient = 0 when $x = 2$ and when $x = 4$</p> <p>For $0 < x < 4$ except when $x = 2$, $y = f(x)$ is an increasing graph, hence the gradient, i.e. $f'(x)$ is greater than 0 in this region.</p> <p>Question 17 B</p> <p>The derivative does not exist at $x = 2$ or at $x = -1$, or at $x = 4$</p> <p>\therefore domain of $f'(x) = R \setminus \{-1, 2, 4\}$</p>
<p>Question 18 B</p> $\frac{dy}{dx} = -3x^2 + 4x + 7 = 0 \text{ for T.P.}$ $(3x-7)(-x-1) = 0$ $\therefore 3x = 7 \text{ or } x = -1 \quad \therefore x = \frac{7}{3} \text{ or } x = -1$ $\therefore \text{turning points exist at } x = \frac{7}{3} \text{ and } x = -1$ <p>When $x < -1$ $\frac{dy}{dx} < 0$</p> <p>When $-1 < x < \frac{7}{3}$ $\frac{dy}{dx} > 0$</p> <p>When $x > \frac{7}{3}$ $\frac{dy}{dx} < 0$</p> <p>Hence, local minimum when $x = -1$ and gradient is positive for $-1 < x < \frac{7}{3}$</p>	<p>Question 19 C</p> $y = \int \frac{dx}{2x+1}$ $y = \frac{1}{2} \int \frac{2dx}{2x+1}$ $y = \frac{1}{2} \log_e(2x+1) + c$
<p>Question 20 D</p> $f(x) = \int (3e^{\pi} \sin \frac{x}{4}) dx$ $f(x) = 3e^{\pi} \int (\sin \frac{1}{4} x) dx$ $f(x) = 3e^{\pi} (-\cos \frac{x}{4}) \div \frac{1}{4} + c$ $f(x) = -12e^{\pi} \cos \frac{x}{4} + c$	<p>Question 21 D</p> <p>Area of trapezium = $\frac{1}{2}(a+b)h$</p> $\therefore \text{Area under graph} = \frac{1}{2}(f(1) + f(1.5))0.5 + \frac{1}{2}(f(1.5) + f(2))0.5$ $f(1) = 1 + 3 = 4$ $f(1.5) = 2.25 + 3 = 5.25$ $f(2) = 4 + 3 = 7$ $\therefore \text{Area under graph} = \frac{1}{4}(f(1) + 2f(1.5) + f(2))$ $\therefore \text{Area under graph} = \frac{1}{4}(4 + 10.5 + 7) = 5.375$

<p>Question 22 B</p> $\int_3^a e^{2x} dx = 21623.037$ $\frac{1}{2} e^{2x} \Big _3^a = 21623.037$ $\frac{1}{2} [e^{2a} - e^6] = 21623.037$ $e^{2a} - e^6 = 43246.074$ $e^{2a} = 43246.074 + e^6$ $e^{2a} = 43649.50279$ $2a = \log_e 43649.50279$ $2a = 10.684$ $a = 5.3$	<p>Question 23 D</p> $\sum \text{Pr} = 1$ $\therefore b = 1 - (0.2 + 0.3 + 0.1)$ $\therefore b = 1 - 0.6$ $\therefore b = 0.4$ $\mu = \sum x \text{Pr}(X = x) = 1.2$ $\therefore -0.4 - 0.3 + 0.4a + 0.1a + 0.4 = 1.2$ $\therefore 0.5a = 1.5$ $\therefore a = 3$
<p>Question 24 C</p> $Z = \frac{x - \mu}{\sigma}$ $1.5 = \frac{a - 10}{4}$ $6 = a - 10$ $a = 16$	<p>Question 25 C Without replacement, hypergeometric</p> <p>Pr at least one green = Pr 1 is green or Pr 2 are green</p> $\text{Pr}(x = 1) + \text{Pr}(x = 2)$ $= \frac{\binom{2}{1} \binom{4}{1}}{\binom{6}{2}} + \frac{\binom{2}{2} \binom{4}{0}}{\binom{6}{2}}$ $= 0.6$
<p>Question 26 A</p>  <p style="text-align: right;">$\sigma = \frac{1}{2}$</p> $\text{Pr}(X > 4) = \text{Pr}(Z > 2)$ $\text{Pr}(X > 4) = 1 - \text{Pr}(Z < 2)$ $Z = \frac{x - \mu}{\sigma}$ $Z = \frac{4 - 3}{\frac{1}{2}} = 2$ $\text{Pr}(X > 4) = 1 - 0.9772$ $\text{Pr}(X > 4) = 0.0228$ $\text{Pr}(X > 4) = 2.28\%$	<p>Question 27 D Binomial</p> $p = 0.4$ $q = 0.6$ $n = 3$ $x = 1$ $\text{Pr}(X = 1) = \binom{3}{1} (0.4)^1 (0.6)^2$ $\text{Pr}(X = 1) = 0.43$

<p>Question 1</p> $2x - 3 \geq 0$ $\therefore 2x \geq 3$ $\therefore x \geq \frac{3}{2}$ $\therefore \text{Domain } \left[\frac{3}{2}, \infty\right) \quad (1 \text{ mark})$ <p>When $x = \frac{3}{2}$ $f(x) = -\frac{1}{2} \times 0 + 4 = 4$</p> <p>When $x \rightarrow \infty$ $f(x) \rightarrow 4 -$ a very large number</p> $\therefore f(x) \rightarrow -\infty$ $\text{Range}(-\infty, 4] \quad (1 \text{ mark})$	<p>Question 2</p> $\frac{\log_a \frac{16}{2}}{\log_a 2} = \frac{\log_a 8}{\log_a 2} \quad (1 \text{ mark})$ $= \frac{\log_a 2^3}{\log_a 2}$ $= \frac{3 \log_a 2}{\log_a 2}$ $= 3 \quad (1 \text{ mark})$
<p>Question 3</p>  <p>Using Pythagorean triad 7:24:25</p> $x = 24$ <p>θ is in the 4th quadrant</p> $\therefore \tan \theta \text{ is negative} \quad (1 \text{ mark})$ $\therefore \tan \theta = -\frac{24}{7} \quad (1 \text{ mark})$	<p>Question 4</p> <p>a.</p> $f(x) = 3 \sin\left(\frac{1}{3}x\right)$ $f'(x) = 3 \times \frac{1}{3} \cos\left(\frac{1}{3}x\right)$ $= \cos\left(\frac{x}{3}\right) \quad (1 \text{ mark})$ <p>b.</p> <p>Minimum value of $3 \sin\left(\frac{x}{3}\right)$, from the amplitude would be -3 (1 mark)</p>
<p>Question 4 b.(continued)</p> $3 \sin\left(\frac{x}{3}\right) = -3$ $\sin\left(\frac{x}{3}\right) = -1 \quad 0 \leq x \leq 8\pi$ $0 \leq \frac{x}{3} \leq \frac{8\pi}{3}$ $\frac{x}{3} = \frac{3\pi}{2}$ $x = \frac{9\pi}{2} \quad (1 \text{ mark})$ <p>Minimum is -3 when $x = \frac{9\pi}{2}$</p>	<p>Question 5</p> <p>a.</p> $\frac{dy}{dx} = -8(3 - 2x) \times (-2)$ $\frac{dy}{dx} = 16(3 - 2x) = 0 \text{ for turning point}$ $\Rightarrow (3 - 2x) = 0$ $\Rightarrow x = \frac{3}{2}$ <p>When $x = \frac{3}{2}$</p> $y = 5 - 4(3 - 3)^2$ $\Rightarrow y = 5 - 0 = 5$ <p>Turning point is $\left(\frac{3}{2}, 5\right)$ (1 mark)</p>

<p>Question 5 b.</p> $y = -4(3 - 2x)^2 + 5$ <p>Graph of $y = x^2$ is translated $\frac{3}{2}$ units to the right parallel to the x axis. (1 mark)</p> <p>It is translated 5 units up parallel to the y axis (1 mark)</p> <p>It is reflected in the y axis because of the minus sign in front of the equation (1 mark)</p> <p>It is dilated by a factor of 16 in the y direction (because of the $-4 \times (-2)^2$ which is the coefficient of x^2 in the expansion (1 mark)</p>	<p>Question 6 a.</p> $E(x) = \sum x \Pr(X = x)$ $= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$ $= \frac{21}{6}$ $= 3.5 \quad (1 \text{ mark})$
<p>Question 6 b.</p> <p>95% confidence limits: $\mu \pm 2\sigma$</p> $\sigma = \sqrt{x^2 p(x) - \mu^2}$ $x^2 p(x) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}$ $x^2 p(x) = \frac{91}{6} = 15.1667$ $\sigma = \sqrt{15.1667 - 12.25} = 1.708 \quad (1 \text{ mark})$ $2\sigma = 3.42$ $\mu \pm 2\sigma = 3.5 \pm 3.42$ $0.08 \leq \mu \leq 6.92$ $0.1 \leq \mu \leq 7.0 \quad (1 \text{ mark})$	<p>Question 7</p> <p>Sampling without replacement is hypergeometric</p> $\Pr(X < 2) = \Pr(x = 0) + \Pr(X = 1) \quad (1 \text{ mark})$ $\Pr(X < 2) = \frac{\binom{6}{0} \binom{24}{10}}{\binom{30}{10}} + \frac{\binom{6}{1} \binom{24}{9}}{\binom{30}{10}} \quad (1 \text{ mark})$ $\Pr(X < 2) = 0.3264 \text{ to four decimal places} \quad (1 \text{ mark})$

<p>Question 8 a.</p> $y = e^{\cos x}$ <p>Let $u = \cos x$</p> $\frac{du}{dx} = -\sin x$ $y = e^u$ $\frac{dy}{du} = e^u$ $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ $\frac{dy}{dx} = e^u (-\sin x)$ $\frac{dy}{dx} = -\sin x e^{\cos x} \quad (1 \text{ mark})$	<p>Question 8 b.</p> $\int -\sin x e^{\cos x} dx = e^{\cos x} + c \text{ where } c \text{ is a constant}$ $\int \sin x e^{\cos x} dx = -e^{\cos x} - c \quad (1 \text{ mark})$ $\therefore \int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx = -e^{\cos x} \Big _0^{\frac{\pi}{2}}$ $\therefore \int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx = (-e^{\cos \frac{\pi}{2}}) - (-e^{\cos 0})$ $\therefore \int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx = -e^0 + e^1 = e - 1 \quad (1 \text{ mark})$
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END OF SUGGESTED SOLUTIONS
2003 Mathematical Methods Trial Examination 1

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