THE HEFFERNAN GROUP

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Student Name……

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2003

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of 4 questions.
All questions should be answered.
There is a total of 55 marks available.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.

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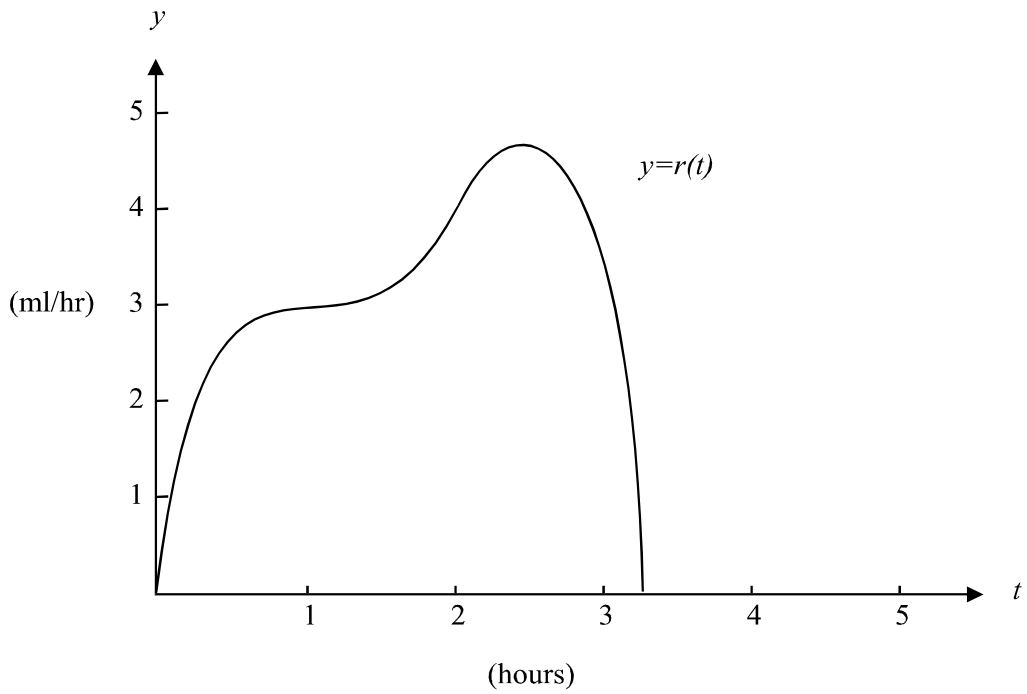
Question 1

Rainfall at a meteorological research station is recorded and analysed. The rate at which rain falls, r, in millimeters per hour, is recorded against time, t, in hours. For a particular episode of rainfall,

$$r(t) = -t^4 + 6t^3 - 12t^2 + 10t, \quad t > 0$$

The graph of $y = r(t)$ is shown on the diagram below.



a. For how long did this episode of rainfall last? (Express your answer correct to the nearest minute.)

2 marks

b. Show that $r'(t) = -4(t-1)^2(t-2\cdot5)$.

3 marks

c. What was the maximum rate at which the rain was falling?

2 marks

d. A point of inflection occurs on the graph at $t = 2$ hours. Explain whether this is a stationary point of inflection.

1 mark

e. **i.** A second point of inflection exists on the graph of $y = r(t)$. Find the t coordinate of this point of inflection and explain why it is a point of inflection.

1 mark

ii. Write down the values of t for which $r'(t)$ is positive.

1 mark

f. For how long during this episode of rainfall, was $r(t)$ greater than 2ml/hour? (Express your answer in hours correct to 1 decimal place.)

2 marks

The amount of rain to fall during this episode of rainfall is represented by the area between the graph of $y = r(t)$ and the t-axis.

g. Find, using calculus, the amount of rain to fall during the first three hours of this episode of rainfall.

2 marks
Total 14 marks

Question 2

A group of five friends including Sue go for a horse ride at a horse stud which stables a large number of riding horses. Each member of the group is randomly assigned a horse. Seventy percent of the horses at the stud are bays (brown coat with black mane and tail).

a. **i.** What is the expected number of people in Sue's group to be randomly assigned a bay coloured horse?

1 mark

ii. Find the probability that 2 people in Sue's group are assigned a bay coloured horse. (Express your answer correct to 4 decimal places.)

2 marks

iii. Find the probability that at least one person in Sue's group would be randomly assigned a bay coloured horse. (Express your answer correct to 4 decimal places.)

2 marks

At the horse stud, the height of the horses is normally distributed with a mean of 150 cm and a standard deviation of 8 cm.

b. **i.** Show that the probability that Sue is assigned a horse which has a height greater then 160 cm, is 0.1056.

1 mark

ii. What is the probability that everybody in Sue's group is each randomly assigned a horse which has a height greater than 160cm. (Express your answer correct to 5 decimal places.)

1 mark

c. What is the probability that exactly two people in Sue's group are assigned a horse which has a height greater than 160cm? (Express your answer correct to 4 decimal places.)

3 marks

d. Sue's group of five friends includes 2 males. Whilst on their ride, the group come to a narrow bridge which they must cross one at a time. They use a random method to decide the order in which they will cross.
Find the probability that the two males in the group are amongst the first three to cross the bridge.

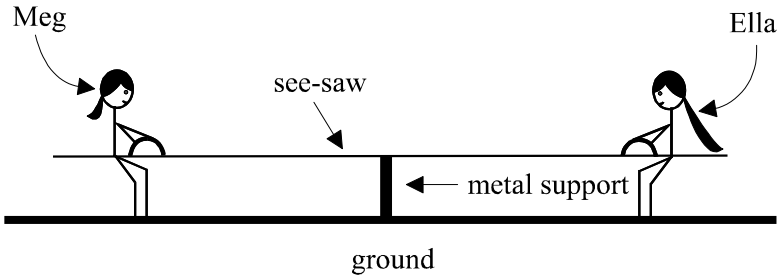
2 marks

e. A large number of Shetland ponies are kept at the horse stud. The height of these Shetland ponies at the horse stud is normally distributed with 19% of the ponies less than 95 cm high and 19% of ponies greater than 107 cm high.
Find the mean and standard deviation of the distribution.

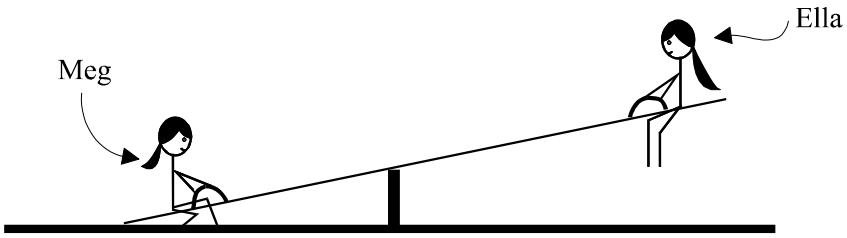
4 marks
Total 16 marks

Question 3

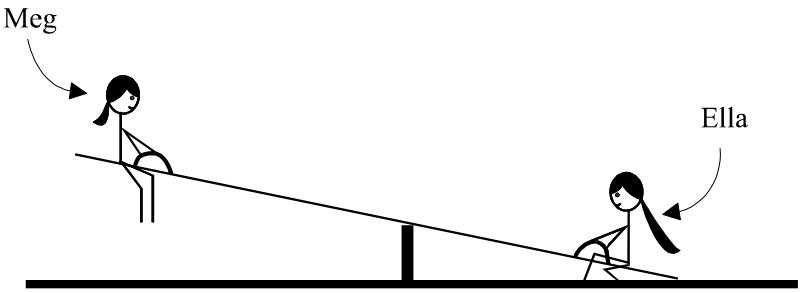
Meg and Ella are having a see-saw. To start with, the girls are seated at either end of the horizontal see-saw.



Then, Ella goes up until Meg's end of the see-saw touches the ground.



Then, Meg goes up until Ella's end of the see-saw touches the ground.



The two girls continue to go up and down in turn on the see-saw. During the see-sawing, the vertical height above the ground, of Ella's end of the see-saw at time *t* seconds, is given by h_E centimetres where

$$h_E = 60\sin\left(\frac{\pi t}{2}\right) + 60, \quad 0 \le t \le 41 \cdot 5$$

a. What is the height of the metal support under the see-saw?

1 mark

b. What is the maximum height that Ella's end of the see-saw reaches above the ground?

1 mark

c. After the two girls start their see-sawing, how long is it before the see-saw is horizontal again?

1 mark

d. How high off the ground is Ella's end of the see-saw when the see-sawing finishes? (Express your answer correct to 2 decimal places.)

1 mark

e. If h_M is the vertical distance above the ground of Meg's end of the see-saw, write down a rule for h_M which involves a sine function.

2 marks

f. During the first 10 seconds of see-sawing, when is Ella's end of the see-saw exactly 90 cm off the ground?

3 marks

Ella feels sick if the see-saw goes too fast. That is, if the rate at which h_E changes, is greater than 1 metre per second.

g. **i.** Using calculus, find an expression for the rate at which h_E changes with respect to time.

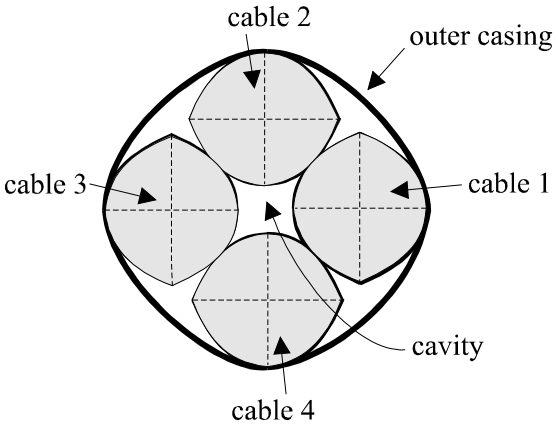
1 mark

ii. Hence explain whether or not Ella will feel sick during the see-sawing.

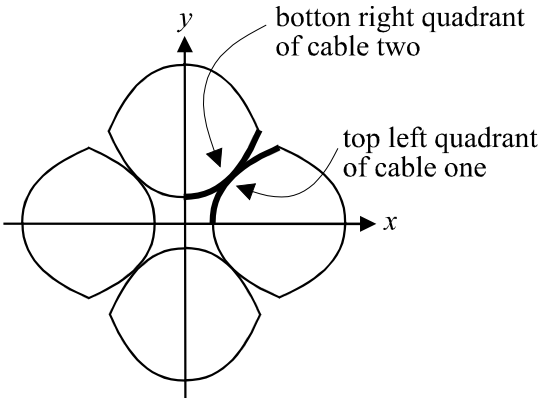
2 marks
Total 12 marks

Question 4

The cross-sectional area of a piece of cabling is shown in the diagram below.



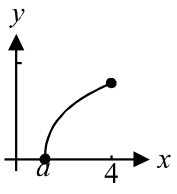
There are four main cables inside the outer casing. Each of these four cables have one point of contact with two other cables. The cross sectional area of each of the four cables shown in the diagrams are symmetrical about the vertical and horizontal dotted lines indicated.
The diagram below shows a cross-sectional view of the cabling with a set of Cartesian axis positioned so that the origin passes through the centre of the cavity between the four cables. The cross-sectional area of the cavity is symmetrical about the x and y axis.



The outer edge of the top left quadrant of the cross-section of cable 1 is given by

$$f:[a,4]\to R,\ f(x)=\log_e(x-1)+2\ .$$

The graph of $y=f(x)$ is shown below.



a. Show that $a = 1 + e^{-2}$.

1 mark

The inverse function f^{-1} models the outer edge of the bottom right quadrant of the cross-section of cable 2.

b. **i** Show that $f^{-1}(x) = 1 + e^{x-2}$.

1 mark

ii. Find the domain of $f^{-1}(x)$.

1 mark

iii. Write down the range of $f^{-1}(x)$.

1 mark

c. Find the equation of the tangent to the function $f(x)$ at the point where $x = 2$.

2 marks

d. Hence find the coordinates of the point of contact between the outer edge of cable 1 and cable 2. Explain your answer.

2 marks

e. Show that $\int_0^2 \left(1 + e^{x-2}\right) dx = 3 - \frac{1}{e^2}$.

2 marks

f. Hence find the area of the cavity (shown in the diagram), between the four cables. Express your answer as an exact value.

3 marks
Total 13 marks