GENERAL COMMENTS

The number of students presenting for Mathematical Methods Examination 2 in 2002 saw a decrease on the 17 468 who sat in 2001. The entire range of marks from 0 to 55 was awarded. The paper provided opportunities for good students to show what they knew; there were excellent papers presented by very capable students who achieved perfect or near-perfect scores. As in past years, a number of students were unable to achieve more than a couple of marks.

The paper tested students' ability to use a graphics calculator well and it was apparent that a range of students was not able to do so. There was much evidence of poor calculator use, leading to answers which were incorrect in the last decimal place. It is important that students are taught to use the calculator accurately and efficiently, and to be aware of limitations of results produced by graphics calculators (for example, asymptotes required in Question 1).

An important decision for students in the examination is when to use the calculator and when not to. Hints are usually provided in the wording of the question. Where an *exact* answer is required, it is probable that the calculator will only be useful to check the answer (and there is evidence that some students are doing this well). On the other hand, answers required to a number of decimal places can usually be found with a calculator, and, indeed, may have to be, as there were questions asked which the student cannot solve by analytical means (for example, Question 4bi).

Students should be sure that their work is clearly legible. It is best if all work is presented in pen, except for graphs which should be single curves drawn in pencil. Care should be taken writing superscripts and subscripts – carelessness often led to errors through students misreading their own work. Carelessness with the use of brackets and fractions was apparent, for example, in Question 1aiii, $\frac{x - 1.1}{-0.5}$ was re-written incorrectly as $-\frac{1.1 - x}{0.5}$ or $\frac{-x - 1.1}{0.5}$.

Many students did not gain marks in questions because they

- did not answer the question asked
- gave decimal answers when an exact answer was required
- gave the wrong number of decimal places
- misread the question in other ways
- did not pay enough attention to detail in sketching graphs
- were not sufficiently careful with algebra.

In Questions 1c, 3bi and 3bii students were required to *show* a given result. Generally, this was not handled well. This is a skill which should be taught and practised.

In these questions it is important that students make it clear that each line follows from the one before it.

SPECIFIC INFORMATION

Question	Marks	%	Response
Question 1	This question was a straightforward one in that it required the sketch graph of a function, the rule and graph for the inverse of this function, then application of logarithm laws. Students tended to rely on their graphics calculator for the graphs, often resulting in a graph with the wrong domain. Setting the window to the given domain may help to prevent this, but a careful reading of the question is the best remedy. Asymptotes were often omitted (they are not drawn by the calculator) or labelled incorrectly (the *y*-axis has equation $x = 0$, not $y = 0$; a label '*y*-axis' was not rewarded as this is not an *equation*). Often, the *x*-axis was also labelled as an asymptote. Many graphs failed to approach asymptotes correctly, either touching the axis or curling away from it. A significant number of students presented a graph which showed an open circle centred on the graph and including the axis. This is not an acceptable means of indicating an asymptote. A number of students gave approximate answers for the end-points of graphs and the domain in 1aiv, although exact answers, such as $1.1 - 0.5\log_e(5)$ were required. It is expected that correct set notation is used in the statement of a domain, such as in 1aiv. Question 1aii, where an explanation was required, was generally well answered. Many algebraic errors were evident in 1b and there were a number of approximate answers presented. Many students attempted to answer 1c by considering a specific value of *x* and demonstrating that the time taken in this case was increased by (usually approximately) √2. This is faulty logic, the equivalent of arguing 'Here is a black dog, therefore all dogs are black'. Some marks were awarded if logarithm laws were used correctly.		
	ai		Graph of *f*
	0/3	21	
	1/3	30	
	2/3	26	
	3/3	24	

	(Average mark 1.53	
	aii 0/1 37 1/1 63 (Average mark 0.63)	Function is 1:1
	aiii 0/2 18 1/2 23 2/2 59 (Average mark 1.41)	$e^{\left(\frac{1.1-x}{0.5}\right)}$
	aiv 0/1 80 1/1 20 (Average mark 0.20)	$[1.1-0.5\log_e(5),\infty)$
	av 0/2 49 1/2 29 2/2 22 (Average mark 0.72)	Graph of f^{-1}
	b 0/2 24 1/2 36 2/2 39 (Average mark 1.14)	$a=0.5, b=\frac{0.2}{\log_e(1.5)}$
	c 0/3 63 1/3 16 2/3 12 3/3 9 (Average mark 0.67)	A sound response to Question 1c was: $t_1 = 1.1 - 0.5\ \log_e(x)$ $t_2 = 1.1 - 0.5\ \log_e(0.5x)$ $t_2 - t_1 = -0.5\ \log_e(0.5x) + 0.5\log_e(x)$ $= 0.5\ \log_e(\frac{x}{0.5x})$ $= 0.5\ \log_e(2) = \log_e(\sqrt{2})$
Question 2	Students generally handled the easier parts of this question well, using a calculator in the appropriate places. There were the usual problems of answers given to the wrong number of decimal places and using a three-decimal place approximation in the next part of the question, giving rise to a small, but important error. One way to avoid this is to store the answer from the earlier part (for example, more than 6 decimal places) in the calculator and recall it when needed.	
	a 0/2 17 1/2 10 2/2 74 (Average mark 1.57)	Question 2a was a standard application of the normal distribution and was handled well; some wrong answers were obtained using '2 standard deviations ≈ 95%'. 0.023
	b 0/4 42 1/4 19 2/4 6 3/4 2 4/4 32 (Average mark 1.61)	Question 2b required students to use transformation to the standard normal distribution and solve resulting equations; some students failed to see that symmetry could be used directly to find the mean and consequently they had to solve simultaneous equations. Many used negative signs incorrectly in an attempt to establish the equations and the incorrect value of 1.46 for Z was frequently given, instead of 1.405, probably a result of reading the table incorrectly. Mean = 23.5; standard deviation = 3.2
	c 0/2 30 1/2 6 2/2 64 (Average mark 1.33)	Question 2c was a standard application of the binomial distribution and was handled well by those that recognised the binomial distribution. 0.088

	di 0/2 1/2 2/2 (Average mark 0.3)	 83 3 14	Question 2di required the use of the principle of total probability. Given the proportion of Jojo butterflies with antennae shorter than 20 mm is 0.1370 and that of Fhaise butterflies is 0.5 (from earlier in the question), the total proportion is $0.1370 \times 0.2 + 0.5 \times 0.8 = 0.427$. Very few students used a tree diagram. 0.427
	dii 0/2 1/2 2/2 (Average mark 0.25)	 83 9 8	While 2dii can be regarded as a conditional probability problem, it is easy to answer using proportions. Of the proportion from part i, $0.5 \times 0.8 = 0.4$ is due to Fhaises. Hence, the probability is $\frac{0.4}{0.1370 \times 0.2 + 0.5 \times 0.8} = 0.936$ 0.936
Question 3	**a** 0/2 1/2 2/2 (Average mark 0.78)	 50 20 29	Many students did not seem to understand the question in part a and found either the roots of the function or of the derivative and many others wrote the derivative but did not equate it to zero. The question uses language which should be familiar. $4x^3 - 1.5x^2 - 5x + 1.5 = 0$
	bi 0/3 1/3 2/3 3/3 (Average mark 1.53)	 40 10 7 43	Question 3bi is a standard question and many students worked through it with no difficulty. A sound response is: $\frac{dy}{dx} = 4x^3 - \frac{3}{2}x^2 - 5x + \frac{3}{2}$, so when $x = 1$, the gradient of the tangent $= 0.5(8 - 3 - 10 + 3) = -1$. Then the gradient of the normal $= -\frac{1}{-1} = 1$. When $x = 1$, $y = -0.5$. The equation to the normal is then $y + 0.5 = x - 1$ so $y = x - 1.5$.
	bii 0/4 1/4 2/4 3/4 4/4 (Average mark 1.46)	 45 6 19 17 13	There were a number of possible approaches to part ii; a number of students used a mixture of methods and were not able to reason effectively, although they could handle the mathematics. Most methods were based on solving the equations to the curve and the normal simultaneously. $x - 1.5 = 0.5(2x^4 - x^3 - 5x^2 + 3x)$ so $x^4 - 0.5x^3 - 2.5x^2 + 0.5x + 1.5$ and $(x - 1)(x - 1.5)(x + 1)^2 = 0$ It can now be argued that the double root at $x = -1$ indicates that the normal is a tangent at this point. Alternatively, the graph indicates that, of the roots of the above equation, the x-co-ordinate of B is -1. When $x = -1$, the gradient of the curve, found from the derivative or a calculator, is 1 and the equation to the tangent is the same as that of the normal. Students who assumed that the normal was a tangent to the curve at another point usually lost a mark.
	ci 0/2 1/2 2/2 (Average mark 0.84)	 47 22 32	In part c, there was evidence that students did not understand what was required. For part ci, many gave a number (presumably from a calculator evaluation of the area) others indefinite integrals, or the sum or difference of definite integrals, where all that was required was $\int_{-1}^{1} (\frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x) - (x - 1.5))dx$

	cii 0/1 74 1/1 26 (Average mark 0.26)	Many students wrote the correct integral and then proceeded to evaluate it as was required in part ii. It should be noted that the answer to part cii was required accurate to two decimal places; a fraction is not in this form. 1.73
Question 4	This question gave students a chance to demonstrate their calculator skills and students who did not set correct windows or used a trace function or a table function to find zeros and intercepts frequently lost marks when their answers were not accurate to the required number of decimal places. Nonetheless, students were often able to pick up some easy marks on this question.	
	ai 0/1 15 1/1 85 (Average mark 0.84)	Question 4a required careful interpretation of a circular function and most students were successful. 21 metres
	aii 0/2 21 1/2 16 2/2 64 (Average mark 1.42)	4.5 seconds, 9 metres
	bi 0/1 57 1/1 43 (Average mark 0.42)	Question 4b required careful use of a calculator but many students faltered. 58.03 seconds
	bii 0/1 40 1/1 60 (Average mark 0.6)	6
	biii 0/2 34 1/2 24 2/2 42 (Average mark 1.07)	55 seconds
	ci 0/2 41 1/2 10 2/2 49 (Average mark 1.07)	Many students successfully differentiated the correct expression in 4ci, using the product and chain rules. $\frac{\pi}{3}e^{0.04t}\cos(\frac{\pi t}{3})+0.04e^{0.04t}\sin(\frac{\pi t}{3})$
	cii 0/3 52 1/3 21 2/3 7 3/3 20 (Average mark 0.95)	While the equation to be solved in Question 4cii could be done analytically, most students chose (wisely) to use their calculators. A number did not answer the question, failing to write down the equation they had evidently solved on the calculator. $\frac{\pi}{3}e^{0.04t}\cos(\frac{\pi t}{3})+0.04e^{0.04t}\sin(\frac{\pi t}{3})=0$ 4.61 m after 58.54 seconds
	d 0/2 80 1/2 7 2/2 13 (Average mark 0.32)	Questions 4d and 4e needed a careful analysis of the graph to determine where the gradient was too large. Effective use of a graphics calculator was required and this presented a challenge to many students. [0, 59.769)
	e 0/3 83 1/3 8 2/3 1 3/3 8 (Average mark 0.35)	a = 0.953