

Mathematical Methods GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2002 examination was 17 728, which was 1.36% more than the 17 487 in 2001. Almost 12% scored 90% or more of the available marks (compared with 14.6% in 2001) and 263 received full marks (compared with 282 in 2001).

The overall quality of responses was similar to that of recent years. There were many very good responses and it was rewarding to see the quite substantial number of students who worked through the paper to obtain full marks. The percentage who scored very few marks and who appeared to attempt little or nothing in Part 2 was slightly less than in recent years. There is little evidence that failure to attempt Part 2 is due to lack of time. It was also noticeable that quite a few students did not answer Questions 1 and 2 on Part 2, the probability questions, yet attempted all the other questions in Part 2.

Students should be familiar with instructions that appear on the examination booklet. Special emphasis should be given to the following:

- a decimal approximation will not be accepted if an exact answer is required to a question
- where an exact answer is required to a question, appropriate working must be shown
- where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Students should be carefully advised with respect to the pre-written notes that may be brought into the examination. These might usefully include material such as the difference between sampling with and without replacement with an example of each; one or more solutions to a typical circular function equation, key points for integral and differential calculus; key reminders for the sketching of curves axial intercepts, turning points and asymptotic behaviour; reminders of how calculator functions will provide intersections between graphs, intersections of graphs with the axes and numerical evaluation of derivatives and integrals.

As noted in previous reports, there were difficulties associated with poor algebraic skills and setting out and use of mathematical notation, especially with respect to brackets. This was particularly noticeable in Questions 4b, 5a and 8b. Students continue to have difficulty in expressing an answer to a specified degree of accuracy.

The use of graphics calculators showed little improvement from 2001. Students need to be aware that an instruction to use calculus requires a derivative or anti-derivative expression to be shown, and an evaluation of an expression on the graphics calculator without showing this expression will not obtain any marks.

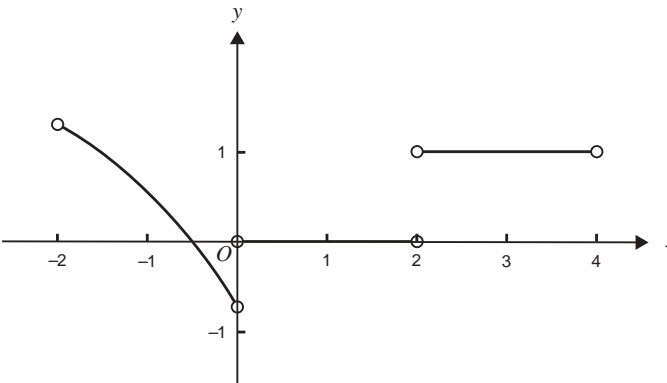
Part 1 – Multiple-choice questions

This table indicates the approximate percentage of students choosing each distractor. The correct answer is the shaded alternative.

Question	A	B	C %	D	E	Question	A	B	C %	D	E
1	1	2	9	2	86	15	12	13	21	48	6
2	11	16	12	14	47	16	4	4	72	17	3
3	3	9	10	74	4	17	61	21	3	9	6
4	7	2	3	15	73	18	10	59	12	11	8
5	4	7	84	4	1	19	58	15	16	8	3
6	56	1	6	19	18	20	9	80	5	4	3
7	82	7	5	4	2	21	7	57	17	8	11
8	10	16	64	3	7	22	3	32	3	50	12
9	26	56	7	6	5	23	9	5	7	75	4
10	61	14	13	9	3	24	58	29	3	5	5
11	2	3	10	80	5	25	39	24	13	15	9
12	13	5	29	50	3	26	16	49	20	9	6
13	10	15	57	6	12	27	14	19	10	19	38
14	3	64	5	21	7						

Part 2 – Short-answer questions

Question	Marks	%	Response
Question 1	a		Correct response: Number of heads Probability
	0/2	29	0 $0.125 = \frac{1}{8}$
	1/2	9	1 $0.375 = \frac{3}{8}$
	2/2	62	2 $0.375 = \frac{3}{8}$
	(Average mark 1.32)		3 $0.125 = \frac{1}{8}$
			Many students did not realise that for a probability distribution of a discrete random variable, the probabilities must sum to one, and that individual probabilities must always be less than or equal to one. This question was not well done given that it could have been completed from first principles without knowledge of the binomial distribution.
	b		Correct response:
	0/1	30	Expected number of heads = $0 \times 0.125 + 1 \times 0.375 + 2 \times 0.375 + 3 \times 0.125$ or $np = 1.5$
	1/1	70	Many students realised that $\mu = np$ or used $\mu = \sum xPr(X=x)$ to obtain an answer. Quite a few responses gave the expected number of heads as greater than the number of trials.
(Average mark 0.70)			
Question 2	0/2	23	Correct response: $X =$ number of defectives in sample of 5 Hypergeometric distribution with parameters $N = 100$, $n = 5$ and $D = 5$ Batch is accepted if $X = 0$
1/2	31	$Pr(X = 0) = \frac{{}^{95}C_5}{{}^{100}C_5} = 0.770$	
2/2	46	Most students realised that the hypergeometric distribution was to be applied, but had difficulty identifying the parameters. Some students also chose to find $Pr(X = 1)$ or $1 - Pr(X = 0)$. Rounding off correctly to three decimal places created a problem for the many students who did not know that zero could be a final digit for a specific accuracy.	
(Average mark 1.22)			
Question 3	a		Correct response: Period = 8
	0/1	27	The response was disappointing as many students thought that $\frac{\pi}{4}$ was
1/1	73	the answer, having obtained this from 'period = $\frac{2\pi}{n}$ '.	
(Average mark 0.73)			
	3b		Correct response: Amplitude = 1
	0/1	25	For a very straightforward question, the response was disappointing. The most common incorrect responses were -2 or -1 . The appropriate knowledge could have been incorporated into the student's summary notes.
1/1	75		
(Average mark 0.75)			
Question 4	a		Correct response: $x = -4.984$
	0/1	60	Many students lacked the persistence to work through this question, leaving their response in exact form as a logarithmic expression. Often the negative sign was lost. As seen often in past years many students do not have the required algebra skills to simplify expressions. Those who obviously used a graphics calculator used it efficiently and well.
1/1	40		
(Average mark 0.40)			
	b		Correct response:
	0/1	50	$\log_e \left(\frac{(3x+1)^2}{x} \right)$
1/1	50	Some well-presented solutions were given. However, a significant number of students obtained the correct answer then attempted to simplify	
(Average mark 0.50)			

		and came unstuck with the algebra. The x on the denominator was cancelled into just one of the two or three terms on the numerator; subtraction and division were confused. The answer was often seen as the quotient of two logarithm expressions, and some students gave the answer in exponential form. Quite a few students also put the expression equal to zero and then attempted to solve for x .
Question 5	a 0/1 76 1/1 24 (Average mark 0.24)	Correct response: $y = -\frac{1}{2(x-3)} + 1$ This question proved too difficult for most students. Commonly, 1 and 3 were interchanged, and the negative sign was frequently missing.
	b 0/2 34 1/2 20 2/2 46 (Average mark 1.11)	Correct response: Domain $\mathbb{R} \setminus \{3\}$ Range $\mathbb{R} \setminus \{1\}$ Students were required to use their answer to part a. to obtain the marks for this question and overall it was handled satisfactorily. Some students gave only one answer, which was assumed to be the domain. Notation used in responses was poor. Students should be familiar with correct use of mathematical notation for expressing sets such as domain and range.
Question 6	a 0/1 67 1/1 33 (Average mark 0.33)	Correct response: 0.084, 0.963 A straightforward question where it was expected that students would use the graphics calculators to obtain their answers. Incorrect rounding of answers was a common problem in all three parts of the question. Some students gave answers outside the specified domain, or only gave one answer.
	b 0/1 50 1/1 50 (Average mark 0.50)	Correct response: -5.940 The most common source of error was students working with their calculator in degree mode rather than radian mode.
	c 0/1 62 1/1 38 (Average mark 0.38)	Correct response: (0, 0.523) The most common incorrect response was for a single value to be given rather than an interval. Of those students who attempted the question, many had the correct response.
Question 7	a 0/2 58 1/2 21 2/2 21 (Average mark 0.62)	Correct response:  Quite a number of students did not attempt this question. Those who did, showed a lack of understanding of the concept involved. Graphs of f were obtained by reflecting in the axes, the line with equation $y = x$ and, in some cases, rotations. Some graphs were graphs of relations, not functions. Too few realised that, in the interval $(0, 2)$, the gradient was zero and that it was 1 in the interval $(2, 4)$. Many students were careless in indicating the inclusion or exclusion of endpoints.

	<p>b</p> <p>0/1 54 1/1 46 (Average mark 0.46)</p>	<p>Correct response: Domain: $(-2, 4) \setminus \{0, 2\}$ or $(-2, 0) \cup (0, 2) \cup (2, 4)$</p> <p>Quite a few students were able to find the correct domain of their graph. However, quite a few simply stated the same domain for the original function and its derivative function. Again incorrect notation was seen here, in particular the use of \cap instead of \cup.</p>
<p>Question 8</p>	<p>a</p> <p>0/2 17 1/2 8 2/2 75 (Average mark 1.58)</p>	<p>Correct response: For intersection $2x^2 + 4x - 5 = 3x + 1$ gives $2x^2 + x - 6 = 0$ and $(2x - 3)(x + 2) = 0$</p> <p>So, line and parabola intersect when $x = -2, \frac{3}{2}$</p> <p>Students were able to answer this question algebraically or by using the graphics calculator. For quite a few students this was the only mark they obtained on Part 2 of the paper. Some students also thought they were required to give the answer as co-ordinates. In a few cases, students attempted to solve each separate equation for x.</p>
	<p>b</p> <p>0/3 40 1/3 21 2/3 17 3/3 23 (Average mark 1.22)</p>	<p>Correct response:</p> $\text{Area} = \int_{-2}^{\frac{3}{2}} (3x + 1 - (2x^2 + 4x - 5)) dx$ $= \int_{-2}^{\frac{3}{2}} (-2x^2 - x + 6) dx$ $= \left[-\frac{2x^3}{3} - \frac{x^2}{2} + 6x \right]_{-2}^{\frac{3}{2}}$ $= \frac{343}{24}$ $= 14.292 \text{ square units correct to 3 decimal places.}$ <p>The setting up of the definite integral was not well done. Many students could not cope with the fact that part of the area to be calculated was below the x-axis. For some it involved dividing the region into three or four sections and trying to evaluate the area of each. This was made more difficult due to the fact that the x-intercepts for the parabola were irrational and by confusion about signed areas. Some students ignored the instruction to use calculus, which was demonstrated by stating an anti-derivative, and simply used the graphics calculator to obtain the answer without showing either an integral or anti-derivative. Marks were not awarded in these cases.</p> <p>Poor algebra and notation were often presented; such as misuse or non-use of brackets, disappearing and re-appearing negative signs, lack of 'dx' and transcription errors. Some students anti-differentiated by substituting into the original expression, differentiating some terms and anti-differentiating others or differentiating all terms. A number of students with the correct anti-derivative were unable to correctly substitute to obtain the correct answer.</p> <p>More successful students were able to show the integral, then the anti-derivative and then use the graphics calculator to obtain the final answer, correct to 3 decimal places.</p>