Mathematical Methods (CAS) GA 2: Written examination 1

GENERAL COMMENTS

Seventy-eight students sat for this pilot examination in 2002. Marks ranged from 12 to 49 out of a maximum of 50. Student responses showed that the paper was accessible and provided opportunities for students to demonstrate what they knew. There were excellent papers presented by several students with 12.8% of students scoring 90% or more of the available marks. The mean mark for the paper was 31.5, comprised of a mean of 18.8 marks on the multiple-choice part (27 in total) and a mean of 12.7 marks on the short-answer part (23 in total). Seventy-two per cent of the students scored over half of the available marks for the paper.

Overall, the symbolic facility of the CAS calculators was used well, especially for the multiple-choice questions, especially Questions 14, 19 and 20. It was disappointing to see that only 54 students managed the correct response to Question 9. Students need to know the difference between factoring over the rational number field and the real number field. There were many very good responses and some students were able to work through most questions completely to obtain full marks. There was little evidence to suggest that not making a reasonable attempt on Part 2 was due to lack of available time.

Students should be familiar with instructions such as:

- a decimal approximation will not be accepted if an exact answer is required to a question
- appropriate working should be shown if more than 1 mark is available.

It was pleasing to see that where working was shown, it was generally shown using correct mathematical notation. Some students still have difficulty in expressing an answer to a specified accuracy.

SPECIFIC INFORMATION

Part 1 – Multiple-choice questions

Content that needs further attention is the application of integration to calculate the area under a curve and simple cases of areas between curves – Question 22 was not very well done. Similarly students should be familiar with the transformation between a given normal distribution and the standard normal distribution.

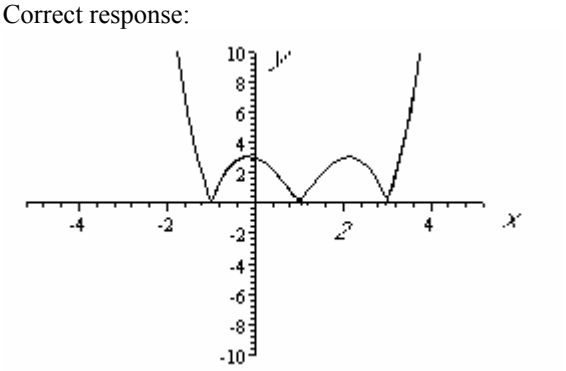
This table indicates the approximate percentage of students choosing each distractor. The correct answer is the shaded alternative.

Question	A	B	C	D	E
	%				
1	1	3	8	0	**88**
2	9	0	13	13	**65**
3	5	8	4	**79**	4
4	1	1	19	11	**68**
5	1	4	**92**	3	0
6	**67**	0	0	13	20
7	**86**	8	4	1	1
8	0	1	**94**	4	1
9	17	**70**	8	5	0
10	12	5	**65**	9	9
11	1	0	6	**90**	3
12	12	9	15	**64**	0
13	8	14	**67**	6	5
14	1	**94**	1	4	0

Question	A	B	C	D	E
	%				
15	9	4	20	**59**	8
16	1	0	**82**	14	3
17	**67**	13	3	11	6
18	9	**62**	9	10	10
19	**86**	1	3	9	1
20	1	**90**	8	1	0
21	13	8	12	**59**	8
22	1	44	2	**40**	13
23	10	**62**	9	12	7
24	**52**	38	0	4	6
25	**40**	21	10	16	13
26	19	**42**	22	9	8
27	17	11	3	19	**50**

Part 2 – Short answer questions

Question	Marks	%	Response
Question 1	**a**		Correct response: $c = 100$.
	0/2	38	This was not done as well as expected. A method mark was awarded for
	1/2	15	working such as $\int_{100}^{\infty} \frac{c}{x^2} dx = 1$ since f is a probability density function. Very
	2/2	46	little other working was shown. Most students used the CAS facility of the
	(Average mark 1.08)		calculator to determine c. Some students showed a suitable formulation but no subsequent manipulation or evaluation.

	b 0/2 37 1/2 26 2/2 37 (Average mark 1)	Correct response: $M = 200$ Similarly this was not done as well as expected. A method mark was awarded for working such as solve $\int_{100}^{M} \frac{100}{x^2} dx = 0.5$ for M. Some students correctly wrote the method of working, but then seemed to make no effort to carry out the corresponding process. Some students tried to solve $\int_{0}^{M} \frac{100}{x^2} dx = 0.5$ for M in part **b.**, and ran into difficulty because of the lower terminal, making this integral improper (and divergent). Students need to take notice of where the density function is zero for this sort of question and related manipulation and calculation.
Question 2	**a** 0/1 34 1/1 66 (Average mark 0.65)	Correct response: 0.486 A fairly straightforward probability calculation – it was surprising that more students did not get this correct.
	b 0/1 68 1/1 32 (Average mark 0.32)	Correct response: 0.034 This was not well done. Some students included the binomial coefficient, clearly not understanding the difference between obtaining exactly one success in five tries and obtaining the first success on the fifth try. Some were not able to relate the question to first principle probability concepts.
	c 0/2 87 1/2 4 2/2 9 (Average mark 0.21)	Correct response: 0.930 This was not done well. A method mark was awarded if the student had written either of $\frac{18}{37} + \frac{19}{37} \times \frac{18}{37} + \left(\frac{19}{37}\right)^2 \frac{18}{37} + \left(\frac{19}{37}\right)^3 \frac{18}{37}$ or $1 - \left(\frac{19}{37}\right)^4$
Question 3	0/2 12 1/2 21 2/2 68 (Average mark 1.56)	Correct response:  This was generally well done. The main errors were smoothing out the cusps (which indicate non-differentiability of the function at these points) and one or other of the tails having the incorrect direction of curvature (concavity).
Question 4	**a** 0/2 13 1/2 17 2/2 70 (Average mark 1.56)	Correct response: $f(-1) = 1 \quad -a + b - c + d = 1$ $f(1) = \frac{5}{3} \quad a + b + c + d = \frac{5}{3}$ $f'(1) = -2 \quad 3a + 2b + c = -2$ $f'(0) = 0 \quad c = 0$ This was quite well done. Errors occurred when students mixed up the x and y values, or when they tried to rearrange the equations they obtained.

	b 0/2 21 1/2 17 2/2 62 (Average mark 1.41)	Correct response: $\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{5}{3} \\ -2 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{5}{3} \\ -2 \end{bmatrix}, c = 0$ or $\begin{bmatrix} -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & \frac{5}{3} \\ 3 & 2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & \frac{5}{3} \\ 3 & 2 & 0 & -2 \end{bmatrix}, c = 0$ This was generally well done. A few students gave the transpose of the required matrix, and a few gave a response similar to the second one listed above but with an extra column in the first matrix. The equation in part a. above giving $c = 0$ caused a few problems. Some students were not able to write the system of equations in a suitable matrix form.
	c 0/2 29 1/2 27 2/2 44 (Average mark 1.14)	Correct response: $a = \frac{1}{3}, b = -\frac{3}{2}, c = 0, d = \frac{17}{6}$ $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{17}{6}$ Students did not make the best use of CAS functions in this question. Some did not give the coefficients in exact form. They should have been aware whether their CAS needed to be in exact or approximate mode. Some students did not write down the rule for *f*. No student appeared to attempt this by hand.
Question 5	**a** 0/1 68 1/1 32 (Average mark 0.31)	Correct response: $y = -\frac{1}{2(x-3)} + 1$ This was not done well. Most students simply wrote down an answer without any interim working shown. Common errors were: dilating by a factor of 2 from the *y*-axis, translation of 3 units parallel to the *y*-axis, translation of 1 unit parallel to the *x*-axis, an incorrect translation of 3 units parallel to the *x*-axis, and an incorrect reflection in *y*-axis.
	b 0/2 19 1/2 28 2/2 54 (Average mark 1.34)	Correct response: Domain R\{3} or $(-\infty, 3) \cup (3, \infty)$ or $\{x \in \mathrm{R} : x \neq 3\}$ Range R\{1} or $(-\infty, 1) \cup (1, \infty)$ or $\{x \in R : x \neq 1\}$ This was quite well done but there was quite a wide variation in the use of notation. Some incorrect responses for the domain included: R{3}, $(-\infty, 3) \cap (3, \infty)$. Students should be familiar with the relevant mathematical conventions and notations for expressing domains and ranges or intervals over which certain conditions apply, for example where a function is increasing.
Question 6	**a** 0/2 26 1/2 14 2/2 59 (Average mark 1.32)	Correct response: $\frac{\pi}{18}, \frac{5\pi}{18}$ This was quite well done. Some students missed the second solution and some gave solutions outside the domain. Others gave a general solution, such as $\frac{(12n+5)\pi}{9}$, without taking the domain restrictions into consideration. Students should be able to identify values of the parameter which leads to solutions in a given interval.

	b 0/1 43 1/1 57 (Average mark 0.56)	Correct response: −5.940 Many students did not give their answer to the required accuracy, particularly by leaving off the final 0.
	c 0/1 72 1/1 28 (Average mark 0.28)	Correct response: $\left(0, \frac{\pi}{6}\right)$ or $\left[0, \frac{\pi}{6}\right)$ Not well done. A few students included $\frac{\pi}{6}$ in the interval.