

Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 2: Analysis Task

Reading time 15 minutes
Writing time 1 hour 30 minutes

Student's	Name:	Sent to the second seco
Teacher's	Name:	

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
4	4	55

Materials

Question and answer booklet of 10 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

The task

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your name and your teacher's name in the space provided on the front of this booklet.

All written responses should be in English.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2002 VCE Mathematical Methods Units 3 & 4 Examination 2.



Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 2: Analysis Task

Formula Sheet

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium:

 $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder:

 $\pi r^2 h$

area of a triangle: $\frac{1}{5}bc\sin A$

 $x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$

 $\int \frac{1}{x} dx = \log_e x + c \text{, for } x > 0$ $\int \sin ax dx = -\frac{1}{a} \cos ax + c$

 $\cos ax dx = \frac{1}{a}\sin ax + c$

 $\int e^{ax} dx = \int_{a}^{1} e^{ax} + c$

volume of a cone:

$$\frac{1}{3}\pi r^2 h$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}\log_e x = \frac{1}{x}$$

$$\frac{d}{dx}(\sin ax) = a\cos ax$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax$$

$$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

 $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{2}$

chain rule:

 $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation:

 $f(x+h) \approx f(x) + hf'(x)$

Statistics and probability

$$Pr(A) = 1 - Pr(A')$$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean:

$$\mu = E(X)$$

variance:

$$var(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$$

	Pr(X=x)	mean	variance
general	p(x)	$\mu = \sum x p(x)$	$\sigma^2 = \Sigma (x - \mu)^2 \rho(x)$
	(***)	**************************************	$= \sum x^2 p(x) - \mu^2$
binomial	$^{n}C_{x}p^{x}(1-p)^{n-x}$	np	np(1-p)
hypergeometric	$\frac{{\stackrel{D}{\cdot}}C_x^{N-D}C_{n-x}}{{}^{N}C_n}$	$nrac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$

Continuous distributions

normal

If X is distributed N(μ , σ^2) and $Z = \frac{X - \mu}{\sigma}$, then Z is distributed N(0, 1).

Table 1: Normal Distribution – cdf

X	0	1	2	3	4	5	5	7	8	9	1	2	3	4	5	6	7	3	9
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Q.J	£179	.6217	.6255	.6293	.6331	.6368	.5406	.6443	.6480	.6517	1 4	8	di di	5	19	23	26	30	34
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1,1	.8543	.8665	.0586	8708	.8729	.8749	.8770	.8790	.8810	.8530	2	4	8	83	10	12	14	16	19
1,2	.8849	,0869	.8888	.8907	.8925	.8944	.8962	.8980	.0917	9015	1 2	4	6	7	9	1	13	15	16
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END OF FORMULA SHEET

Instructions

Answer all questions.

A decimal approximation will not be accepted if an exact answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown.

The instruction use calculus requires students to show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

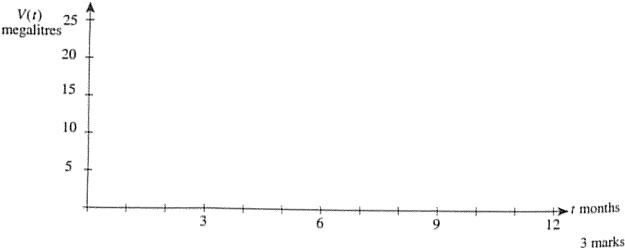
Question 1

The volume of water, V(t) megalitres, in a small reservoir is known to be modelled by the equation

$$V(t) = 10\sin(\frac{t}{2} + 3) + 15 \text{ for } 0 \le t \le 12$$

where t is the number of months in a certain year.

a. On the axes below, sketch a graph of V(t) versus t for the entire year. Clearly show important features including endpoints and turning points to one decimal place.



b. Find the function for the rate of change of volume with respect to time.

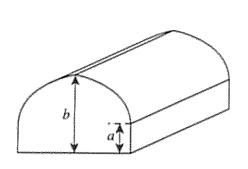
1 mark

c. Calculate the exact values of t when the volume is at its minimum and maximum values during the year.

4 marks

d.	Correct to the nearest 0.1 month, for how long during the year is the volume of water in the reservoir less than 7.5 megalitres?
	2 marks
	Total 10 marks

The design of a new stadium roof is being considered by an architect. A diagram of the proposed stadium (Figure 1) and a cross-sectional view of the stadium on Cartesian axes (Figure 2) are shown.



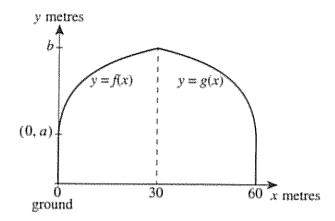


Figure 1

Figure 2

The equation of the left side of the curved roof (y = f(x)) is given by $f: [0, 30] \to \mathbb{R}$, where $f(x) = 5\log_e(x+1) + 10$.

where $f(x) = 5\log_e(x+1) + 10$. a. Calculate the value of a (the height of the side wall).

l mark

b. Calculate (to the nearest centimetre) the maximum height of the roof above the ground (b).

1 marks

The right side of the roof is to be symmetrical to the left side about the line x = 30. Its equation, y = g(x), can be deduced by reflecting the graph of y = f(x) in the y axis and translating it 60 units to the right.

c. Deduce the equation for g(x).

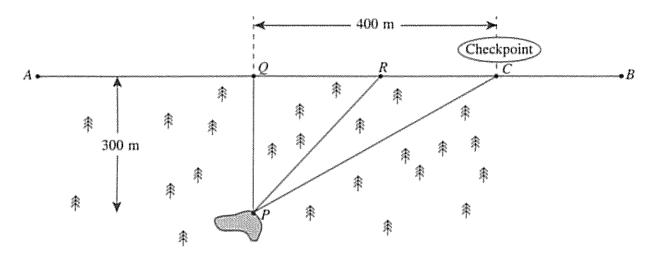
2 marks

d. State the domain and range of g(x).

2 marks

uc a	Calci	ct needs to know the slope of the roof at various points so that roofing materials α ulate an expression in terms of x for the slope of the roof at any x value	•
	i.	when $0 < x < 30$.	
	ii.	when $30 < x < 60$.	1 mark
			oogleggegeer gever seer sprease op en aan ee ook aan make valle van de sprease steer de sprease van de sprease
			l murk
	Calc	culate the exact slope of the roof when	
	i.	x == 15.	
			l mark
	ii.	x = 45.	
The g.	Wr	tect considered bricking in one entire end wall enclosed under the roof span. ite an expression in terms of $f(x)$ for the exact area $(A \text{ m}^2)$ of one end of the stad	1 mark um enclosed
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g.	Usi	tect considered bricking in one entire end wall enclosed under the roof span. ite an expression in terms of $f(x)$ for the exact area $(A \text{ m}^2)$ of one end of the stadder the entire cross sectional span of the roof.	um enclosed 2 marks

Paolo, an experienced orienteer, is standing by a pond P in a heavily wooded forest. He has 5 minutes to report to checkpoint C, along a straight track AB, otherwise he will be disqualified – and he must walk, not run.



Q is a point on the road directly opposite his position. The distances from P to Q and from Q to the checkpoint are as shown in the diagram. Paolo averages 12 km/h when walking along the track and 6 km/h through the forest.

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*				to the nearest minute, will Paolo take to walk from the pond to the

Paolo wishes to get to the checkpoint in the quickest time possible. He believes he can do this by walking through the forest to a point R and then along the track to the checkpoint.

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ii.	Sketch the graph of the time function, clearly showing the endpoints, and hence verify to 4 decimal places.	
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*******		2 m
Will	Paolo be disqualified? Give reasons for your answer.	

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Isaias wants to enter his rally car into the gruelling Arnhem desert off-road race. As this race requires precision driving and fierce concentration, the organisers put the applicants through a preliminary driving test. The probability that an applicant passes this test on their first attempt is 0.6.

a.	4 3 3 8	What is the probability that 6 out of 10 applicants pass the test on their first attempt?
		1 mark
	ii.	What is the probability that at least 6 out of 10 applicants pass the test on their first attempt?
	iii.	1 mark What is the probability that at most 6 out of 10 applicants pass the test on their first attempt?
b.		1 mark oup of 250 candidates attempt the preliminary driving test. Calculate the mean and standard ation of the number of candidates who pass on their first attempt.
		2 marks
	oping	es the preliminary driving test and is invited to participate in the rally. The probability of engine trouble for every day's full driving is 1% and is independent for each day. It is the exact probability that a rally car will not have engine trouble on any particular day?
		1 mark

	Express your answer correct to 4 decimal places.
	l ma
•	What is the probability that a car will have developed engine trouble during the first <i>D</i> days?
	l ma
Ť	
vег га	discovered that engine trouble in the particular make of car that Isaias is driving is caused by the deating of an electronic sensor. The manufacturer of the sensor has found that the temperature at which domly chosen sensor fails is normally distributed with a mean of 94.5°C and a standard deviation is
ver rui .7%	leating of an electronic sensor. The manufacturer of the sensor has found that the temperature at which domly chosen sensor fails is normally distributed with a mean of 94.5°C and a standard deviation
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ver rui .7º	leating of an electronic sensor. The manufacturer of the sensor has found that the temperature at which domly chosen sensor fails is normally distributed with a mean of 94.5°C and a standard deviation of the sensors will operate at 100°C? Express your answer correct to the nearest

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END OF QUESTION AND ANSWER BOOKLET



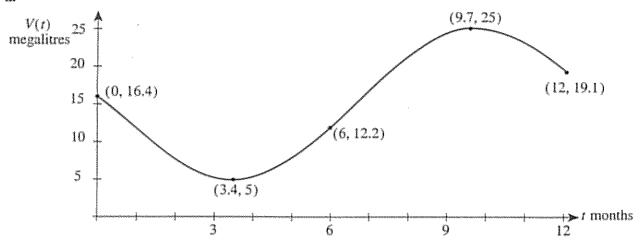
Trial Examination 2002

VCE Mathematical Methods Units 3 & 4

Examination 2: Analysis Task

Suggested Solutions

Ω.



Basic shape and location [A] Endpoints, coordinates and labelling [A][A]

b. Rate of change = V'(t)

$$=5\cos\left(\frac{t}{2}+3\right), \ 0 \le t \le 12$$
 [A]

c. Minimum and maximum occur when $5\cos\left(\frac{t}{2}+3\right) = 0$.

$$\therefore \cos\left(\frac{t}{2} + 3\right) = 0 \tag{M}$$

$$\therefore \frac{t}{2} + 3 = \cos^{-1}(0)$$

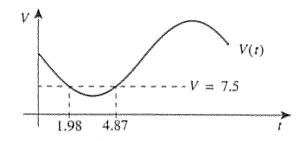
$$= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$
[M]

$$\therefore \frac{t}{2} = \frac{\pi}{2} - 3, \frac{3\pi}{2} - 3, \frac{5\pi}{2} - 3$$

(Note that $\frac{\pi}{2} - 3 < 0$.)

$$\therefore t = 3\pi - 6 \text{ (min)}, 5\pi - 6 \text{ (max)}.$$
 [A][A]

d.



When V = 7.5, t = 1.98 and 4.87.

The volume is less than 7.5 during the year [M]

for
$$4.87 - 1.98 = 2.9$$
 months. [A]

a. When
$$x = 0$$
, $f(0) = 5\log_a 1 + 10$.

As
$$\log_e 1 = 0$$
, $f(0) = 10$.

$$\therefore a = 10$$
 and so the height of the wall is 10 m. [A]

b.
$$b = f(30)$$

$$= 5\log_{4}(30+1) + 10$$

$$=5\log_e 31+10$$

The maximum height above the ground is 27.17 m.

c. Reflection in y axis: g(x) = f(-x)

Translation 60 units to the right: g(x) = f(x - 60)

Combining these gives:

Reflected curve in y axis:
$$g(x) = 5\log_e(-x+1) + 10$$
 [M]

Translated curve (after reflection): $g(x) = 5\log_{e}(-(x-60)+1)+10$

$$\therefore g(x) = 5\log_e(-x+61)+10$$
 [A]

d. The domain of
$$g(x)$$
 is [30, 60] or $30 \le x \le 60$. [A]

The range of g(x) is [10, 27.17] or [10, $5 \ln 31 + 10$]. [A]

e.
$$\mathbf{i}$$
 $f(x) = \frac{5}{x+1}$, $0 < x < 30$. [A]

ii.
$$g'(x) = 5\left(\frac{-1}{-x+61}\right)$$
 or $\frac{-5}{-x+61}$, $30 < x < 60$. [A]

c. i
$$f(15) = \frac{5}{16}$$
 [A]

ii.
$$g'(45) = \frac{-5}{-45+61}$$

$$=\frac{5}{16}$$

g.
$$A = 2 \int_0^{30} f(x) dx$$
 Terminals [A]
 $2 \times \text{integral with } dx$ [A]

~30

h.
$$A = 2 \int_0^{30} (5\log_e(x+1) + 10) dx$$

$$= 10 \int_0^{30} (\log_e(x+1) + 2) dx$$
 [M]

$$= 10 \left[(x+1)\log_{c}(x+1) - x + 2x \right]_{0}^{30}$$
 [A]

 $= 10(31\log_2 31 + 30)$

$$= 310\log_{e} 31 + 300$$
 [A]

i.
$$1364.5 \text{ m}^2$$

a. PQC is a 3-4-5 right angled triangle

$$\therefore PC = \sqrt{(300)^2 + (400)^2}$$

= 500 m.

= 500 m.

$$T_{PC} = \frac{0.5}{6}$$
= 0.083 hr
= 5 min.



and Jimn.

b.
$$T_{\text{total}} = T_{\text{forest}} + T_{\text{track}}$$

 $= \frac{0.3}{6} + \frac{0.4}{12}$
 $= 0.05 + 0.033$
 $= 0.083$
 $= 5 \text{ min}$

[A]

[A]

[M]

c.
$$T_{PRC} = T_{PR} + T_{RC}$$
$$d_{PR} = \sqrt{300^2 + x^2}$$
$$d_{RC} = 400 - x$$

Hence
$$T_{PRC} = \frac{\sqrt{300^2 + x^2}}{6000} + \frac{400 - x}{12000}$$
 [A]
= $\frac{2\sqrt{300^2 + x^2} + 400 - x}{12000}$ hr

and so
$$T_{PRC} = \frac{2\sqrt{300^2 + x^2} + 400 - x}{12000} \times 60 \text{ min}$$

$$= \frac{2\sqrt{90000 + x^2} + 400 - x}{200} \text{ min}$$
[A]

d. Minimum time is where $\frac{dT}{dx} = 0$. [M]

$$T = \frac{2(90000 + x^2)^{1/2}}{200} + \frac{400 - x}{200}$$
$$= \frac{1}{100}(90000 + x^2)^{1/2} + 2 - \frac{1}{200}x.$$

$$\frac{dT}{dx} = \frac{1}{200} (90000 + x^2)^{-1/2} \times 2x - \frac{1}{200}$$
$$= \frac{1}{200} \left(\frac{2x}{\sqrt{90000 + x^2}} - 1 \right).$$

[A]

$$\frac{dT}{dx} = 0$$
 when $\frac{2x}{\sqrt{90000 + x^2}} - 1 = 0$.

$$\therefore \frac{2x}{\sqrt{90000 + x^2}} = 1$$

$$\therefore 2x = \sqrt{90000 + x^2}$$

$$\therefore 4x^2 = 90000 + x^2$$

$$3x^2 = 90000$$

$$xx^2 = 30000$$

$$\therefore x = 173.2051 \text{ m.}$$
 [A]

$$T_{\min} = \frac{2\sqrt{90000 + 30000} + 400 - 173.2051}{200}$$

$$= 4.598$$

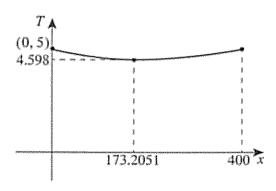
$$= 4.6 \min \text{ (to 1 d.p.)}.$$
[A]

e. i
$$T'(170) = \frac{1}{200} \left[\frac{2(170)}{\sqrt{90000 + 170^2}} - 1 \right] = -0.699 \times 10^{-5}$$
 [A]

$$T'(180) = \frac{1}{200} \left[\frac{2(180)}{\sqrt{90000 + 180^2}} - 1 \right] = 0.0000145$$
 [A]

Since the gradient changes from negative to positive, the time found in d. is a minimum. [A]

ii. A graphic calculator is useful here.



General shape and minimum [A]

Endpoints [A]

f. All possible routes will give a time within [4,598, 5] and so Paolo will not be disqualified. [A]

a. I Let the random variable X be the number of applicants who pass on their first attempt. X has a binomial distribution with n = 10 and p = 0.6.

$$Pr(X = 6) = {10 \choose 6} (0.6)^6 (0.4)^4$$
$$= 0.2508.$$
 [A]

Alternatively, using the graphic calculator, binompdf(10, 0.6, 6) = 0.2508.

ii.
$$Pr(X \ge 6) = Pr(X = 6) + Pr(X = 7) + Pr(X = 8) + Pr(X = 9) + Pr(X = 10)$$

= 0.2508 + 0.21499 + 0.12093 + 0.04031 + 0.0060466
= 0.6331. [A]

Alternatively, using the graphic calculator, 1 - binomedf(10, 0.6, 5) = 0.6331.

iii.
$$Pr(X \le 6) = 1 - Pr(X > 6)$$

= $1 - (0.21499 + 0.12093 + 0.04031 + 0.0060466)$
= $1 - 0.382277$
= 0.6177 . [A]

Alternatively, using the graphic calculator, binomcdf(10, 0.6, 6) = 0.6177.

b.
$$E(X) = np$$

= 250×0.6
= 150

$$SD(X) = \sqrt{np(1-p)}$$

= $\sqrt{150(0.4)}$
= 7.746

c.
$$1 - 0.01 = 0.99$$
 [A]

d.
$$(0.99)^1(0.99)^1 = (0.99)^2 = 0.9801.$$
 [A]

e.
$$1 - (0.99)^d$$
. [A]

f. Let T = overheating temperature.

This is a normal distribution with $\mu = 94.5$ and $\sigma = 5.7$.

So,
$$Pr(T \le 100) = Pr\left(Z \le \frac{100 - 94.5}{5.7}\right)$$
 [A]

$$= Pr(Z \le 0.965).$$

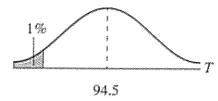
Using graphic calculator or tables, $Pr(T \le 100) = 0.8328$. [A]

The proportion of sensors that will operate (not overheat) at 100°C is

$$1 - 0.8328 = 0.1672$$

= 17% .

g. We require the temperature at which 1% fail.



So,
$$Pr(T < t) = 0.01$$
.

Using the graphic calculator or reverse tables to look up 0.99, z = -2.326

[A]

[A]

and so
$$\frac{t - 94.5}{5.7} = -2.326$$

$$\therefore t = (-2.326 \times 5.7) + 94.5$$
$$= 81.24^{\circ}\text{C}.$$

To the nearest ${}^{\circ}C$, $t = 81 {}^{\circ}C$.