Neap.
advancing education

Trial Examination 2002

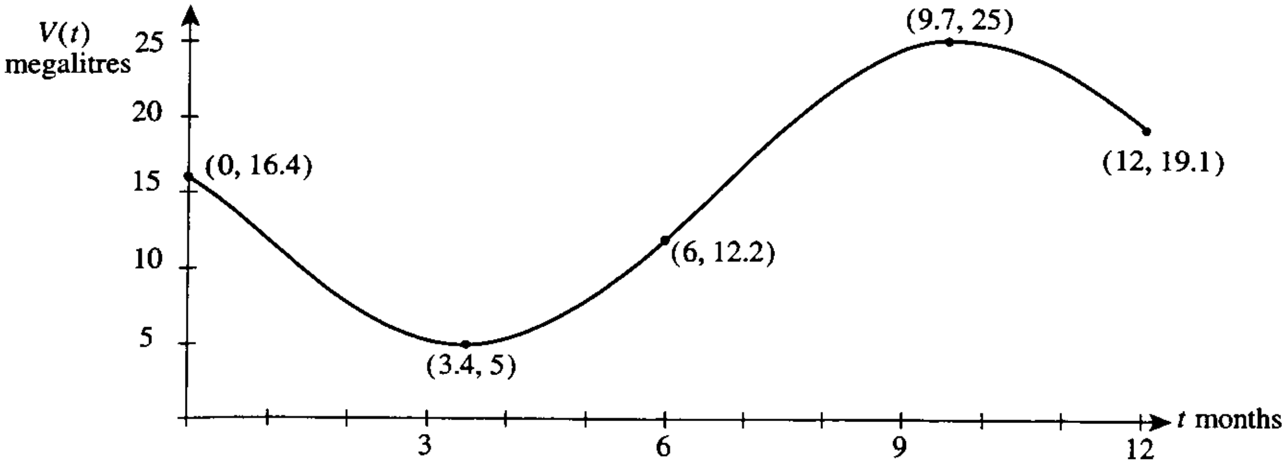
VCE Mathematical Methods
Units 3 & 4

Examination 2: Analysis Task

Suggested Solutions

Question 1

a.



Basic shape and location [A]
Endpoints, coordinates and labelling [A][A]

b. Rate of change $= V'(t)$

$$= 5\cos\left(\frac{t}{2}+3\right),\ 0 \le t \le 12$$ [A]

c. Minimum and maximum occur when $5\cos\left(\frac{t}{2}+3\right) = 0$.

$$\therefore \cos\left(\frac{t}{2}+3\right) = 0$$ [M]

$$\therefore \frac{t}{2}+3 = \cos^{-1}(0)$$

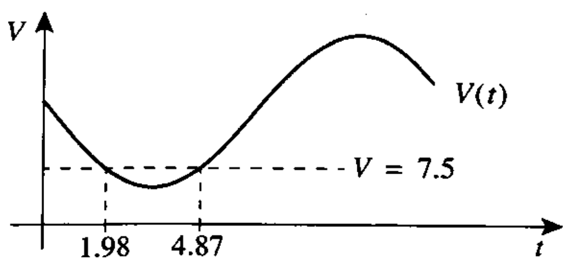
$$= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$ [M]

$$\therefore \frac{t}{2} = \frac{\pi}{2}-3, \frac{3\pi}{2}-3, \frac{5\pi}{2}-3$$

(Note that $\frac{\pi}{2}-3 < 0$.)

$$\therefore t = 3\pi - 6 \text{ (min)}, 5\pi - 6 \text{ (max)}.$$ [A][A]

d.



When $V = 7.5$, $t = 1.98$ and 4.87.

The volume is less than 7.5 during the year [M]
for $4.87 - 1.98 = 2.9$ months. [A]

Question 2

a. When $x = 0$, $f(0) = 5\log_e 1 + 10$.

As $\log_e 1 = 0$, $f(0) = 10$.

$\therefore a = 10$ and so the height of the wall is 10 m. [A]

b. $b = f(30)$

$= 5\log_e(30 + 1) + 10$

$= 5\log_e 31 + 10$

≈ 27.17 m. [A]

The maximum height above the ground is 27.17 m.

c. Reflection in y axis: $g(x) = f(-x)$

Translation 60 units to the right: $g(x) = f(x - 60)$

Combining these gives:

Reflected curve in y axis: $g(x) = 5\log_e(-x + 1) + 10$ [M]

Translated curve (after reflection): $g(x) = 5\log_e(-(x - 60) + 1) + 10$

$\therefore g(x) = 5\log_e(-x + 61) + 10$ [A]

d. The domain of $g(x)$ is $[30, 60]$ or $30 \le x \le 60$. [A]

The range of $g(x)$ is $[10, 27.17]$ or $[10, 5\ln 31 + 10]$. [A]

e. **i** $f'(x) = \dfrac{5}{x+1}$, $0 < x < 30$. [A]

ii. $g'(x) = 5\left(\dfrac{-1}{-x+61}\right)$ or $\dfrac{-5}{-x+61}$, $30 < x < 60$. [A]

f. **i** $f'(15) = \dfrac{5}{16}$ [A]

ii. $g'(45) = \dfrac{-5}{-45+61}$

$= -\dfrac{5}{16}$ [A]

g. $A = 2\displaystyle\int_0^{30} f(x)dx$

Terminals [A]

$2 \times$ integral with dx [A]

h. $A = 2\displaystyle\int_0^{30} (5\log_e(x + 1) + 10)dx$

$= 10\displaystyle\int_0^{30} (\log_e(x + 1) + 2)dx$ [M]

$= 10\Big[(x + 1)\log_e(x + 1) - x + 2x\Big]_0^{30}$ [A]

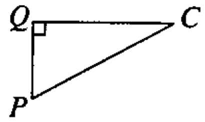
$= 10(31\log_e 31 + 30)$

$= 310\log_e 31 + 300$ [A]

i. $1364.5\ \text{m}^2$ [A]

Question 3

a. PQC is a 3-4-5 right angled triangle



$$\therefore PC = \sqrt{(300)^2 + (400)^2}$$
$$= 500 \text{ m.}$$

$$\therefore T_{PC} = \frac{0.5}{6}$$
$$= 0.08\dot{3} \text{ hr}$$
$$= 5 \text{ min.}$$

[A]

b. $T_{\text{total}} = T_{\text{forest}} + T_{\text{track}}$

$$= \frac{0.3}{6} + \frac{0.4}{12}$$ [M]

$$= 0.05 + 0.0\dot{3}\dot{3}$$
$$= 0.083$$
$$= 5 \text{ min}.$$

[A]

c. $T_{PRC} = T_{PR} + T_{RC}$

$$d_{PR} = \sqrt{300^2 + x^2}$$
$$d_{RC} = 400 - x$$

Hence $T_{PRC} = \dfrac{\sqrt{300^2 + x^2}}{6000} + \dfrac{400 - x}{12000}$ [A]

$$= \frac{2\sqrt{300^2 + x^2} + 400 - x}{12000} \text{ hr}$$

and so $T_{PRC} = \dfrac{2\sqrt{300^2 + x^2} + 400 - x}{12000} \times 60 \text{ min}$

$$= \frac{2\sqrt{90000 + x^2} + 400 - x}{200} \text{ min}$$ [A]

d. Minimum time is where $\dfrac{dT}{dx} = 0$. [M]

$$T = \frac{2(90000 + x^2)^{1/2}}{200} + \frac{400 - x}{200}$$
$$= \frac{1}{100}(90000 + x^2)^{1/2} + 2 - \frac{1}{200}x.$$

$$\frac{dT}{dx} = \frac{1}{200}(90000 + x^2)^{-1/2} \times 2x - \frac{1}{200}$$
$$= \frac{1}{200}\left(\frac{2x}{\sqrt{90000 + x^2}} - 1\right).$$ [A]

$\frac{dT}{dx} = 0$ when $\frac{2x}{\sqrt{90000 + x^2}} - 1 = 0$.

$\therefore \frac{2x}{\sqrt{90000 + x^2}} = 1$

$$\begin{aligned} \therefore 2x &= \sqrt{90000 + x^2} \\ \therefore 4x^2 &= 90000 + x^2 \\ \therefore 3x^2 &= 90000 \\ \therefore x^2 &= 30000 \\ \therefore x &= 173.2051 \text{ m.} \end{aligned}$$ [A]

$T_{min} = \frac{2\sqrt{90000 + 30000} + 400 - 173.2051}{200}$ [A]

$= 4.598$

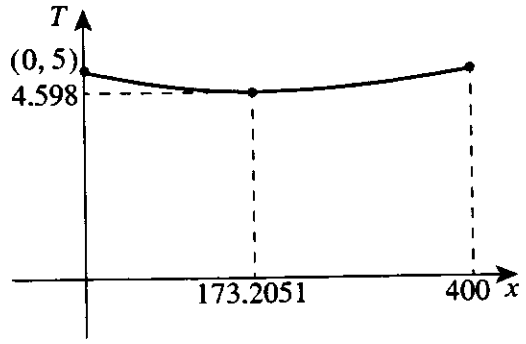
$= 4.6$ min (to 1 d.p.).

e. i $T'(170) = \frac{1}{200}\left[\frac{2(170)}{\sqrt{90000 + 170^2}} - 1\right] = -0.699 \times 10^{-5}$ [A]

$T'(180) = \frac{1}{200}\left[\frac{2(180)}{\sqrt{90000 + 180^2}} - 1\right] = 0.0000145$ [A]

Since the gradient changes from negative to positive, the time found in **d.** is a minimum. [A]

ii. A graphic calculator is useful here.



General shape and minimum [A]
Endpoints [A]

f. All possible routes will give a time within [4.598, 5] and so Paolo will not be disqualified. [A]

 TEVMM34EXAM2S02.FM

Question 4

a. i Let the random variable X be the number of applicants who pass on their first attempt. X has a binomial distribution with $n = 10$ and $p = 0.6$.

$$\Pr(X = 6) = \binom{10}{6}(0.6)^6(0.4)^4$$
$$= 0.2508.$$ [A]

Alternatively, using the graphic calculator, binompdf(10, 0.6, 6) = 0.2508.

ii.
$$\Pr(X \geq 6) = \Pr(X = 6) + \Pr(X = 7) + \Pr(X = 8) + \Pr(X = 9) + \Pr(X = 10)$$
$$= 0.2508 + 0.21499 + 0.12093 + 0.04031 + 0.0060466$$
$$= 0.6331.$$ [A]

Alternatively, using the graphic calculator, 1 – binomcdf(10, 0.6, 5) = 0.6331.

iii.
$$\Pr(X \leq 6) = 1 - \Pr(X > 6)$$
$$= 1 - (0.21499 + 0.12093 + 0.04031 + 0.0060466)$$
$$= 1 - 0.382277$$
$$= 0.6177.$$ [A]

Alternatively, using the graphic calculator, binomcdf(10, 0.6, 6) = 0.6177.

b.
$$E(X) = np$$
$$= 250 \times 0.6$$
$$= 150$$ [A]

$$SD(X) = \sqrt{np(1-p)}$$
$$= \sqrt{150(0.4)}$$
$$= 7.746$$ [A]

c. $1 - 0.01 = 0.99$ [A]

d. $(0.99)^1(0.99)^1 = (0.99)^2 = 0.9801.$ [A]

e. $1 - (0.99)^d.$ [A]

f. Let T = overheating temperature.
This is a normal distribution with $\mu = 94.5$ and $\sigma = 5.7$.

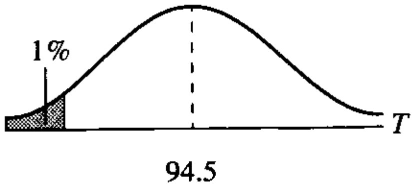
So, $\Pr(T \leq 100) = \Pr\left(Z \leq \frac{100 - 94.5}{5.7}\right)$ [A]

$$= \Pr(Z \leq 0.965).$$

Using graphic calculator or tables, $\Pr(T \leq 100) = 0.8328$. [A]

The proportion of sensors that will operate (not overheat) at 100°C is
$1 - 0.8328 = 0.1672$
$= 17\%.$ [A]

g. We require the temperature at which 1% fail.



So, $\Pr(T < t) = 0.01$. [M]

Using the graphic calculator or reverse tables to look up 0.99, $z = -2.326$ [A]

and so $\frac{t - 94.5}{5.7} = -2.326$

$$\therefore t = (-2.326 \times 5.7) + 94.5$$
$$= 81.24°\text{C}.$$

To the nearest °C, $t = 81°\text{C}$. [A]