MAM 61 2002 EXAM 1 NEAP

SECTION A

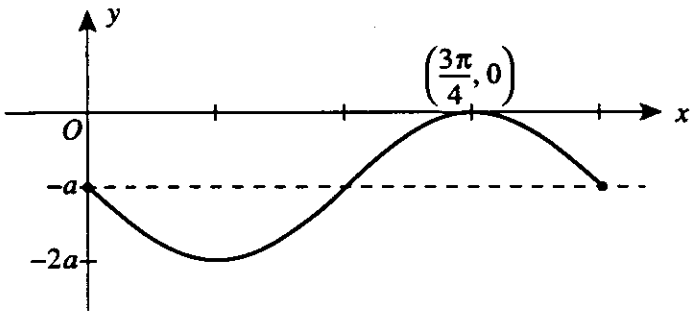
Specific Instructions for Part I

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1



The graph shown above is best represented by the equation

A. $y = -2a\sin 2(x-a)$

B. $y = a\cos 2x - a$

C. $y = -2a\cos 2x - a$

D. $y = -a\sin\frac{1}{2}x - a$

E. $y = -a\sin 2x - a$

Question 2

The coefficient of the x^2 term in the expansion of $(3x-2)^6$ is

A. 4320

B. −4320

C. 2160

D. −2160

E. 1080

Question 3

The range of the function $f(x) = \log_2 x$ for $x \in (0, 2]$ is

A. R

B. $R^+ \cup \{0\}$

C. $[0, 1]$

D. $(-\infty, 1]$

E. $(-\infty, 2]$

Question 4

$\log_e x - 3\log_e 2x + 2\log_e 3x$ is equal to

A. $\log_e\left(\frac{8}{9}\right)$

B. $\log_e 9 - \log_e 8$

C. $\log_e\left(\frac{9x^3}{8}\right)$

D. $\log_e\left(\frac{9x^6}{8}\right)$

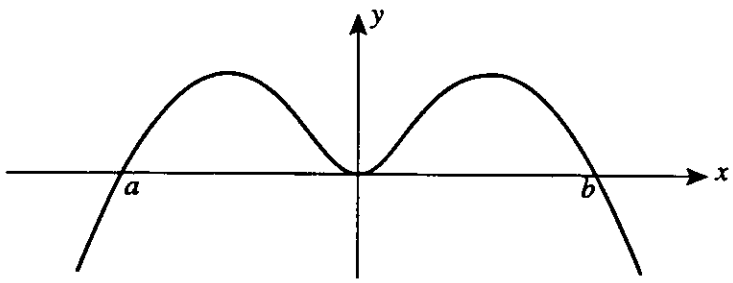
E. $\frac{9}{8}$

Question 5

Which of the following gives the solution or solutions to the equation $\log_2 x(x-1) = 1$?

A. 1 only
B. –2 and 1
C. –1 only
D. 2 only
E. –1 and 2

Question 6



The most likely equation for the polynomial graph shown is

A. $y = x(x-a)(x-b)$

B. $y = x^2(x-a)(x-b)$

C. $y = -x^2(x+a)(x-b)$

D. $y = x^2(a-x)(x-b)$

E. $y = x^2(a-x)(b-x)$

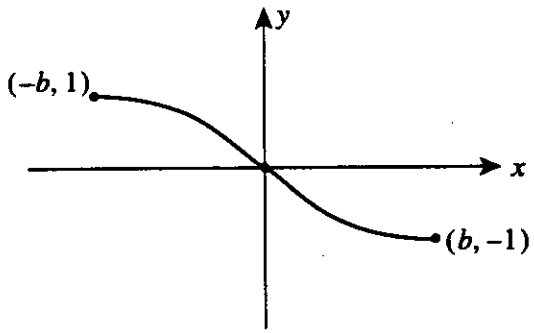
Question 7

The number of solutions to the equation $2\sin 3x = \log_{10} 30x$ is

A. 1
B. 2
C. 3
D. 4
E. 5

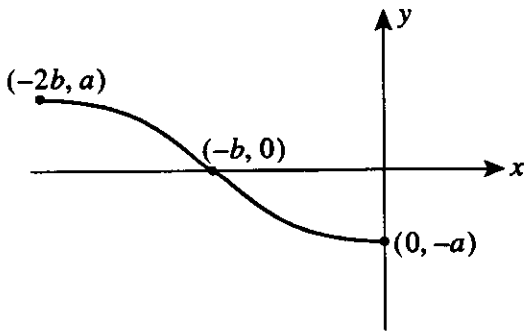
Question 8

The graph of $y = f(x)$ is shown below.

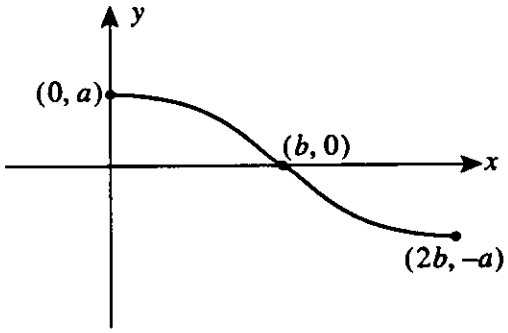


Which of the following graphs best represents $y = af(x - b)$ if $a, b > 0$?

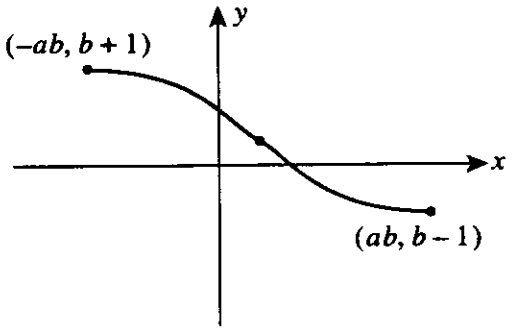
A.



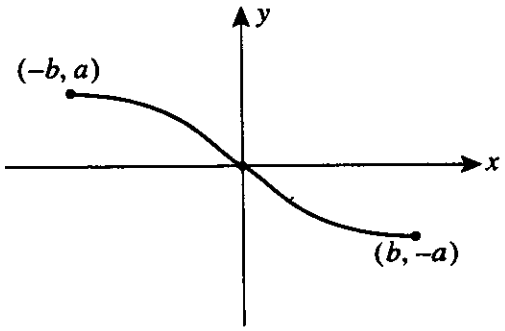
B.



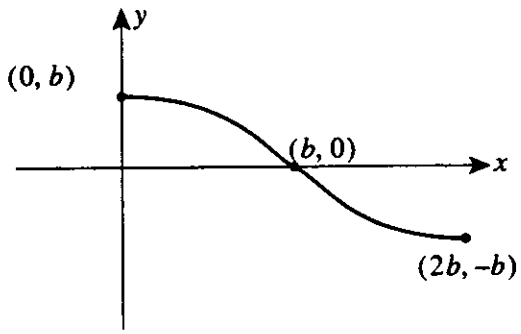
C.



D.

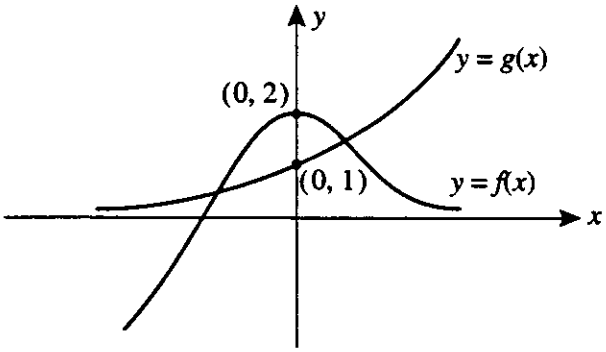


E.



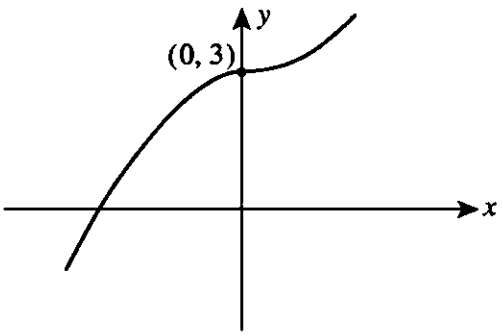
Question 9

The graphs of $y = f(x)$ and $y = g(x)$ are shown on the set of axes below.

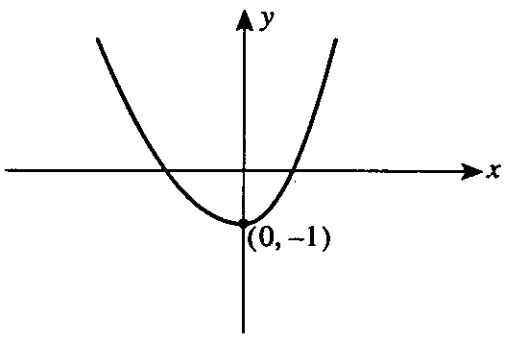


Which of the following is most likely to be the graph of $y = f(x) - g(x)$?

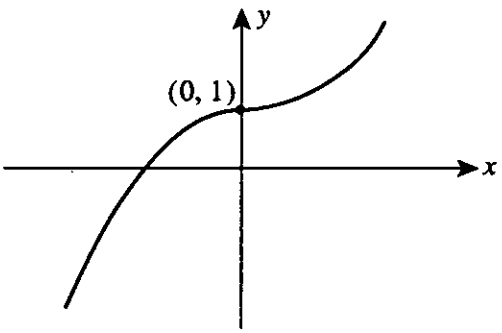
A.



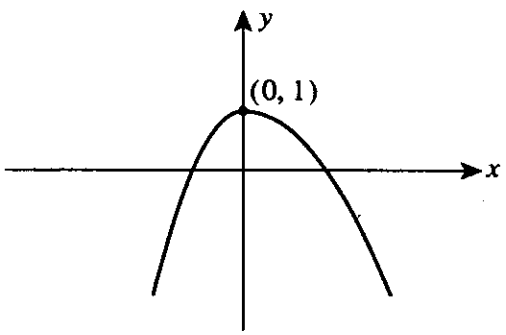
B.



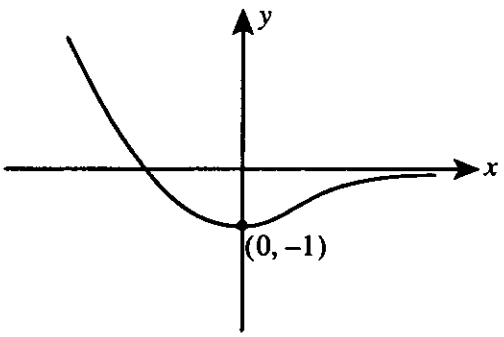
C.



D.

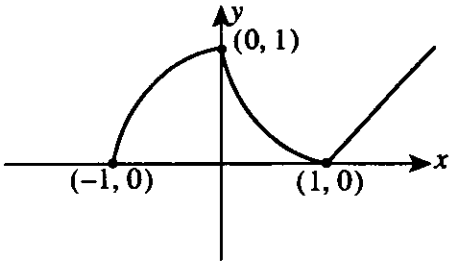


E.



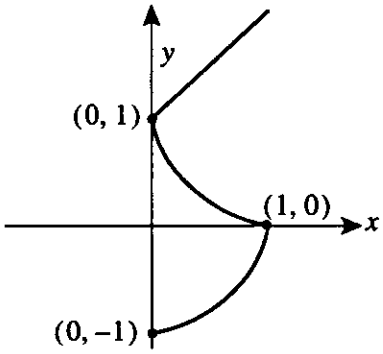
Question 10

The graph $y = f^{-1}(x)$ is shown on the set of axes below.

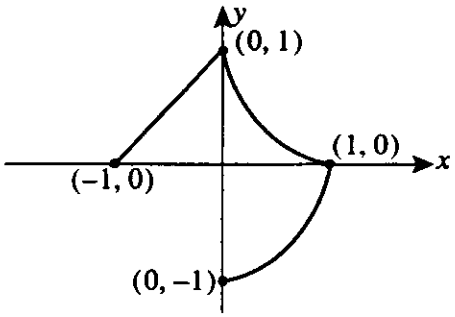


Which of the following is most likely to be the graph of $y = f(x)$?

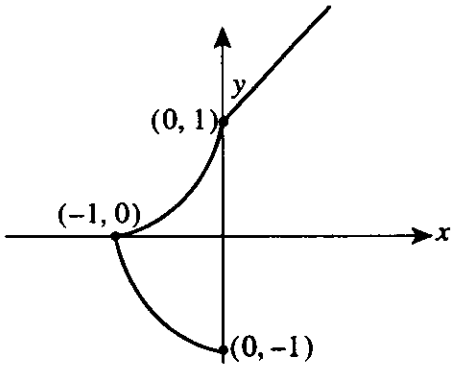
A.



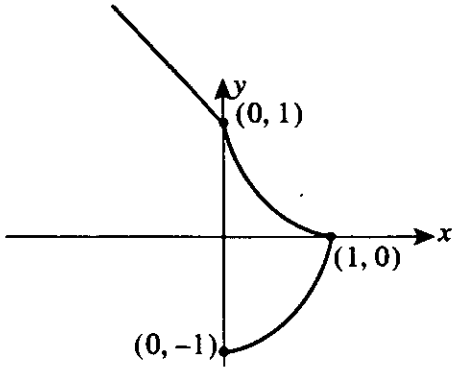
B.



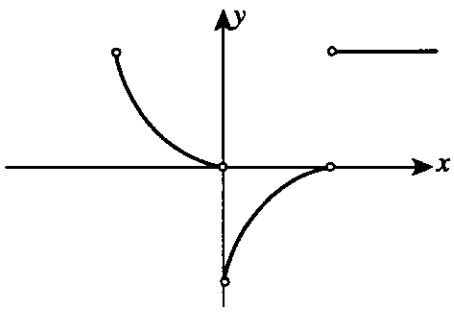
C.



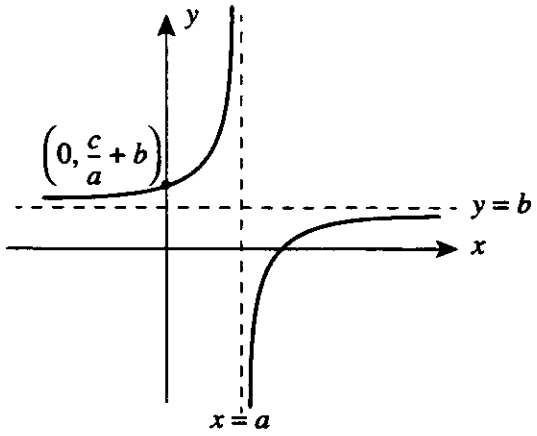
D.



E.



Question 11



The most appropriate equation for the graph shown above, if a, b and $c \in R^+$, is

A. $y = \frac{c - bx}{a - x}$

B. $y = \frac{c}{b - x} + a$

C. $y = \frac{c}{x - b} + a$

D. $y = \frac{c}{a - x} + b$

E. $y = \frac{c}{x - a} + b$

Question 12

$f(x)$ is e^{x^2+1}. Which of the following is $f'(x)$?

A. $2xe^{x^2+1}$

B. $2xe^{2x}$

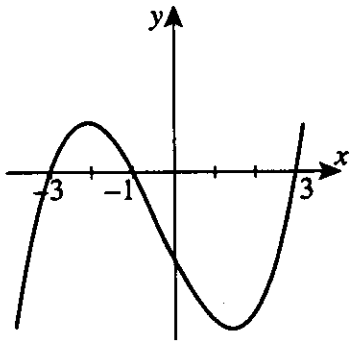
C. $2x$

D. $(x^2 + 1)e^{x^2}$

E. $(x^2 + 1)e^{2x}$

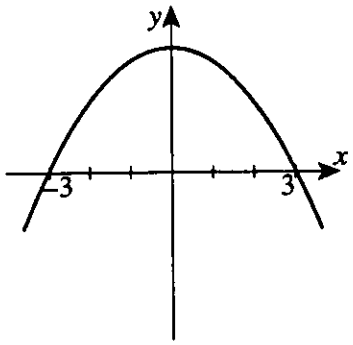
Question 13

The graph of $y = f(x)$ is shown below.

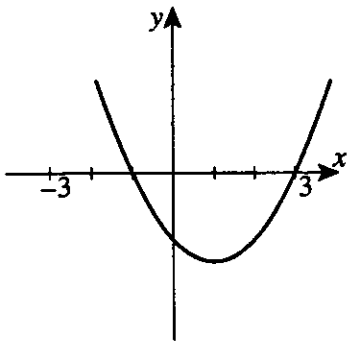


Which one of the following is most likely to be the graph of the derivative function with the equation $y = f'(x)$?

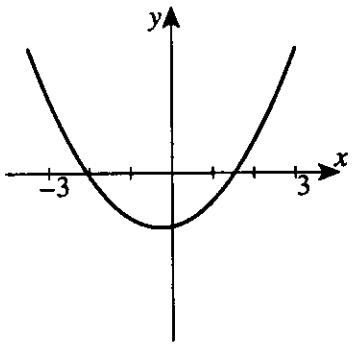
A.



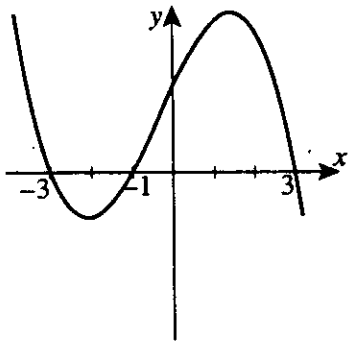
B.



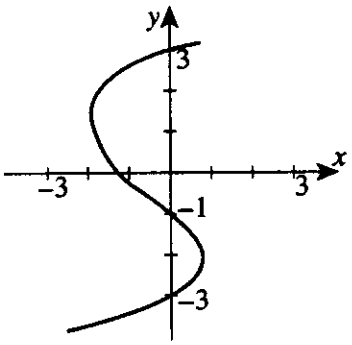
C.



D.



E.



Question 14

The derivative of $\log_e\left(\frac{1}{\sin x}\right)$ with respect to x is

A. $-\frac{1}{\tan x}$

B. $\frac{1}{\tan x}$

C. $-\tan x$

D. $\frac{1}{\cos x}$

E. $\tan x$

Question 15

If $y = \frac{\sin \pi x}{e^x}$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

A. 0

B. -1

C. π

D. $-\pi$

E. $\frac{1}{\pi}$

Question 16

The derivative of $e^x \log_e 2x$ is

A. $e^x\left(\frac{1}{x} + \log_e 2x\right)$

B. $e^x(1 + \log_e 2x)$

C. $-e^x(1 + \log_e 2x)$

D. $e^x x + \log_e 2x$

E. $e^x\left(\frac{1}{2x} + \log_e 2x\right)$

Question 17

The equation of the normal to the curve of the function with equation $y = e^{-\frac{1}{2}x} - 1$ at the point where $x = 0$ is

A. $y = x$

B. $y = -2x$

C. $y = 2x$

D. $y = -\frac{1}{2}x$

E. $y = \frac{1}{2}x$

Question 18

Using the left rectangle approximation with rectangles of width 1, the area of the region bounded by the curve $y = x^2 + 1$, the x axis, and the lines $x = 1$ and $x = 5$ is approximated by

A. 30 sq. units

B. 34 sq. units

C. 46 sq. units

D. $45\frac{1}{3}$ sq. units

E. 58 sq. units

Question 19

An antiderivative of $\frac{1}{2(5x+2)^2}$ is

A. $\frac{1}{10(5x+2)}$

B. $\frac{1}{10(5x+2)^3}$

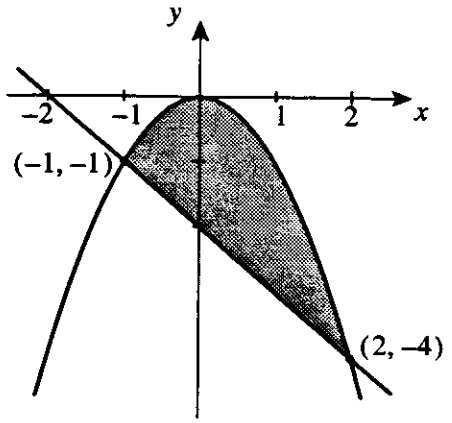
C. $\frac{-1}{10(5x+2)^3}$

D. $\frac{-10}{10(5x+2)^3}$

E. $\frac{-1}{10(5x+2)}$

Question 20

The graphs of function $y = -x^2$ and $y = -(x + 2)$ are shown in the diagram below.



The area shaded (in square units) is equal to

A. $1\frac{1}{2}$

B. $4\frac{1}{2}$

C. $-4\frac{1}{2}$

D. $5\frac{1}{6}$

E. $\frac{5}{6}$

Question 21

If $\int_a^{\frac{3\pi}{4}} \sin 2x dx = 0$ then a is most likely to be

A. $\frac{\pi}{4}$

B. 0

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

E. π

Question 22

A is a binomial random variable with $\mathrm{E}(A) = 2$ and $\mathrm{Var}(A) = \frac{3}{2}$. As a result $\Pr(A = 2)$ is equal to

A. $\left(\frac{1}{4}\right)^2$

B. ${}^4C_2\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^2$

C. ${}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

D. ${}^8C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6$

E. ${}^8C_2\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^2$

Question 23

On a school committee composed of eight members, five favour the proposition that students should wear protective hats throughout the entire year, while three consider it only necessary during term 1 and term 4.

What is the probability that a randomly chosen subcommittee of four will contain **exactly** three who favour all year protection?

A. $\frac{1}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{4}{7}$

E. $\frac{5}{7}$

Question 24

If Z is a normal random variable with $\mu = 0$ and $\sigma = 1$, then $\Pr(-0.3 < Z < 0.5)$ is equal to

A. $\Pr(Z < 0.5) - \Pr(Z > -0.3)$

B. $\Pr(Z < 0.5) - 1 + \Pr(Z > -0.3)$

C. $\Pr(Z < 0.5) + \Pr(Z < -0.3)$

D. $\Pr(Z < -0.3) - \Pr(Z < 0.5)$

E. $\Pr(Z < -0.3) + \Pr(Z > 0.5)$

Question 25

Jasmeet recently purchased a bug zapper to hang out under her patio. Over a period of 20 days, she has been daily counting the number of insects zapped in the collection tray at the bottom of her device. The results are given in the table below.

Number of insects (x)	0	1	2	3	4	5	6
Number of days zapper contained x	1	2	1	4	7	4	1

The mean number of insects, per day, that Jasmeet found was equal to

A. 1.95
B. 3
C. 3.5
D. 3.9
E. 6.1

Question 26

Joel, a keen golfer, is selecting some tees from a gift box he has recently received. The box contains only 35 yellow and 25 red tees. He randomly selects 10 tees.

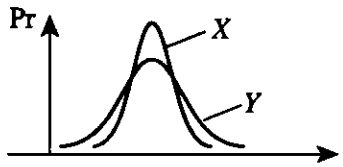
The expected number of yellow tees and the variance for the number of yellow tees that Joel selects is closest to

A. 5, 1.4
B. 5, 2.4
C. 6, 1.4
D. 6, 2.1
E. 6, 2.4

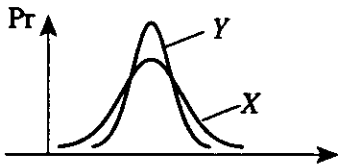
Question 27

A sample of 5 is selected from a display containing dark and milk chocolates. X is the number of dark chocolates in the sample when the sample is consumed and Y is the number of dark chocolates in the sample when it is made with replacement. Which of the following graphs best represents the distributions described?

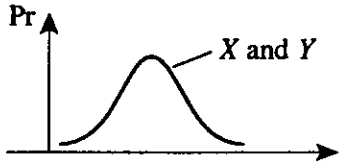
A.



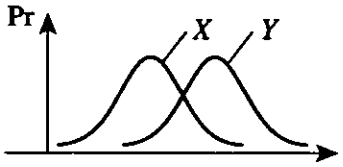
B.



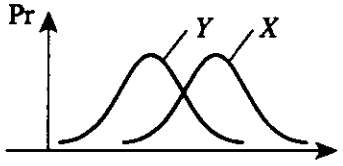
C.



D.



E.



END OF MULTIPLE-CHOICE QUESTION BOOKLET

Question 1

Consider the function $f(x) = \log_e(2x-1)$.

a. State the maximal domain of $f(x)$. (1)

b. Determine the equation of $f^{-1}(x)$. (2)

c. State the range of $f^{-1}(x)$. (1)

Question 2

Calculate the **exact** solutions to the equation $\sqrt{3} + 2\sin\frac{x}{2} = 0$ for $-2\pi \le x \le 0$. (3)

Question 3

Consider the equation $y = x^2\log_e 3x$.

Find

a. $\frac{dy}{dx}$ (2)

b. the exact value of the gradient of the tangent when $x = \frac{1}{3}$. (1)

c. the equation of the tangent when $x = \frac{1}{3}$. (2)

Question 4

Given $y = e^x$, find in terms of h an expression without exponential or logarithmic notation, for the approximate increase in y as x is increased from $\log_e 2$ to $h + \log_e 2$. (2)

Question 5

A recent telephone survey concluded that 54% of Australians favour the Government's decision to support stem-cell research.

a. If 1769 calls were made, what would be the expected number of non-supporters of the Government decision? (1)

b. If 5 people were telephoned at random, what is the probability that at least two will favour the Government decision? Express your answer to the nearest percentage. (2)

Question 6

For a particular class, the marks in a Maths Methods test are normally distributed with a mean of 67. The probability that a randomly selected student from the class will obtain a mark of less than 50 is 0.1.

a. What is the standard deviation of the marks in the class? Express your answer to two decimal places. (3)

b. Chrissy, a student in the class, decides to attempt some self-paced learning in order to improve her score. She purchases a computer program sold as a box of 6 disks. As a promotional gimmick, one or more of these disks can be 'tagged' for a prize. It is known that these disks have a chance of 0.13 of being tagged for a prize. Find, to 4 decimal places, the probability that Chrissy purchases a box containing fewer than three prize disks, if she knows it contains at least one prize disk. (3)