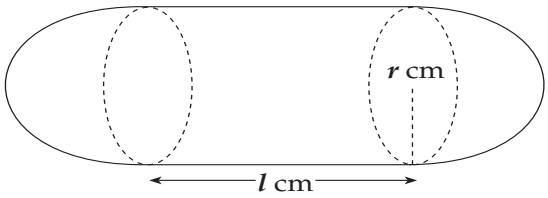
Question 1

A plastic lolly container is in the shape of a cylinder with a hemispherical lid at each end as shown below. Let r be the radius and l the height of the cylinder in centimetres.



The volume, V cm^3, of the container is given by the rule

$$V = \pi r^2 l + \frac{4}{3}\pi r^3$$

a. Write an expression for the surface area, A cm^2, of the container in terms of r and l. The surface area of a sphere is $4\pi r^2$.

1 mark

b. If the surface area of the container is 400 cm^2, show that $l = \dfrac{200 - 2\pi r^2}{\pi r}$ cm.

2 marks

c. Hence, show that $V = 200r - \dfrac{2}{3}\pi r^3$.

2 marks

d. State the domain (possible r values) of the function in **part c**. Give exact values.

2 marks

e. Find the largest value of r, correct to one decimal place, which gives a volume of 350 cm^3.

1 mark

f. Determine the exact values of r and l which give the maximum volume of the container? Interpret your results.

4 marks

g. What is the exact value for the maximum volume of the container?

2 marks

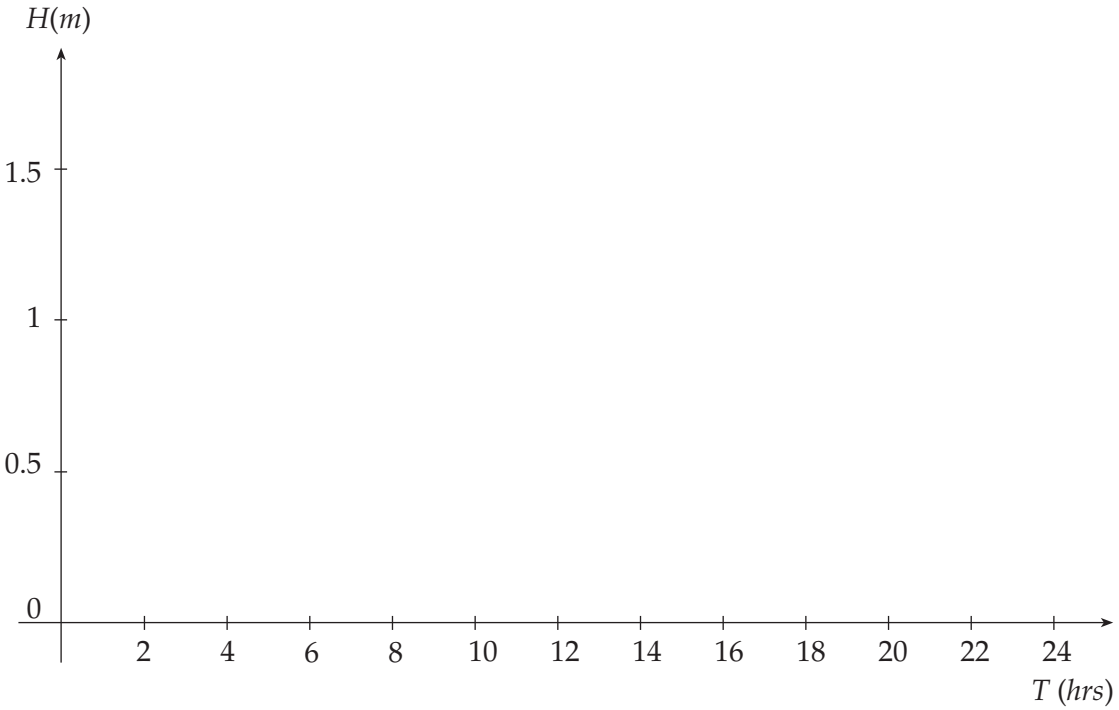
Total 14 marks

Question 2

Assume that the height of the tide, H metres, at Port Lincoln, is modelled using a function of the form $H = a\sin(n(T - 1.5)) + c$, where a, n and c are constants and T hours is the time after midnight on Monday September 30th.

The first high tide at Port Lincoln on Monday September 30th was recorded at 5:06 am when the height of the tide was 1.49 metres. The next low tide was recorded at 12:18 pm when the height was 1.01 metres.

a. Mark, and label, this information on the axes provided.



2 marks

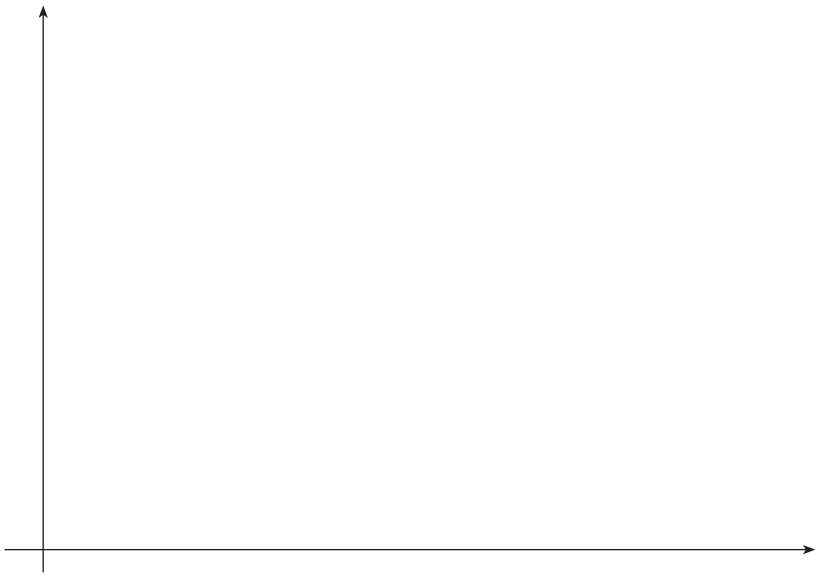
b. Calculate the values of a and c for the trigonometric model defined above.

2 marks

c. Show that the value of n is $\frac{5\pi}{36}$.

2 marks

d. Sketch this relationship for 1 period on the axes provided starting at $T = 0$. (Label the endpoints.) 2 marks



e. Determine the earliest time, to the nearest minute, when the height of the tide is 1.2 metres on Monday September 30th and find the rate in m/hr correct to 4 decimal places, at which the height is changing at this time.

2 marks

During a lunar month (28 days), the difference between the levels of successive high and low tides changes. The spring tides, which happen near the days of the New Moon and the Full Moon, have the greatest range, while the neap tides near the days of the First and Last Quarter have the smallest range.

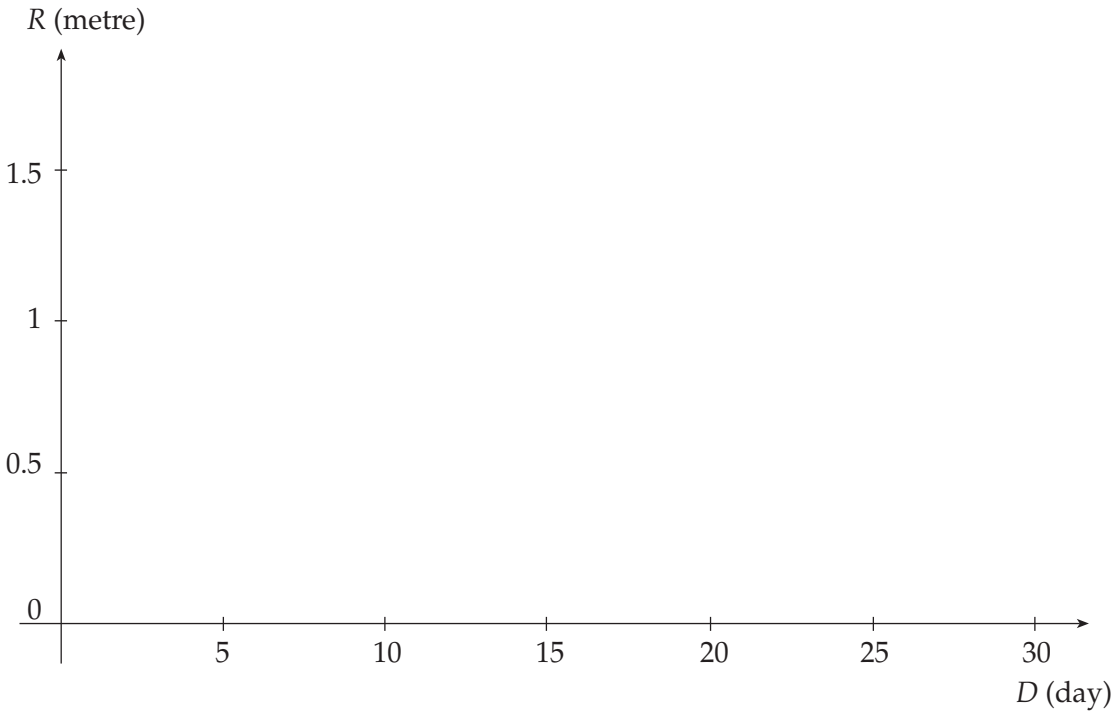
f. If the height of the high tide near the New Moon, day 0, was 1.72 metres and the low tide height was 0.54 metres, calculate the largest range.

1 mark

g. If the height of the high tide near the First Quarter, day 7, was 1.08 metres and the low tide height was 0.88 metres, calculate the smallest range.

1 mark

h. Use a cosine function to model the range of the tides R metres in terms of the days, D, after the New Moon. Sketch this function on the axes below.

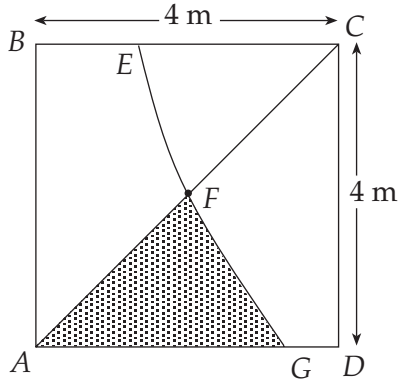


2 marks

Total 14 marks

Question 3

A stained glass window *ABCD* is being designed for an Art Gallery as shown below. The window is 4m by 4m.



If *AB* represents the *y*–axis and *AD* the *x*–axis, then the curve *EG* is represented by the equation $y = f(x) = 3e^{2-x} - 1$ and the line *AC* by the equation $y = g(x) = x$.

a. Find exact values for the coordinates of the points *E*, *F* and *G*.

3 marks

b. Show that the exact area, in square metres, of the blue glass *AFG* (shaded) required for the window is $4 - \log_e 3$.

4 marks

c. The designer decided to add more panels to the window. Find the rule for $f^{-1}(x)$ and draw the curve on the diagram above, clearly showing the exact values for the coordinates of the endpoints.

4 marks

d. Find the area, in square metres correct to one decimal place, of the new panel which is adjacent to the blue panel and inside the triangle *ACD*. Write down the rule you used to find the area.

3 marks

Total 14 marks

Question 4

A certain company markets chocolates in 500 gm bags. The weights of the bags are normally distributed with a mean of 505 gm and a variance of 100 gm. (State all answers to **four decimal places**).

a. Find the probability that a randomly selected bag will weigh less than 500gm.

2 marks

The Consumer Protection Authority receives a number of complaints and is about to investigate. As the company will be fined heavily if too many bags weigh less than 500 gm, the packaging process is changed to ensure that 95% of the bags weigh at least 500 gm.

b. What would be the new mean weight required if the variance remains at 100gm?

2 marks

c. What standard deviation would be acceptable if the mean weight is to remain at 505 gm?

2 marks

d. Which alternate is preferable from the company's point of view? Give a reason for your choice.

2 marks

The company decides to alter their process so that the bags have a mean weight of 510 gm and a variance of 40 gm. The bags are packed for distribution to the retailer in boxes with 25 bags in each box.

e. Five bags are selected at random from a box for checking. What is the probability that 3 or more will weigh less than 500 grams?

3 marks

f. A box is rejected if three or more of the random sample of five are found to be under 500 grams. Given that a box contains 5 underweight bags, what is the probability that the box will be rejected?

2 marks

Total 13 marks

