Part 1: Multiple-choice questions

Question 1

For the function $f(x) = 7 - 6\sin(4x)$, the amplitude and period respectively are

A. $7, \frac{\pi}{3}$

B. $-6, 8\pi$

C. $-6, \frac{\pi}{2}$

D. $7, \frac{\pi}{2}$

E. $6, \frac{\pi}{2}$

Question 2

Exact solutions to the equation $4\sin^2(2\theta) - 3 = 0$, where $-\pi \le \theta \le \pi$ are

A. $\theta = -0.5\sin^{-1}(\frac{3}{4}), 0.5\sin^{-1}(\frac{3}{4})$ only

B. $\theta = -\frac{\pi}{3}, \frac{\pi}{3}$ only

C. $\theta = -\frac{\pi}{6}, \frac{\pi}{6}$ only

D. $\theta = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

E. $\theta = -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}$ only

Question 3

The graph of $y = \cos x$ is transformed by doubling the amplitude, doubling the period and then translating 1 unit vertically down. The new function would be

A. $y = 2\cos(x) - 1$

B. $y = 2\cos(2x) - 1$

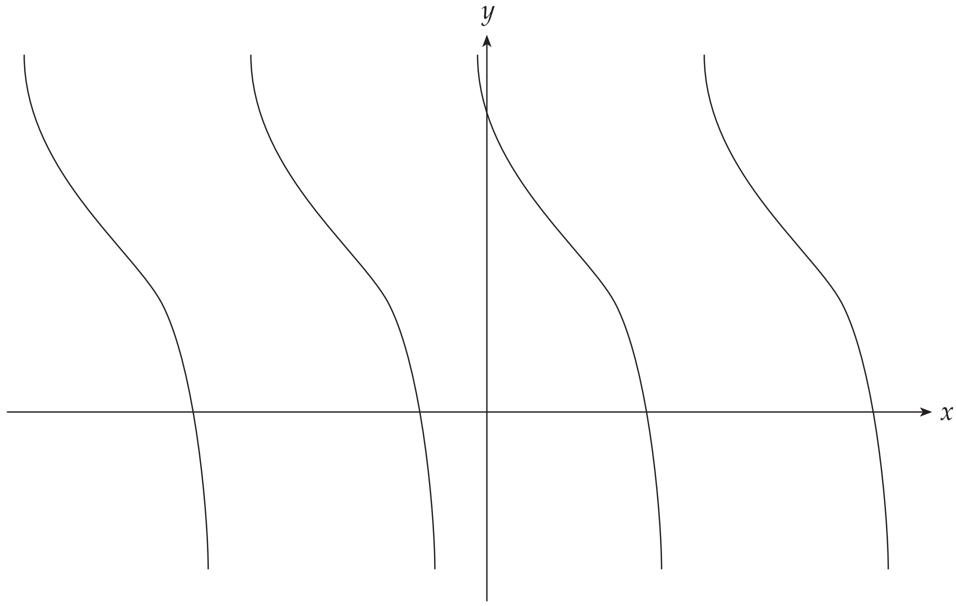
C. $y = 0.5\cos(2x) - 1$

D. $y = 2\cos(0.5x) - 1$

E. $y = 2(\cos(0.5x) - 1)$

Question 4

If a, b and c are positive constants, a possible equation for the function shown could be



A. $f(x) = a - b\tan(x)$

B. $f(x) = a + b\tan(x + c)$

C. $f(x) = b - a\tan(x + c)$

D. $f(x) = a + b\tan(x)$

E. $f(x) = a\tan(x - b) + c$

Question 5

If $y = -3x^2 + 6x - 3a$, where a is a constant, then the y coordinate of the turning point is

A. $-1 + a$

B. $1 - a$

C. $-3 + 3a$

D. $3 - 3a$

E. a

Question 6

The term independent of x in the expansion $(2x^2 - \frac{3}{x})^6$ is

A. 90

B. –90

C. 324

D. 4860

E. –4860

Question 7

If $2\log_2(x) - \log_2(x+4) = 1$ then x equals

A. –2

B. 4

C. –2 or 4

D. $\frac{1 \pm \sqrt{17}}{2}$

E. $\frac{1 + \sqrt{17}}{2}$

Question 8

If $9^x - 3^{(x+1)} = 54$ then x equals

A. –6

B. –2

C. 2

D. 9

E. –6 or 9

Question 9

Let $h: D \to R, h(x) = \dfrac{2}{(3x-5)^2} + 1$ where D is the maximal domain of h. The smallest value of b such that $g: (b, \infty) \to R$ with $g(x) = h(x)$ is a one to one function is

A. 2

B. $1\frac{2}{3}$

C. 1

D. $-1\frac{2}{3}$

E. 5

Question 10

The inverse function, $f^{-1}(x)$ of $f:(-\infty, -1] \to R, f(x) = -4(x+1)^2$ is

A. $f^{-1}: (-\infty, 0] \to R, f^{-1}(x) = \dfrac{\sqrt{-x}}{2} - 1$

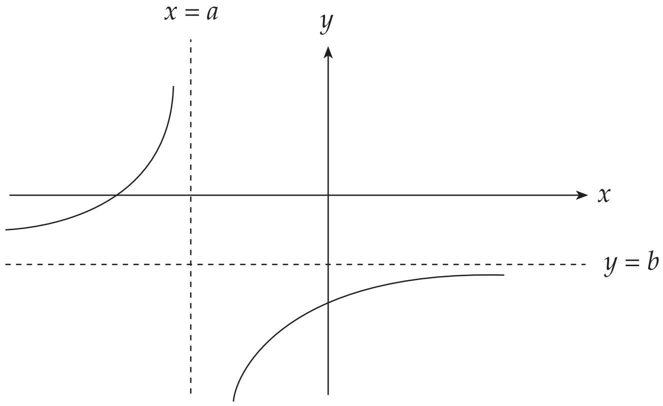
B. $f^{-1}: (-\infty, 0] \to R, f^{-1}(x) = \dfrac{\sqrt{x}}{2} - 1$

C. $f^{-1}: (-\infty, 0] \to R, f^{-1}(x) = \dfrac{\pm\sqrt{-x}}{2} - 1$

D. $f^{-1}: (-\infty, 0] \to R, f^{-1}(x) = \dfrac{-\sqrt{-x}}{2} - 1$

E. $f^{-1}: (-\infty, -1] \to R, f^{-1}(x) = \dfrac{-\sqrt{-x}}{2} - 1$

Question 11



The rule for the above graph could be

A. $y = \dfrac{-1}{x+a} + b$

B. $y = \dfrac{1}{x+a} + b$

C. $y = \dfrac{-1}{x-a} + b$

D. $y = \dfrac{-1}{x-a} - b$

E. $y = \dfrac{-1}{x+a} - b$

Question 12

The curve of a cubic function, $f(x)$, in the form $f\colon R \to R$, where $f(x) = A(x-B)^3 + C$ and A, B and C are **positive** real constants will have a

A. positive gradient for all $x \in R$.

B. local minimum followed by a local maximum.

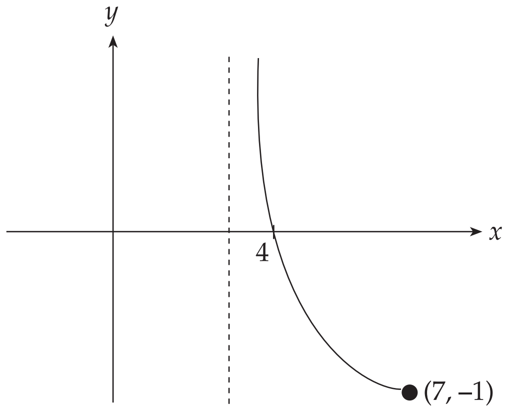
C. a local maximum followed by a local minimum.

D. a stationary point of inflection or a local maximum and local minimum.

E. a stationary point of inflection.

Question 13

The rule for the function of the graph shown below is of the form $y = A\log_e(x + B)$, where A and B are real constants.



The values of A and B are

A. $A = -\frac{1}{\log_e 4}$ $B = -3$

B. $A = -\frac{1}{\log_e 10}$ $B = 3$

C. $A = -1$ $B = -3$

D. $A = -1.789$ $B = 2.477$

E. $A = -0.721$ $B = 3$

Question 14

The average rate of change of the function $f(x) = (x - 1)e^x$ with respect to x over $[0, 2]$ is

A. $\frac{e^2 + 1}{2}$

B. $2e^2$

C. xe^x

D. e^2

E. $\frac{e^2}{2}$

Question 15

The equation of the normal to the curve of the function with equation $y = \frac{x}{\cos(x)}$ at the point where $x = \pi$ is

A. $y = x$

B. $y = x + 2\pi$

C. $y = -x$

D. $y = -x + 2\pi$

E. $y = x - 2\pi$

Question 16

The derivative of $\log_e(\tan(x))$ is

A. $\frac{\sec(x)}{\tan(x)}$

B. $\frac{\sec(x)}{\sin(x)}$

C. $\sec^2(x)\tan(x)$

D. $\sec^2(x)$

E. $\frac{\tan(x)}{\sec^2(x)}$

Question 17

If $y = \left(\sqrt{x^2 + 1}\right)^3$ then $\frac{dy}{dx}$ equals

A. $3xy^{\frac{1}{3}}$

B. $3xy^3$

C. $\frac{3}{2}\sqrt{x^2 + 1}$

D. $2x\sqrt{x^2 + 1}$

E. $6x\sqrt{x^2 + 1}$

Question 18

For the curve of the function with equation $y = (x - 1)^3(x + 2)$, the largest subset of R for which the gradient of the graph is positive is

A. $(-\infty, -2)$

B. $(-\infty, -1.25)$

C. $(-1.25, \infty)$

D. $(-1.25, 1) \cup (1, \infty)$

E. $(1, \infty)$

Question 19

The approximate area, in square units correct to two decimal places, bounded by the graph of $y = 10^x$ and the x–axis, using the left end point between $x = 0$ and $x = 3$ and using rectangular strips of width 0.5 is

A. 231.01

B. 433.86

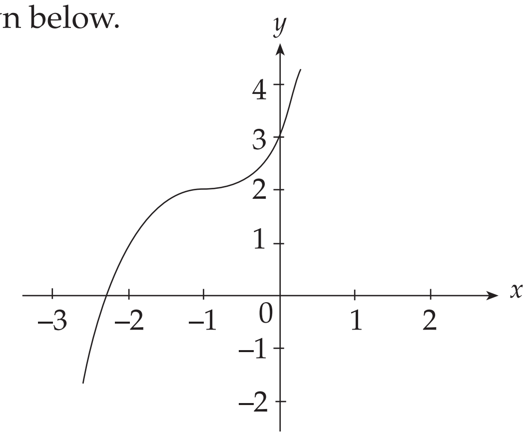
C. 480.76

D. 610.50

E. 730.51

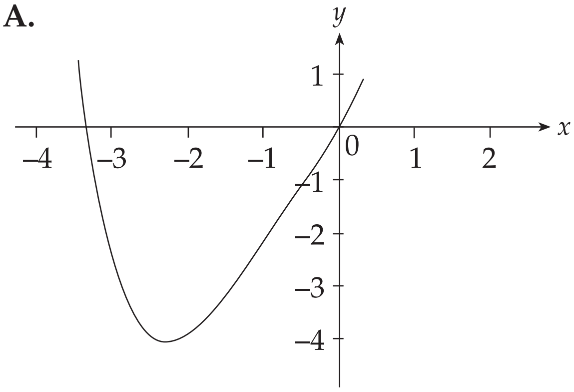
Question 20

The graph of $y = f(x)$ is shown below.

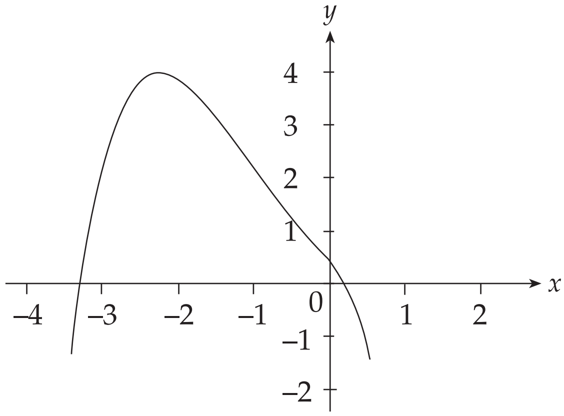


If h is a function such that $h'(x) = f(x)$, then the graph of h could be

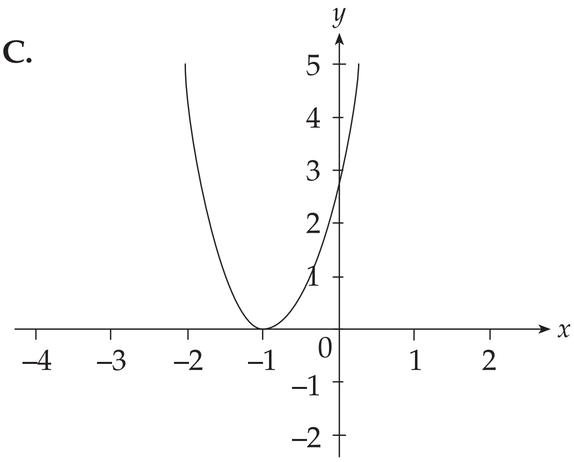
A.



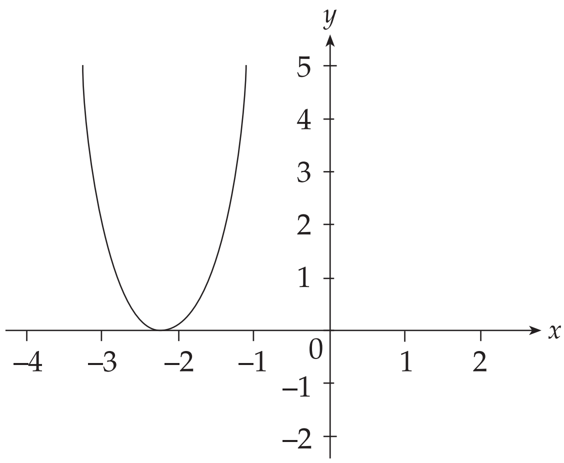
B.



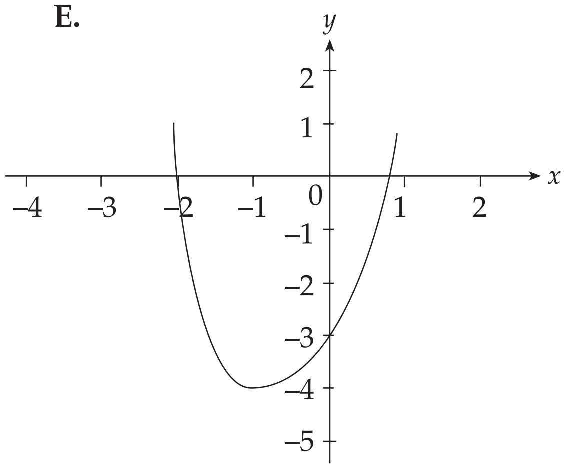
C.



D.



E.



Question 21

Note: c is a real constant.

$\int \frac{e^{3x}+1}{e^x}\,dx$ equals

A. $\frac{e^{2x}}{2} - e^x + c$

B. $\frac{e^{2x}}{2} + e^{-x} + c$

C. $2e^{2x} + e^{-x} + c$

D. $2e^{2x} - e^{-x} + c$

E. $\frac{e^{2x}}{2} - \frac{1}{e^x} + c$

Question 22

If $\int_1^a \frac{1}{(x-2)^3}\,dx = -\frac{1}{2}$ then a equals

A. $2 + \frac{\sqrt{2}}{2}$

B. $2 \pm \frac{\sqrt{2}}{2}$

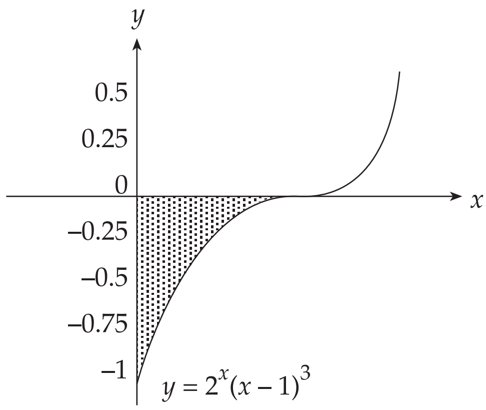
C. $2 - \frac{\sqrt{2}}{2}$

D. $2 + \sqrt{2}$

E. $2 - \sqrt{2}$

Question 23

The area, in square units, of the shaded region (the region bounded by the curve and the axes) shown correct to three decimal places is



A. 0.289

B. 0.578

C. 0.587

D. 0.876

E. –0.289

Question 24

A random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	a	$2a$	$4a$	$3a$

The value of $E(2X - 1)$ is

A. 0.1

B. 1.5

C. 1.9

D. 2.5

E. 2.8

Question 25

An examination paper consists of 33 multiple–choice questions, each question having 5 possible answers. A student randomly guesses the answer to every question. The probability of her getting 20 correct is

A. $^{33}C_{20}\,(0.2)^{20}\,(0.8)^{13}$

B. $^{33}C_{5}\,(0.2)^{20}$

C. $\dfrac{20}{33}$

D. $^{33}C_{20}\,(0.2)^{13}\,(0.8)^{20}$

E. $^{33}C_{20}\,(0.2)^{5}$

Question 26

There are b identical black socks and n identical navy socks in a drawer. Two socks are taken from the drawer at random in the dark. The probability of obtaining a pair is

A. $\dfrac{b(b-1)}{(b+n)}$

B. $\dfrac{(b-n)}{(b+n)}$

C. $\dfrac{b}{(b+n)}$

D. $\dfrac{b(b-1)+n(n-1)}{(b+n)(b+n-1)}$

E. $\dfrac{(b^2+n^2)}{(b+n)^2}$

Question 27

The height (H) of trees in a plantation is known to be normally distributed with a mean of 6 metres. If $\Pr(H > 6.5) = 0.05$ then the standard deviation of the distribution is closest to

A. 0.092

B. 0.304

C. 0.526

D. 0.962

E. 3.290

PART II

SHORT ANSWER QUESTIONS (23 marks)

Question 1

Find exact solutions for $4\cos^2(x) + 4\sin(x) = 1$ given that $0 < x < 2\pi$.

3 marks

Question 2

Let $f: R\backslash\{-1\} \rightarrow R$, where $f(x) = \dfrac{x^2 + 2x + 2}{(x+1)^2}$.

a. Express $f(x)$ in the form $\dfrac{A}{(x+1)^2} + B$, where A and B are positive integers.

b. Hence, if $f(x)$ is dilated a factor of 2 from the x–axis and then translated 1 unit to the right, write down the equation for this new function $f_1(x)$.

c. State the range of $f_1(x)$.

2 + 1 + 1 = 4 marks

Question 3

The curve with equation $y = x^3 + bx^2 + cx + d$ has a stationary point at (1, 2) and a y–intercept of 1.

a. Find b, c and d.

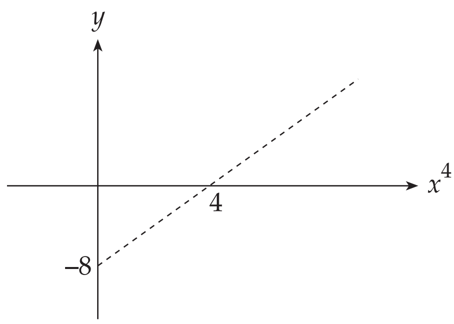
b. Write y in the form $A(x - B)^3 + C$, where A, B and C are positive integers.

c. Hence, show that the x–intercept is $\sqrt[3]{-2} + 1$.

3 + 1 + 1 = 5 marks

Question 4

An experiment was conducted to find the relationship between two variables x and y. The graph of y against x^4 is plotted below and was found to be linear.



a. Find a rule for y in terms of x.

b. Factorise or use another method to find exact solutions to $y = 0$.

c. Find the exact area bounded by the quartic and the x–axis.

2 + 2 + 2 = 6 marks

Question 5

For a discrete random variable Y, the probability function is defined by $f(Y) = \frac{Y}{10}$. Complete the distribution table and find $E(Y)$ and hence find the standard deviation of Y.

Y	0	1	2	3	4
$f(Y)$					

3 marks

Question 6

The probability that a person dies from a certain disease is 0.4. What is the probability correct to 4 decimal places that out of 10 randomly selected patients, at least 3 will die as a result of the disease?

2 marks