Year 2002

VCE

Mathematical Methods

Trial Examination 2

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STUDENT NUMBER									Letter
Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2002

MATHEMATICAL METHODS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*
4	4

Directions to students

Materials
Question and answer book of 9 pages.
Working space is provided throughout the book.
There is a detachable sheet of miscellaneous formula supplied.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the book during reading time.
Please ensure that your **student number** is written in the space provided on the front cover of this book.
Answer **all** questions
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 55 marks available for the examination.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
All written responses should be in English.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax\ dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax\ dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 1

A manufacturer of electronic equipment has fixed costs of $500 per day. Production costs are 25 cents for each item manufactured, while miscellaneous additional costs come to $10x^2$ cents per day where x is the number of electronic items produced per day.

i. What is the total cost of production per day in cents?

1 mark

ii. What is the daily cost of production ,C , per item in cents?

1 mark

iii How many electronic items should be produced each day so that the daily production cost per unit is minimised? Give your answer to the nearest whole number.

3 marks

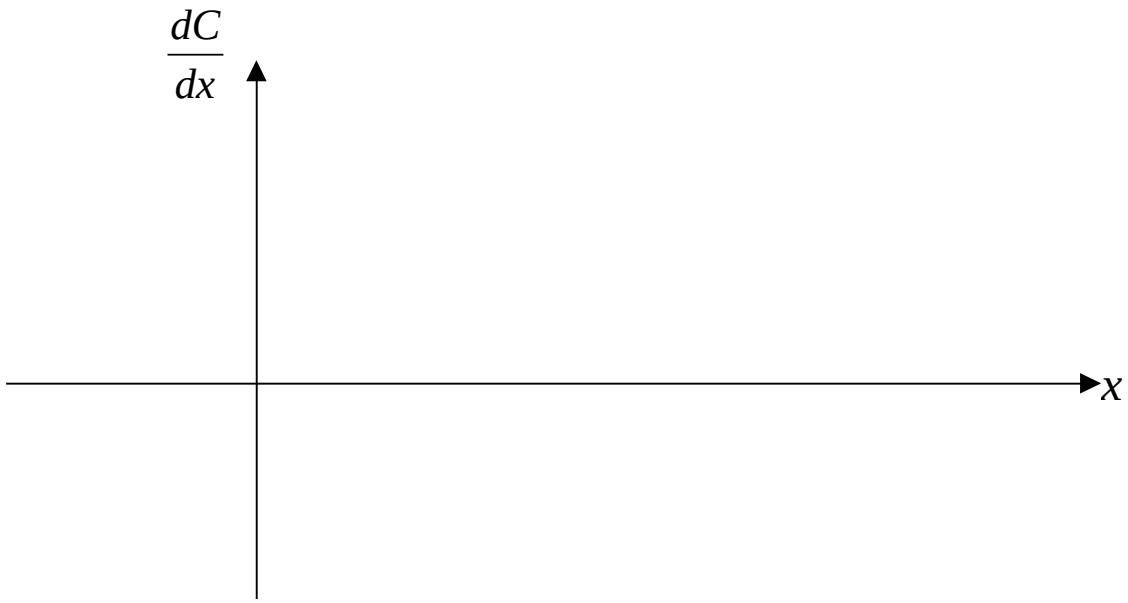
Question 1 (continued)

iv What is the domain of $\frac{dC}{dx}$?

_____

1 mark

v. Sketch the graph of $\frac{dC}{dx}$ versus x on the axes below, labeling the asymptotes and any intersections with the axes.



4 marks

(Total = 10 marks)

Question 2.

A parabola with equation $y = Ax^2$, where A is a constant, has the line $y - 20x = 20$ as a tangent.

a. What is the gradient of the tangent line?

_____

_____

1 mark

b. Find the value of A.

_____

_____

_____

_____

_____

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_____

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_____

3 marks

c. Find the coordinates of the point of tangency.

_____

_____

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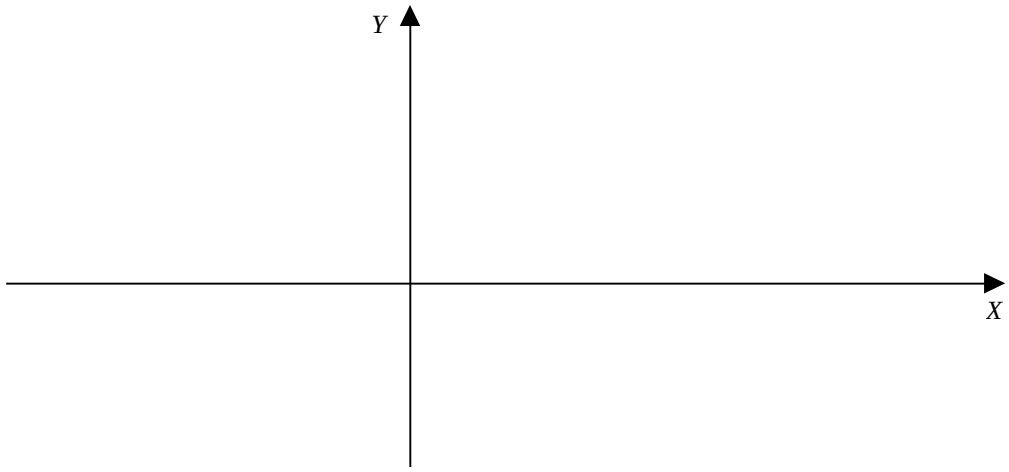
2 marks

Question 2 (continued)

d. Sketch the parabola and the tangent on the set of axes below, showing all intercepts with the axes and the point of contact.



5 marks

e. (i) On the graph above, shade the area bounded by the tangent, the parabola and the *Y* axis.

1 mark

e. (ii) Find this shaded area.

3 marks

f. Another straight line with a gradient of 2 cuts the tangent line at the point where $x = -3$. What is the equation of this line?

1 mark

(Total = 16 marks)

Question 3

A box contains ten tennis balls, seven of which have never been used. For the first set of tennis, two balls are selected at random from the box.

a. What is the probability that exactly one of the balls selected for the first set of tennis has never been used? Give your answer to three decimal places.

2 marks

b. Both balls used for the first set of tennis had never been used and after the first set of tennis, the two balls are returned to the box**.** Emily selects two balls from the box for the second set of tennis. What is the probability that both of these balls have never been used? Give your answer to three decimal places.

2 marks

c. Another box contains seven tennis balls, three of which have never been used. If two balls are taken out at random, to be used in the next set of tennis, what is the expected number of balls that have never been used in the sample? Give your answer to three decimal places.

2 marks

d. From a box containing six tennis balls, four of which have never been used, Harry removes a ball, checks if it has been used or not and then returns it to the box. He does this three times altogether. What is the probability that he will withdraw exactly two balls that have never been used? Give your answer to three decimal places.

2 marks

Question 3 (continued)

e. (i) When Laura and Yan play a set of tennis, Yan has an 80% chance of winning the set. If they play three sets what is the probability that Laura will win at least one set? Give your answer to two decimal places.

3 marks

e. (ii) What is the expected number of sets that Yan will win if Laura and Yan play ten sets?

1 mark

f. (i) The Longlife tennis ball manufacturing company claims that their tennis balls have a lifetime of 30 hours with a standard deviation of two hours. What is the probability that a randomly selected ball will have a lifetime greater than 27 hours?
Give your answer to four decimal places.

2 marks

Question 3 (continued)

f. (ii) If the probability that at least one ball, from a box containing a certain number of these balls, will have a lifetime greater than 30 hours is 0.9844, how many balls are in the box?

_____

_____

_____

_____

_____

_____

_____

_____

2 marks

(Total = 16 marks)

Question 4.

A cup of hot coffee at temperature $T\ ^0$C loses heat when placed in a cooler environment.
The equation that models this situation is

$T = T_0 + Ae^{kt}$ where A and k are constants, T_0 is the temperature of the environment in degrees celsius and t is the time that has elapsed in minutes.

If a cup of coffee at 100 ^{0}C is placed in an environment at 20 ^{0}C and it cools to 70 ^{0}C in 10 minutes,

a. (i) What is the value of T when $t = 0$?

_____

1 mark

a. (ii) Hence, show that $A = 80$.

_____

_____

_____

2 marks

a. (iii) What is the value of T when $t = 10$?

_____

1 mark

a. (iv) Hence, or otherwise, show that the value of the constant, k , to two decimal places, is – 0.05.

_____

_____

_____

_____

_____

_____

_____

_____

3 marks

Question 4 (continued)

b. (i) If the change in temperature $(T - T_0)$ for the same cup of coffee in the same environment equals $\theta\,^0$C, find $\frac{d\theta}{dt}$

2 marks

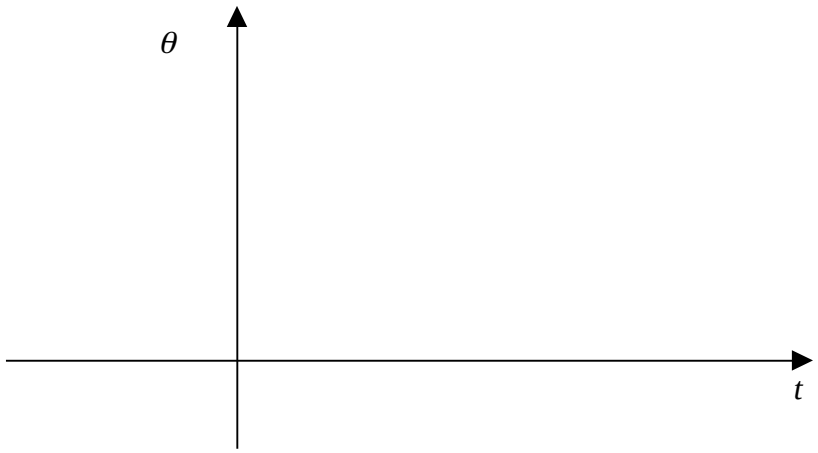
b. (ii) Show that the rate of change of θ is negative for all values of t.

1 mark

b. (iii) What is the rate of change of θ when $t = 10$? Give your answer to three decimal places.

1 mark

c. Sketch the graph of θ against t, showing any intercepts with the axes.



2 marks

(Total = 13 marks)

END OF QUESTION AND ANSWER BOOK
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