Question 1

i.

Total Cost = $50{,}000 + 25x + 10x^2$ (1 mark)

ii.

$$C = \frac{50{,}000 + 25x + 10x^2}{x}$$

(1 mark)

iii.

$$C = 50{,}000x^{-1} + 25 + 10x$$

$$\frac{dC}{dx} = -50{,}000x^{-2} + 10 = 0 \text{ for turning point}$$

$$\frac{50{,}000}{x^2} = 10$$ (1 mark)

$$10x^2 = 50{,}000$$

$$x^2 = 5{,}000$$

$x = 70.7$ (1 mark)

If $x < 70.7, \frac{dC}{dx} < 0$

If $x > 70.7, \frac{dC}{dx} > 0$ (1 mark)

$\therefore$ minimum cost when $x = 70.7$

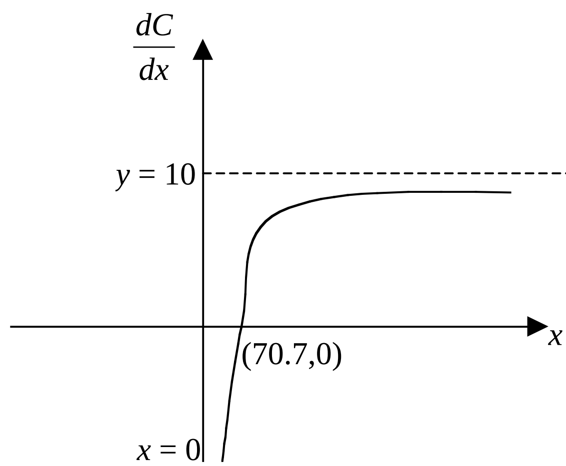
$x = 71$ to nearest whole number.

iv.

$x > 0$

(1 mark)

v.



- 1 mark for shape
- 1 mark for x intercept
- 1 mark for labeling vertical asymptote
- 1 mark for labeling horizontal asymptote

Question 2

a.

$y = 20x + 20$

Gradient of tangent $=$ 20

(1 mark)

b.

$y = Ax^2$

$y = 20x + 20$

At point of tangency

$Ax^2 = 20x + 20$ (1 mark)

$Ax^2 - 20x - 20 = 0$

At point of tangency, only one solution

$b^2 - 4ac = 0$ (1 mark)

$400 + 80A = 0$

$80A = -400$

$A = -5$ (1 mark)

c.

$y = -5x^2$

$y = 20x + 20$

$-5x^2 = 20x + 20$

$5x^2 + 20x + 20 = 0$

$x^2 + 4x + 4 = 0$

$(x + 2)^2 = 0$

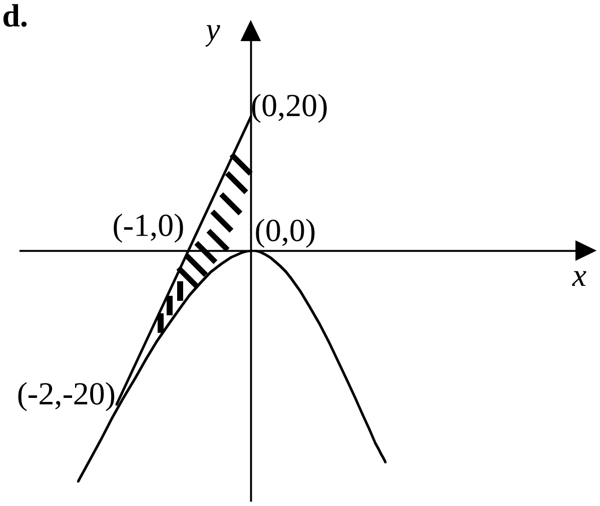
$x = -2$ (1 mark)

When $x = -2, y = -5 \times -2^2$

$y = -20$ (1 mark)

(-2,-20)

d.



$y = 20x + 20$

X intercept when $y = 0$

$20x + 20 = 0$

$20x = -20$

$x = -1$

- 1 mark for parabola positioned correctly
- 1 mark for line
- 1 mark for point (0,20)
- 1 mark for point (-1,0)
- 1 mark for point (-2,-20)

Question 2

e.(i)
1 mark for shading correct region in graph on page 2.

e.(ii)

$A = \int_{-2}^{0} (20x + 20 + 5x^2)dx$ (1 mark)

$A = 10x^2 + 20x + \frac{5x^3}{3}]_{-2}^{0}$ (1 mark)

$A = 0 - [40 - 40 - \frac{40}{3}]$

$A = 13\frac{1}{3}$ sq units. (1 mark)

f.

$y = 2x + c$

$y = 20x + 20$

When $x = -3, y = -40$

$-40 = 2 \times -3 + c$

$-40 = -6 + c$

$c = -34$

$\therefore y = 2x - 34$

(1 mark)

Question 3

a.

$\Pr = \frac{\binom{7}{1}\binom{3}{1}}{\binom{10}{2}}$ (1 mark)

$\Pr = 0.467$ (1 mark)

b.

$\Pr = \frac{\binom{5}{2}\binom{5}{0}}{\binom{10}{2}}$ (1 mark)

$\Pr = 0.222$ (1 mark)

c.

$\text{Mean} = n \times \frac{D}{N}$

$n = 2, N = 7, D = 3$ (1 mark)

$\text{Mean} = 2 \times \frac{3}{7} = 0.857$ (1 mark)

d.

$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$n = 3, x = 2, p = \frac{2}{3}$ (1 mark)

$\Pr(X = 2) = \binom{3}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^1$

$\Pr(X = 2) = 0.444$ (1 mark)

Question 3 (continued)

e. (i)

Pr. Laura wins a set = $\frac{1}{5}$

Pr. Laura wins at least one set = 1 – Pr Laura wins 0 sets (1 mark)

Pr. Laura wins at least one set $=1-\binom{3}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^3$ (1 mark)

Pr. Laura wins at least one set $= 1 - \frac{64}{125}$

Pr. Laura wins at least one set $= \frac{61}{125} = 0.49$ (1 mark)

e.(ii)

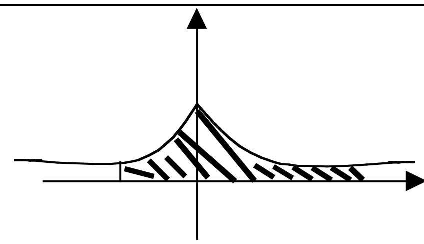
Binomial

Mean = np

Mean = $10 \times \frac{4}{5}$

Mean = 8 (1 mark)

f.(i)



$\Pr(X > 27) = 1 - \Pr(X < 27)$

$= 1 - \Pr(Z < \frac{27-30}{2})$ (1 mark)

$= 1 - \Pr(Z < -1.5)$

$= 1 - \Pr(Z > 1.5)$

$= 1 - [1 - \Pr(Z < 1.5)]$

$= \Pr(Z < 1.5)$

$= 0.9332$ (1 mark)

f.(ii)

$1 - \Pr(X=0) = 0.9844$

$\Pr(X=0) = 0.0156$

Let there be n balls in the box

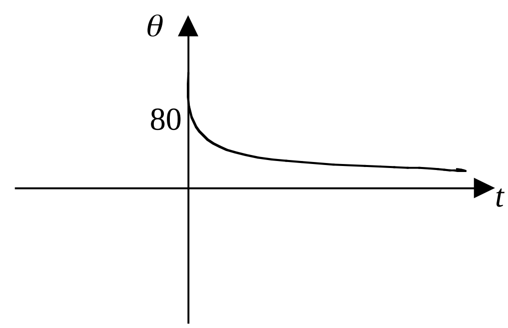
$(0.5)^n = 0.0156$ (1 mark)

$n\log_{10}(0.5) = \log_{10}0.0156$

$n = \frac{\log_{10} 0.0156}{\log_{10} 0.5}$

$n = 6$ (1 mark)

Question 4

a.(i) 100 ^{0}C (1 mark)	**a.(ii)** $100 = 20 + A \times e^{0}$ (1 mark) $100 = 20 + A$ $A = 80$ (1 mark)
a.(iii) From the information given, $T = 70\ ^{0}$C when $t = 10$ (1 mark)	**a.(iv)** $T = T_0 + Ae^{kt}$ $70 = 20 + 80e^{10k}$ $50 = 80e^{10k}$ $0.625 = e^{10k}$ (1 mark) $10k = \log_e 0.625$ (1 mark) $10k = -0.47$ $k = -0.047$ $k = -0.05$ to two decimal places (1 mark)
b.(i) $T - T_0 = Ae^{kt}$ $\theta = 80e^{-0.05t}$ (1 mark) $\dfrac{d\theta}{dt} = -4e^{-0.05t}$ (1 mark)	**b.(ii)** $\dfrac{d\theta}{dt} = -4e^{-0.05t}$ For all values of $t, e^{-0.5t} > 0$ $\therefore -4 \times e^{-0.5t} < 0$ for all values of t (1 mark)
b.(iii) When $t = 10$ $\dfrac{d\theta}{dt} = -4e^{-0.5}$ $= -2.426\ ^{0}$C/ min (1 mark)	**b.(iv)** $\theta = 80e^{-0.05t}$  • 1 mark for shape • 1 mark for point (0,80)

END OF SUGGESTED SOLUTIONS
2002 Mathematical Methods Trial Examination 2

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