MAM 61 2002 EXAM 2 HEFFERNAN

Question 1

At Mario's café, sugar is provided to customers in sachets, which contain about one teaspoon of sugar. Over many years Mario has observed that of every ten customers, four would use no sachets of sugar in their coffee, three would use one sachet, two would use 2 sachets and 1 would use three sachets.

a. Complete the probability distribution table, which describes the number of sachets of sugar (X) a customer would use in their coffee according to Mario. (1)

X	0	1	2	3
$\Pr(X = x)$				

b. Find how many sugar sachets Mario would expect to be used by the next 100 customers who buy a coffee. (2)

c. Find the probability that for the next three customers at Mario's who buy a coffee,

i. each uses one sachet of sugar. (1)

ii. at least one uses no sachets of sugar. (2)

iii. between the 3 of them a total of 2 sachets of sugar are used. (3)

d. The weight of the sachets of sugar is normally distributed with a mean of 5.2grams and a standard deviation of 0.1gram.

i. What is the probability that the next customer at Mario's who uses a sachet of sugar uses one that weighs less than 5 grams? Express your answer correct to 4 decimal places. (1)

ii. Sue comes in for a coffee at Mario's. Using your answer to part i., find the probability that Sue will choose exactly one sachet of sugar that weighs less than 5 grams? Express your answer correct to 4 decimal places. (3)

Question 2

Consider the function g where $g(x)=\dfrac{1}{x-a}+a$ and $a\geq 2$.

a. **i.** Write down the x and y intercepts of the graph of $y=g(x)$. (2)

ii. Hence explain why the least value that the x-intercept can have is $\dfrac{3}{2}$. (1)

iii. Sketch the graph of $y=g(x)$ showing clearly any asymptotes. (2)

iv. Write down the maximal domain of g. (1)

b. State with reasons whether $g(x)$ is a one-to-one or a many-to-one function. (1)

c. On the set of axes below, sketch the graph of $y=g^{-1}(x)$. (2)

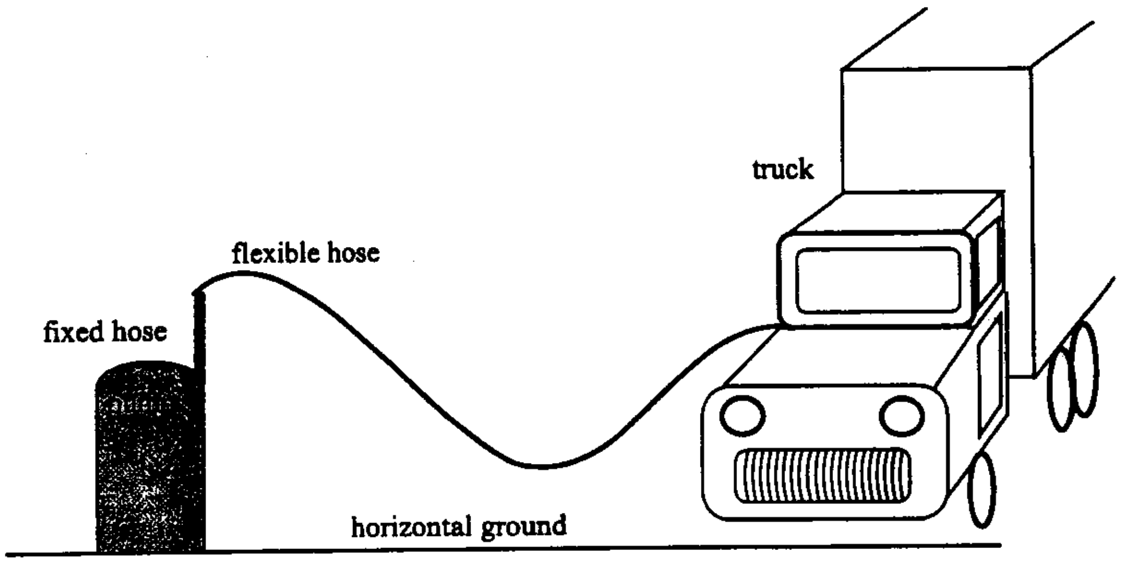
d. Find the rule for $g^{-1}(x)$. (1)

e. **i.** Evaluate $\int_{2a}^{3a} g(x)\,dx$. (2)

ii. Hence show that if $\int_{2a}^{3a} g(x)\,dx=\log_e 2e^5$ then $a=\sqrt{5}$. (2)

Question 3

At a service station a truck is being refueled at a pump.

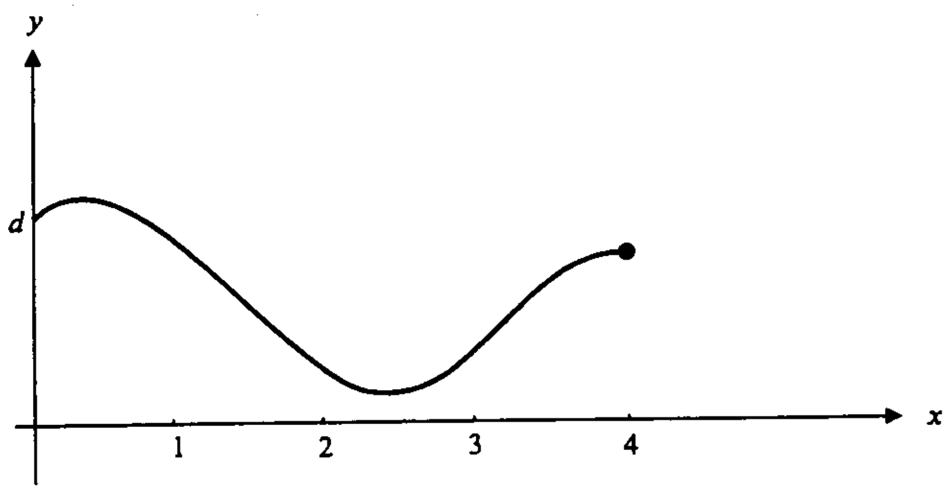


The flexible part of the hose can be modelled by the function $f(x)$ where

$$f(x)=\begin{cases}\dfrac{x^3}{2}-2x^2+1\cdot 125x+2.125, & x\in[0,3]\\ 1+\log_e(x-2), & x\in[3,4]\end{cases}$$

All units are in metres.

The graph of $y=f(x)$ is shown below.



a. Find the value of d. (1)

b. Use calculus to find the minimum distance of the hose from the ground. Express your answer correct to 1 decimal place. (3)

c. Write down the values of x (correct to 1 decimal place where applicable) for which the rate of change of the function $f(x)$ with respect to x is positive. (2)

d. Find the average rate of change of the function over the interval $x \in [3, 4]$. Express your answer as an exact value. (2)

e. By sketching the graph of $y = \frac{1}{x-2}$, for $x > 2$, find the steepest slope of the hose over the interval $x \in [3.5, 4]$. (4)

Question 4

Maree places a piece of corrugated plastic sheeting over timber supports to provide some shelter for her dog. A cross-sectional view with the x-axis representing the timber supports and the y-axis representing one edge of the sheeting is shown in Figure A below.
All units of measurement are in centimetres.

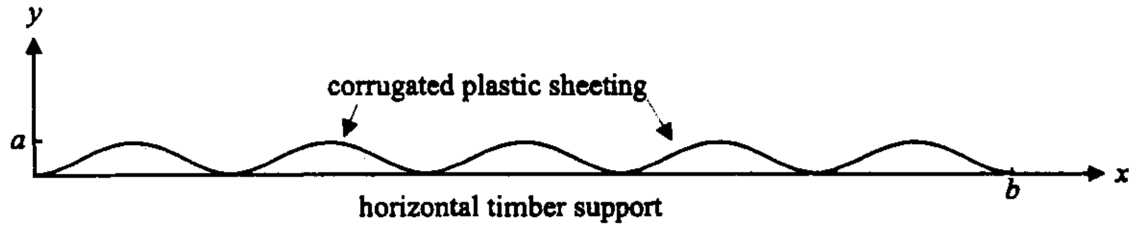


Figure A

In relation to the axes shown, the outline of the corrugated plastic sheeting can be modelled by the function.

$$y = 2\sin\frac{\pi}{6}(x-3)+2, \qquad 0 \le x \le b$$

a. Find a, the maximum height of the sheeting above the horizontal timber support. (1)

b. Find b, the width of the piece of corrugated plastic sheeting. (2)

c. Find the solution(s) to the equation (2)

$$2\sin\frac{\pi}{6}(x-3)+2 = 1, \qquad 6 \le x \le 18$$

Maree uses rust resistant nails, which have caps on their tops in order to secure the corrugated plastic sheeting to the timber. The shaft of the nail enters the timber at the point where the timber makes contact with the corrugated plastic sheeting. Because of the curved nature of the cap on the top of the nail, the sides of the cap come into contact with the corrugated plastic sheeting at points C and D as shown in Figure B below.

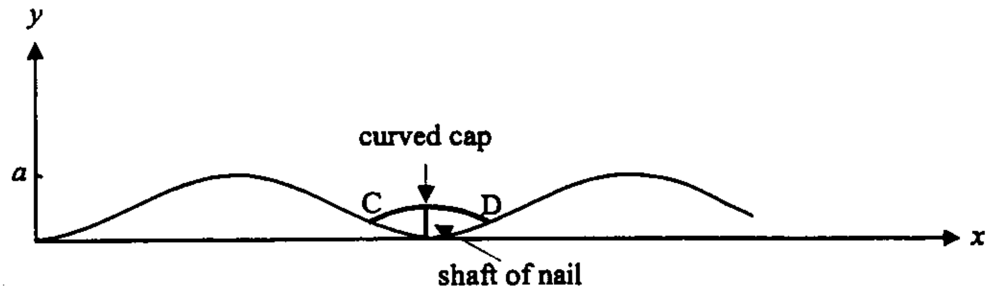


Figure B

d. If the horizontal distance from C to D is 4cm, find the coordinates of points C and D. (2)

The shape of the cap of the nail described in part **d.** can be modeled by the function

$$y = \frac{-(x-12)^2}{4} + 2, \qquad x \in [m, n]$$

e. Find m and n. (1)

f. **i.** Find the derivative of $\cos\frac{\pi}{6}(x-3)$. (1)

ii. Hence show that $\int 2\sin\frac{\pi}{6}(x-3)\,dx = \frac{-12}{\pi}\cos\frac{\pi}{6}(x-3) + c$ where c is a constant. (2)

g. Use the results from part e. and part **f. ii**, to find the area between the cap of the nail and the corrugated plastic sheeting below it as indicated by the shaded area in Figure C below. Express your answer correct to 1 decimal place. (5)

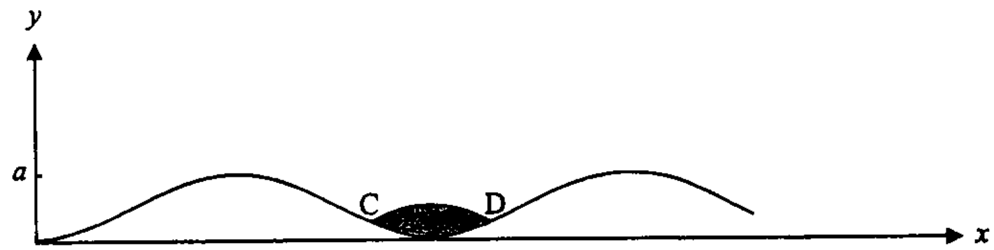


Figure C