SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter
Words										

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2001

MATHEMATICAL METHODS

Written examination 2

(Analysis Task)

Monday 12 November 2001

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

Materials
- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided on the front of this book.
- All written responses must be in English.

Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown.

The instruction **use calculus** requires students to show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The temperature, T degrees Celsius, in a greenhouse at t hours after midnight for a typical November day is modelled by the formula

$$T = 25 - 4\cos\frac{\pi(t-3)}{12}, \quad \text{for } 0 \le t \le 24.$$

Use this model to answer the following questions.

a. State the maximum and the minimum temperatures in the greenhouse.

2 marks

b. At what time does the maximum temperature occur?

1 mark

c. At what times will the temperature equal 23ºC?

2 marks

d. Consider the hottest four-hour interval in the greenhouse for a typical November day. What is the minimum temperature in this four-hour interval?

Give your answer correct to two decimal places.

2 marks

e. In the greenhouse, there is an automatic switch for a root watering system. Whenever the **rate of change** of temperature, with respect to time, is greater than or equal to +0.2ºC per hour, the system switches on. It switches off again when the rate of change of temperature, with respect to time, is less than +0.2ºC per hour.

i. Use calculus to find an expression for the rate of change of temperature with respect to time.

1 mark

ii. Hence find the range of values of t (correct to two decimal places) for which the watering system will be on.

3 marks

Total 11 marks

TURN OVER

Question 2

The Candlelite Company produces scented candles. These candles are sold in boxes of 25, of which 10 candles are jasmine-scented, 8 are musk-scented and 7 are sandalwood-scented.

Jo, Kim and Lee each buy a box of candles.

a. Jo opens her box of 25 candles and takes out four candles at random and lights them. What is the expected number of jasmine-scented candles she takes out?

2 marks

b. Kim opens her box of 25 candles and takes out three candles at random and lights them. Find, correct to three decimal places, the probability that Kim takes out one candle of each type of scent.

2 marks

c. Lee opens his box of 25 candles and takes out four jasmine-scented candles and one musk-scented candle and lights them. He then takes out another candle from the box at random. Find the probability that it will be a sandalwood-scented candle.

1 mark

The Candlelite Company claims that 90% of all the scented candles, regardless of the scent, will burn for at least 30 hours.

d. Assuming that this claim is correct

i. find the probability, correct to three decimal places, that of the 25 scented candles in a box, 24 or more will burn for at least 30 hours.

3 marks

ii. what is the expected number of scented candles in a box of 25 that will burn for more than 30 hours?

1 mark

The scented candles have a burning time which is normally distributed with mean of 32 hours and standard deviation of 1.5 hours.

e. Find, correct to three decimal places, the proportion of these scented candles that will burn for more than 30 hours.

2 marks

f. Lee lights n scented candles. The probability that at least one of these n candles will burn for longer than 32 hours is 0.9375.

Find the value of n.

2 marks

The Candlelite Company can add a chemical to the candles to adjust μ, the mean burning time of the candles. The burning time of the candles, with the chemical added, is also normally distributed with standard deviation 1.5 hours.

g. Find the least value of μ, correct to two decimal places, so that 97% of scented candles produced, with the chemical added, burn for 30 hours or more.

3 marks

Total 16 marks

Working space

Question 3

Let f be the function $f\colon D \to R$, $f(x) = \log_e(3 - 4x)$, where D is the largest possible domain over which f is defined.

a. Find the exact coordinates of the intercepts of the graph of $y = f(x)$ with the x- and y-axes.

2 marks

b. Find D, the largest possible domain over which f is defined.

1 mark

c. Use calculus to show that the rate of change of $f(x)$ with respect to x is always negative.

2 marks

f^{-1} is the inverse of f.

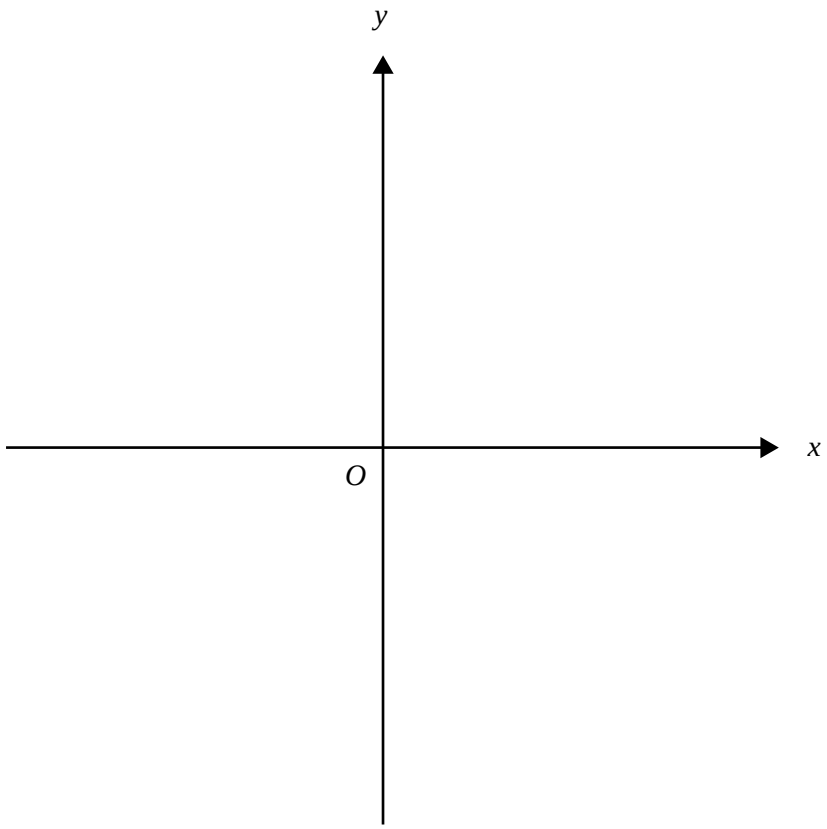
d. **i.** Find $f^{-1}(x)$, the rule for f^{-1}.

2 marks

ii. State the domain of f^{-1}.

1 mark

e. On the axes below, sketch the graph of $y = f^{-1}(x)$. Clearly label any intercept with the x- or y-axis with its exact value. Label any vertical or horizontal asymptote with its equation.



2 marks

f. **i.** Find the derivative of $\frac{1}{4}(3-4x)\log_e(3-4x)$.

2 marks

ii. **Hence** find the exact value of the area bounded by the graph of $y = f(x)$, the x-axis and the y-axis.

3 marks

Total 15 marks

Working space

Question 4

Experiment 1

A colony of viruses is grown in a laboratory by putting a number of the viruses in a dish of nutrient. At first the viruses multiply quickly, but later the numbers decline, approaching a long-term stable population.

The number of viruses in the dish, N million, at time t days may be modelled by the formula

$$N(t) = \frac{1}{2(t+1)^2} - e^{-6t} + 1, \quad t \geq 0.$$

Use this model to answer the following questions.

a. **i.** How many viruses were put in the dish initially?

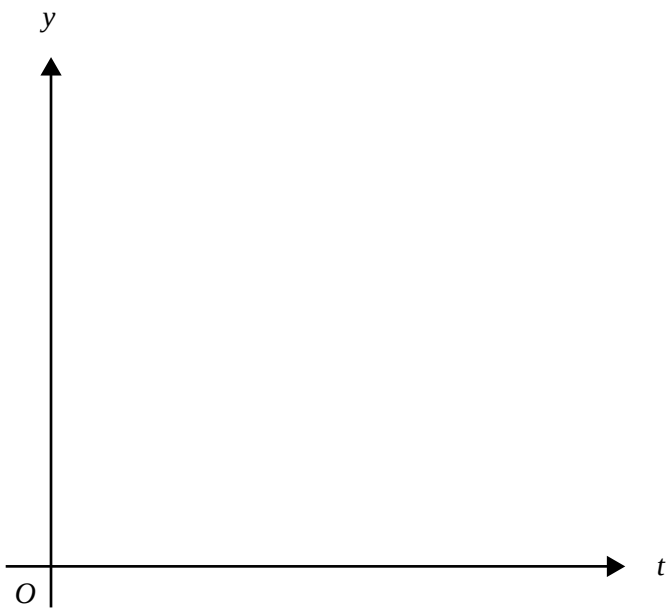
1 mark

ii. What value does $N(t)$ approach as time increases?

1 mark

iii. On the axes below, sketch the graph of $y = N(t)$. Clearly label any t- and y-intercepts. Label any asymptote with its equation.

(You are not required to find the coordinates of any turning point.)



2 marks

b. **i.** Find $\frac{dN}{dt}$

2 marks

ii. Find the number of days after the experiment started, correct to two decimal places, when the maximum number of viruses is present in the dish. Also find this maximum number, correct to the nearest thousand.

2 marks

Experiment 2

A new experiment is started in which 10 million viruses are placed in a dish and a virus-killing drug is then added to the dish. Five days later, there are no viruses present.

In order to find an equation to use as a model for the number of viruses, V million, present after x days, $0 \leq x \leq 5$, a cubic polynomial model is proposed. The graph of the cubic polynomial, $y = V(x)$, must have stationary points at (0, 10) and (5, 0).

c. Find $V(x)$

5 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e(x) + c, \text{for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$ $\qquad$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$ $\qquad$ variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2 p(x) = \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\, {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed N(0, 1).		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET