Mathematical Methods (CAS) pilot study: supplementary questions – multiple choice

Question 1

The largest set of real values of t for which $|t + 6| > 6$ is

A. $\{t \in R: \ t > 0 \text{ or } t < -12\}$

B. $\{t \in R: \ t > 0\}$

C. $\{t \in R: \ t < -12\}$

D. $\{t \in R: \ t > -12\}$

E. $\{t \in R: \ t > 0 \text{ or } t < -6\}$

Question 2

If $f(x) = (x + 2)^4$ then $f(3x + 2)$ is equal to

A. $3f(x) + 2$

B. $f(3x) + 2$

C. $(3x + 2)^4$

D. $(3x + 4)^4$

E. $81x^4 + 8$

Question 3

The three points whose coordinates are (-1, 0), (0, 1) and (1, 0) are transformed by a dilation from the x-axis by a factor of three followed by a vertical translation of 2 units down and a horizontal translation of 3 units to the left. The respective coordinates of the images of these three points after these transformations are

A. $(-6,-2)$, $(-3, -1)$, $(0, -2)$

B. $(-4,-2)$, $(-3,1)$, $(-2, -2)$

C. $(0, -2)$, $(3, -1)$, $(6, -2)$

D. $(-4,2)$, $(-3,5)$, $(-2,2)$

E. $(2, -2)$, $(3,1)$, $(4, -2)$

Question 4

Let $y(x) = \dfrac{f(x)}{e^x}$, where f is a real valued differentiable function. The gradient of the graph of y will be positive:

A. for all values of x in the domain of f

B. only for positive values of x in the domain of f

C. when $f(x) > 0$

D. when $f'(x) > f(x)$

E. when $f(x) > f'(x)$

Question 5

Let $y = \log_e(x^2)$. At $x = -1$, $\dfrac{dy}{dx}$ has the value:

A. –2

B. –1

C. 0

D. 1

E. 2

Question 6

Let $f(x) = e^{x^2}$ and $g(x) = \sin(x^2)$ where f and g are real valued functions. If $h(x) = f(x)\, g(x)$ then $h'(x)$ is:

A. $2xe^{x^2}(\sin(x^2) + \cos(x^2))$

B. $2xe^{x^2}$

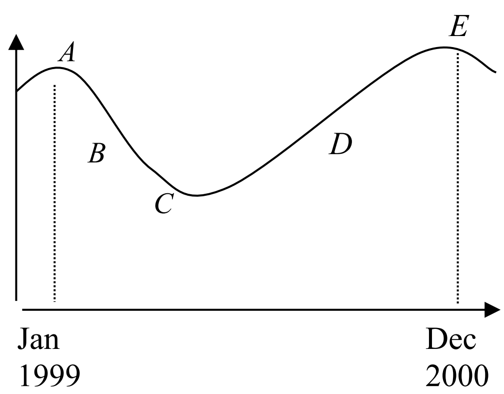
C. $4xe^{x^2}x\cos(x^2)$

D. $e^{2x}\sin(2x)$

E. $2e^{2x}\sin(x)(\sin(x) + \cos(x))$

Question 7

The graph below shows the trend for daily price of shares of a particular company over a 2 year period from 1999 to 2000.



The labelled point on the graph above at which the daily price of shares in the company was increasing most quickly is

A. *A*

B. *B*

C. *C*

D. *D*

E. *E*

Question 8

A plant grows and increases in height with a variable rate of growth. The height, H, in cm, of the plant is given by $H = f(t)$, $t \geq 0$ where f is a differentiable real-valued function of t, the time in days since the plant began to grow. The height of the plant will be increasing most quickly when

A. $\dfrac{dH}{dt} = 0$

B. H is a maximum

C. $\dfrac{dH}{dt}$ is a maximum

D. H is a local minimum

E. $\dfrac{dH}{dt}$ is positive

Question 9

The function f is a polynomial function of degree 4. The derivative function of f has the following properties:

$f'(-1) = 0$ $\quad\quad$ $f'(x) > 0$ for $\{x: x < -3\}$

$f'(-3) = 0$ $\quad\quad$ $f'(x) < 0$ for $\{x: -3 < x < -1\} \cup \{x: x > -1\}$

The graph of $y = f(x)$ has

A. a stationary point of inflection at $x = -1$ and a maximum at $x = -3$.

B. a stationary point of inflection at $x = -1$ and a minimum at $x = -3$.

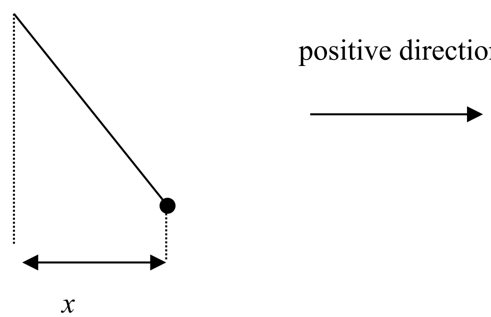
C. a local maximum at $x = -1$ and a local minimum at $x = -3$.

D. a local minimum at $x = -1$ and a local maximum at $x = -3$.

E. a minimum at $x = -1$ and a stationary point of inflection at $x = -3$.

Question 10

A pendulum is swinging such that its horizontal displacement x centimetres from a fixed vertical position at time t seconds, where $t \geq 0$, is given by $x(t) = 5e^{\frac{-t}{10}}\sin(2\pi t)$



Which one of the following statements about the pendulum's initial motion is **correct**?

A. It moves in the negative direction at 0.5 centimetres per second.

B. It moves in the negative direction at π centimetres per second.

C. It moves in the positive direction at $\dfrac{5}{2\pi}$ centimetres per second.

D. It moves in the positive direction at 5 centimetres per second.

E. It moves in the positive direction at 10π centimetres per second.

Question 11

The relative rate of change of $f(x)$ is defined as $\dfrac{f'(x)}{f(x)}$. The relative rate of change of $f(x) = xe^x$ is equal to

A. x

B. $e^x(x+1)$

C. xe^x

D. $\dfrac{x}{x+1}$

E. $\dfrac{x+1}{x}$

Question 12

Economists study the effects on the demand for a product brought about by a change in the price of a product. This is achieved by using the price elasticity of demand function $E(p)$ defined by

$$E(p) = \frac{-pD'(p)}{D(p)}$$

where p is the price of the product in dollars and $D(p)$ is the demand for the product at price p.

If the demand for a product, $D(p)$, is given by $D(p) = \sqrt{200-p}$ where $50 < p < 200$, then the price elasticity of demand, $E(p)$, for $50 < p < 200$ is equal to

A. $\dfrac{-1}{2(200-p)}$

B. $\dfrac{p}{2(200-p)}$

C. $\dfrac{-p}{2(200-p)}$

D. $-p$

E. $\dfrac{-1}{2\sqrt{200-p}}$

Question 13

A train leaves station *A* and takes 2 minutes to travel in a straight line to station *B*. The velocity, *v*, in kms per minute, of the train at time *t* minutes on this journey is given by $v(t) = \sin^2(\frac{\pi t}{2})$. The distance between station *A* and station *B* is

A. 1 km

B. $\frac{2}{\pi}$ km

C. 2 km

D. π km

E. 4 km

Question 14

Consider the probability distribution for the discrete random variable, X, shown in the table below.

x	0	1	2	3
$\Pr(X = x)$	$\frac{1}{2}k^3$	$\frac{1}{2}k^3$	$\frac{3}{4} - 3k^2$	$1 - \frac{1}{4}k$

The value of *k* is

A. 3

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 1

Question 15

If $f(x) = e^{-\frac{x}{m}}$, where $0 < x < m$, and $f(x) = 0$ elsewhere, is a probability density function, then m is equal to

A. $-e^{-1}$

B. $(1 - e^{-1})^{-1}$

C. $1 + e^{-1}$

D. $1 - e^{-1}$

E. -1

Question 16

If a random variable X has probability density function

$$f(x) = 2(1 - x) \text{ for } 0 \le x \le 1 \text{ and } f(x) = 0 \text{ elsewhere}$$

then the median of X is

A. $\dfrac{\sqrt{2} + 2}{2}$

B. 1

C. 0.5

D. 0.701

E. $\dfrac{-\sqrt{2} + 2}{2}$

Question 17

A canteen serves coffee and tea. It is found that 10% of customers who have tea on a particular day choose coffee the next and 60% of customers who choose coffee on a particular day choose tea on the next. It is found that 1000 people use the canteen each day and they all have tea or coffee but not both. On a Monday 500 people have tea and 500 people have coffee. How many people have each drink on the following Wednesday?

A. 275 coffee and 725 tea
B. 800 tea and 200 coffee
C. 360 tea and 640 coffee
D. 825 tea and 175 coffee
E. 640 tea and 360 coffee

Question 18

If a random variable X has probability density function

$f(x) = xe^{-\frac{x}{k}}$ for $0 \le x \le k$ and $f(x) = 0$ elsewhere, then the value of k is

A. $\sqrt{\dfrac{e}{e-2}}$
B. 1
C. e
D. $\sqrt{e}$
E. 2

For **Question 19** and **Question 20,** the proportion of people who respond to a certain mail order catalogue is a continuous random variable X that has the probability density function

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Question 19

The probability, correct to four decimal places, that more than one quarter but fewer than one half of the people contacted will respond to this mail order catalogue is

A. 0.0896

B. 0.1000

C. 0.2375

D. 0.3875

E. 0.4500

Question 20

The mean of X, correct to four decimal places, is

A. 1.0000

B. 0.3667

C. 0.4000

D. 0.5000

E. 0.5333

Mathematical Methods (CAS) pilot study: supplementary questions – multiple choice

Questions 21, 22 and 23 are based on the information in Question 26 of Mathematical Methods Exam 1A, 2000

Andrea throws a netball towards a goal ring. If the ball passes through the ring, she scores a goal. Andrea knows that on average she scores a goal 17 times out of every 20 throws. The result of each throw is independent of the previous throw. Andrea throws the netball 10 times towards a goal ring.

Question 21 (Q26, Mathematical Methods Exam 1A, 2000)

The probability of obtaining more than 8 goals is

A. ${}^{10}C_9(0.15)^1(0.85)^9$

B. ${}^{10}C_9(0.15)^1(0.85)^9 + (0.85)^{10}$

C. ${}^{10}C_8(0.15)^2(0.85)^8 + {}^{10}C_9(0.15)^1(0.85)^9 + (0.85)^{10}$

D. $(0.85)^{10}$

E. $\dfrac{{}^{17}C_8 \times {}^{3}C_2}{{}^{20}C_{10}}$

Question 22

The probability that the first goal she shoots is on her 4th throw is

A. ${}^{4}C_1(0.15)^1(0.85)^3$

B. ${}^{4}C_1(0.15)^3(0.85)^1$

C. $(0.15)^2(0.85)^1$

D. $(0.15)^3(0.85)^1$

E. $(0.15)^4(0.85)^1$

Question 23

The probability that she will have scored her first goal before her fourth attempt is

A. $(0.15)^2(0.85)^1$

B. $(0.15)^1(0.85)^1 + (0.15)^2(0.85)^1$

C. $(0.85)^1 + (0.15)^1(0.85)^1 + (0.15)^2(0.85)^1$

D. $(0.85)^1 + (0.15)^1(0.85)^1 + (0.15)^2(0.85)^1 + (0.15)^3(0.85)^1$

E. ${}^{3}C_1(0.15)^2(0.85)^1$

Question 24

The number of components in a batch that survive a given shock test is a random variable X which has a binomial distribution with mean 15 and standard deviation 3. The probability of a component surviving the given shock test is

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{2}$

D. $\frac{3}{5}$

E. $\frac{4}{5}$

Question 25

The number of components in a batch that survive a given shock test is a random variable X which has a binomial distribution with mean 15 and standard deviation 3. The probability that the sixth component tested will be the first component tested to survive the shock test is

A. ${}^{6}C_{5}\left(\frac{4}{5}\right)^{5}\left(\frac{1}{5}\right)$

B. $\left(\frac{4}{5}\right)^{5}\left(\frac{1}{5}\right)$

C. ${}^{6}C_{5}\left(\frac{3}{5}\right)^{5}\left(\frac{2}{5}\right)$

D. $\left(\frac{3}{5}\right)^{5}\left(\frac{2}{5}\right)$

E. $\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right)$