Mathematical Methods (CAS) pilot study: supplementary questions – multiple choice solutions and comments

Question	Answer	Comments
1	**A**	Solve $t + 6 > 6$ and $t + 6 < -6$ to give $t > 0$ or $t < -12$ respectively.
2	**D**	$f(3x + 2) = ((3x + 2) + 2)^4 = (3x + 4)^4$
3	**B**	(–1,0), (0,1) and (1,0) are transformed to (–1,0), (0,3) and (1,0) respectively by the dilation of a factor 3 from the *x*-axis, then to (–1,–2), (0,1) and (1,–2) respectively by a vertical translation of two units down, and finally to (–4,–2), (–3,1) and (–2,–2) respectively by a horizontal translation of 3 units to the left.
4	**D**	The gradient of the graph of *y* will be positive when $y'(x) = \frac{d}{dx}\left(\frac{f(x)}{e^x}\right) > 0$. $\frac{d}{dx}\left(\frac{f(x)}{e^x}\right) = \frac{e^x f'(x) - e^x f(x)}{e^{2x}}$ $= \frac{f'(x) - f(x)}{e^x}$ Since $e^x > 0$ for all x, so $\frac{d}{dx}\left(\frac{f(x)}{e^x}\right) > 0$ whenever $f'(x) > f(x)$.
5	**A**	$\frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x}$, $x \neq 0$. At $x = -1$, $\frac{dy}{dx} = -2$.
6	**A**	Product rule for differentiation, followed by some re-expression.
7	**D**	The price of the shares was increasing most quickly at the point where the gradient has its largest positive value. This is at *D*.

8	**C**	As the plant is growing at a variable rate of growth, the height will be increasing most quickly when the gradient $\frac{dH}{dt}$ is a maximum.
9	**A**	Since $f'(-1) = 0$ and $f'(x) < 0$ when $x > -1$ or when $-3 < x < -1$, so the graph has a stationary point of inflection at $x = -1$. Since $f'(-3) = 0$ and $f'(x) > 0$ when $x < -3$ and $f'(x) < 0$ when $-3 < x < -1$, so the graph has a local maximum at $x = -3$. Since f is a polynomial function of degree 4, this local maximum must also be a maximum.
10	**E**	$x'(t) = 5e^{\frac{-t}{10}}(\frac{-t}{10}\sin(2\pi t) + 2\pi\cos(2\pi t))$ $x'(0) = 10\pi$, so the pendulum initially moves in the positive direction at 10π centimetres per second. Hence E is correct.
11	**E**	If $f(x) = xe^x$ then $f'(x) = xe^x + e^x = e^x(x+1)$ so $\frac{f'(x)}{f(x)} = \frac{xe^x + e^x}{xe^x} = \frac{x+1}{x}$.
12	**B**	$D(p) = \sqrt{200-p}$, so $D'(p) = \frac{-1}{2\sqrt{200-p}}$ so $E(p) = \frac{-pD'(p)}{D(p)} = \frac{p}{2(200-p)}$
13	**A**	Distance = $\int_0^2 \sin^2(\frac{\pi t}{2})dt = 1$

14	**C**	Since the sum of probabilities must equal 1, $k^3 - 3k^2 - \frac{k}{4} + \frac{7}{4} = 1$. Solutions are $k = -\frac{1}{2}, \ \frac{1}{2}, \ 3$. If $k = 3$, P($X = 0$) >1, an impossibility. If $k = -\frac{1}{2}$, then P($X = 0$) <0, also an impossibility. If $k = \frac{1}{2}$, then all probabilities in the table will lie between 0 and 1, as required for a probability distribution.
15	**B**	For this to be a probability density function, we must have $\int_0^m e^{-\frac{x}{m}} dx = 1$. As $\int_0^m e^{-\frac{x}{m}} dx = m(1 - e^{-1})$ which requires that $m = (1 - e^{-1})^{-1}$
16	**E**	If M is the median of X, then $\int_0^M 2(1-x)dx = 0.5$, and hence M could be $\frac{2 \pm \sqrt{2}}{2}$. But $0 \le x \le 1$ and $\frac{2+\sqrt{2}}{2} > 1$, so $M = \frac{2-\sqrt{2}}{2}$.
17	**D**	This is a Markov chain application, since what happens on a day depends only on what happened the previous day(alternatively a tree diagram could be used). Transition matrix $T = \begin{bmatrix} .90 & .60 \\ .10 & .40 \end{bmatrix}$ and initial state matrix $S_0 = \begin{bmatrix} 500 \\ 500 \end{bmatrix}$ On Wednesday the state matrix is $\begin{bmatrix} .90 & .60 \\ .10 & .40 \end{bmatrix}^2 \begin{bmatrix} 500 \\ 500 \end{bmatrix} = \begin{bmatrix} 825 \\ 175 \end{bmatrix}$

18	**A**	As f is a probability density function, $\int_0^k xe^{-\frac{k}{m}}dx = 1$. Since $\int_0^k xe^{-\frac{k}{m}}dx = k^2(1-\frac{2}{e})$, and $k > 0$, $k = \sqrt{\frac{e}{e-2}}$
19	**C**	$\Pr(0.25 < X < 0.5) = \int_{0.25}^{0.5} \frac{2(x+2)}{5}dx = 0.2375$.
20	**E**	$\mu = \mathrm{E}(X) = \int_0^1 \frac{2x(x+2)}{5}dx = 0.5333$
21	**B**	X = the number of goals in 10 throws, $X \sim \mathrm{Bi}\,(10, 0.85)$. $\Pr(X > 8) = \Pr(X = 9) + \Pr(X = 10)$
22	**D**	Pr(First three throws miss, fourth throw is a goal) $= 0.15 \times 0.15 \times 0.15 \times 0.85$
23	**C**	If she has scored her first goal before her fourth attempt, then she must have scored on either her first, second or third attempt.
24	**B**	$np = 15$, $\sqrt{np(1-p)} = 3$, where n is the number of components in a batch and p is the probability of a component surviving the shock test. Hence, $15(1-p) = 9$, so $(1-p) = \frac{3}{5}$, and $p = \frac{2}{5}$.
25	**D**	$np = 15$, $\sqrt{np(1-p)} = 3$, where n is the number of components in a batch and p is the probability of a component surviving the shock test. From question 26, $p = \frac{2}{5}$. If the sixth component tested is the first to survive the shock test, then the first five components tested must have failed the shock test, hence required probability is $\left(\frac{3}{5}\right)^5 \times \frac{2}{5}$.