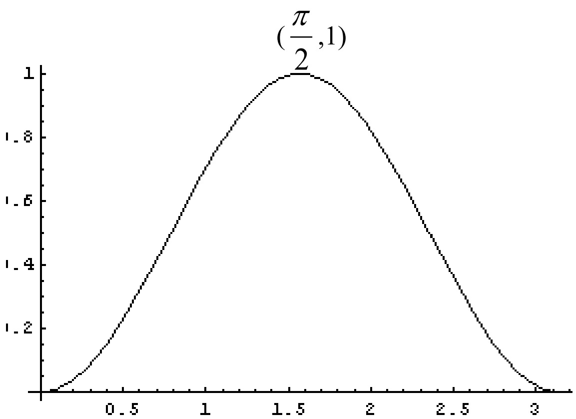
Solution	**Comments**
7. a.  3 marks	Marks awarded for clear identification of maximum and endpoints, and correct shape.
b. Area = $\int_0^{\pi} \sin^2(x)\, dx = \frac{\pi}{2}$. Therefore $a = \frac{2}{\pi}$. 2 marks	Suitable definite integral expression (including terminals) awarded a method mark. CAS should be used to evaluate the integral.
c. $x = \frac{\pi}{2}$ is an axis of symmetry for the graph of $y = \sin^2(x)$. 1 mark	

d. For the mean the following integral may be evaluated or symmetry used: $\frac{2}{\pi}\int_0^{\pi} x\sin^2(x)\,dx = \frac{\pi}{2}$ $E(X^2) = \frac{2}{\pi}\int_0^{\pi} x^2\sin^2(x)\,dx$ Therefore variance = $E(X^2) - [E(X)]^2$ $= \frac{2\pi^2-3}{6} - (\frac{\pi}{2})^2$ $= \frac{\pi^2-6}{12}$. The interval $[\mu - 2\sigma, \mu + 2\sigma]$ $= [\frac{\pi}{2} - 2\sqrt{\frac{\pi^2-6}{12}}\ ,\ \frac{\pi}{2} + 2\sqrt{\frac{\pi^2-6}{12}}\ ]$ = [0.435, 2.707], correct to3 decimal places. 6 marks	Awarded a mark each for the mean, the integral to calculate $E(X^2)$, the variance and the correct interval. Alternatively the variance can be determined by using $E[(X-\mu)^2]$. CAS should be used, the mean and variance can be entered in the CAS as constants, and used in subsequent calculations.

e. $\Pr(\mu - k < X < \mu + k) = 0.95$ $\Pr(X < \mu) = 0.5$. Therefore by symmetry $\Pr(\mu < X < \mu + k) = 0.475$ $\int_{\frac{\pi}{2}}^{k+\frac{\pi}{2}} f(x)dx = 0.475$ $\frac{2k + \sin(2k)}{2\pi} = 0.475$ $k = 1.072$ correct to three decimal places. 3 marks	A method mark awarded for getting $\Pr(\mu < X < \mu + k) = 0.475$ and a mark awarded for evaluating the integral. CAS should be used.
Total: 15 marks	
8. a. i. $\frac{dy}{dx} = 2(x-1)$ for the parabola When $x = a$, $\frac{dy}{dx} = 2(a-1)$. 2 marks	A method mark is awarded for correct $\frac{dy}{dx}$.
a. ii. $m = 2(a-1)$. 1 mark	

b. $(a, (a-1)^2)$. 2 marks	A mark awarded for each coordinate.
c. i. $y-(a-1)^2 = 2(a-1)(x-a)$ $y = 2(a-1)x - a^2 + 1$. 2 marks	A mark awarded for working with correct form for straight line.
c. ii. $2(a-1)x = a^2 - 1$ $x = \frac{1}{2}(a+1)$ as $0 < a < 1$. 2 marks	
d. i. $\int_0^{\frac{a+1}{2}} (2(a-1)x - a^2 + 1)dx$. 1 mark	
d. ii. Area = $\frac{-(a+1)(a^2-1)}{4}$. 1 mark	

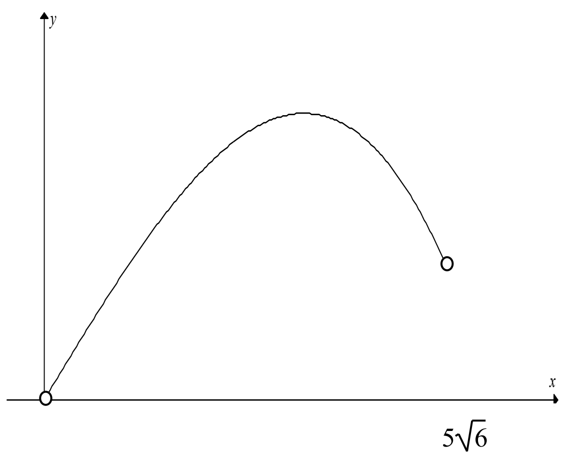
e. Area is a maximum when $a = \frac{1}{3}$. Equation of tangent is $y = \frac{-4}{3}x + \frac{8}{9}$. 3 marks	A method mark awarded for stating a suitable relationship between area and equation of tangent.
Total: 14 marks	
9. a. Let X be the random variable with values for the times of telephone usage in minutes. Pr ($X > 200$) = 0.3085 correct to four decimal places. 2 marks	A method mark would be awarded for recognition of use of the normal distribution, with correct parameters or correct transformation to standard normal form. This may be done using transformation and tables, integration and numerical equation solving, or by using a built in inverse normal function.
b. Let Y be the random variable which gives the number of customers out of the three whose usage exceeds 200 minutes. $\Pr(Y \geq 1) = 1 - \Pr(Y = 0) = (0.6915)^3 = 0.3306$ correct to four decimal places. 3 marks	A method mark would be awarded for recognition of the binomial distribution, with appropriate parameters. Suitable notation indicating use of built in function for computation is also acceptable. A mark would be awarded for 1 – Pr ($Y = 0$) or equivalent statement.

c. Let $T = \begin{bmatrix} .8 & a \\ .2 & 1-a \end{bmatrix}$, $S = \begin{bmatrix} .2 \\ .8 \end{bmatrix}$ solve $TS = S$ for a to obtain $a = \frac{1}{20}$. 2 marks	A mark would be awarded for identification of both the appropriate transition matrix and initial matrix.
d. $T = \begin{bmatrix} .8 & .1 \\ .2 & .9 \end{bmatrix}$, $S = \begin{bmatrix} .2 \\ .8 \end{bmatrix}$ $TS = \begin{bmatrix} 0.24 \\ 0.76 \end{bmatrix}$ Expected marked share is 24% . 2 marks	
Total: 9 marks	
10. a. $f(x) - g(x) = bx - ax^2 - (\frac{4}{20-x})$ $= \frac{-ax^3 + (20a + b)x^2 - 20bx + 4}{x-20}$ $h(x) = -ax^3 + (20a + b)x^2 - 20bx + 4.$ 2 marks	CAS can be used to advantage in this question. Write the difference with common denominator and then collect like terms in the numerator. Method mark awarded for attempt at obtaining common denominator.

b. i. $h(\frac{1}{4})= 0$ implies $\frac{79a}{64} - \frac{79b}{16} + 4 = 0$ and $h(\frac{39}{2}) = 0$. Implies $\frac{1521a}{8} - \frac{39b}{4} + 4 = 0$. 2 marks	Substitution in $h(x)$ to obtain simultaneous equations would be awarded one method mark.
b. ii. $a = \frac{64}{3081}$ and $b = \frac{2512}{3081}$. 2 marks	
c. The third solution is $\frac{79}{2}$. 2 marks	A method mark awarded for substitution and an attempt to solve the equation.
d. Coordinates are $(\frac{1}{4}, \frac{16}{79})$ and $(\frac{39}{2}, 8)$. 2 marks	
e. i. $\int_{0.25}^{19.5} (f(x) - g(x))dx$. 2 marks	Marks awarded for correct terminals and using $f(x) - g(x)$ or equivalent expression.

e. ii. Area = $4\log_e(2)+\frac{1915991}{18486}-4\log_e(79)$. 1 mark	
e. iii. Area is 88.94 correct to two decimal places. 1 mark	
e. iv. Area of triangle = $\frac{5929}{79}$ = 75.05, correct to 2 decimal places. This area is smaller. 84.3% of the actual shape. 2 marks	A mark awarded for the area and a mark awarded for the comparison.
Total: 16 marks	
11. a. i. Surface area of curved cylindrical surface = $2\pi rh$ and surface are of two hemispheres = $4\pi r^2$. 1 mark	

ii. $2\pi rh = 1000\pi - 4\pi r^2$ $h = \dfrac{500 - 2r^2}{r}$. 2 marks	A method mark awarded for evidence of solving the appropriate equation for h.
b. i. $V = \pi r^2(h - \dfrac{4r}{3})$. 1 mark	
ii. 'Show that' requires substitution of the result from **a. ii.** in **b. i.** 2 marks	Method mark awarded for substitution.
c. i. $\dfrac{dV}{dr} = 500\pi - 10\pi r^2$. 1 mark	
c. ii. $500\pi - 10\pi r^2 = 0$ $r = 5\sqrt{2}$. 1 mark	

c. iii. When $r = 5\sqrt{2}$ $V = \dfrac{5000\pi\sqrt{2}}{3}$ Maximum volume is $\dfrac{5000\pi\sqrt{2}}{3}$ cm^3 . 2 marks	Method mark awarded for correct substitution of student's value for r in result from **b. i.**
d.  3 marks	Marks awarded for correct domain and correct shape.
Total: 13 marks	
12. a. i. $R(z) = z\,Q(z) = z(400 - 2z) = 400z - 2z^2$ 1 mark	

a. ii. $P(z) = R(z) - C(z)$ $= 400z - 2z^2 - (0.2z^2 + 4z + 400)$ $= -2.2z^2 + 396z - 400$. 2 marks	Mark awarded for $R(z) - C(z)$.
b. i. $\dfrac{dP}{dz} = -4.4z + 396$ Therefore maximum occurs when $z = 90$. 2 marks	Method mark awarded for correct derivative or equivalent first step, such as completing the square or use of quadratic formula.
b. ii. Price per unit = $400 - 2 \times 90 = 220$ dollars. 2 marks	Mark for substituting student's value.
b. iii. Maximum profit = \$17 420 . 1 mark	
c. $P(z) = 0$ implies $-2.2z^2 + 396z - 400 = 0$ $z = 178.984\ldots$ or $z = 1.015\ldots.$, therefore a profit is obtained for between 2 and 178 inclusive items being produced. 3 marks	Mark awarded for solutions of quadratic equation.

d. $P(z) = -2.2z^2 + 396z - 400 - 22z$ $= -2.2z^2 + 374z - 400$. $\frac{dP}{dz} = -4.4z + 374$, therefore maximum occurs where $z = 85$. 2 marks	Mark awarded for new profit function. Method mark awarded for correct derivative or equivalent first step, such as completing the square oruse of quadratic formula.
e. i. $S(x) = \frac{x+1}{40} + 10$ and $D(x) = \frac{8000}{x+1}$, so $S(x) = D(x)$ implies that: $x^2 + 402x - 319599 = 0$. Therefore $x_0 = 399$, or $x_0 = -801$ but $x_0 > 0$, so $x_0 = 399$. 2 marks	Mark awarded for obtaining a correct equation.
e. ii. $p_0 = 20$. The price is \$20. 1 mark	

iii. $\int_0^{x_0} D(x)dx - p_0 x_0$ $= \int_0^{399} \frac{8000}{x+1}dx - 399 \times 20$ $= 8000\log_e(400) - 7980$ $= 39\,952$, to the nearest dollar. 3 marks	Mark awarded for correct substitution of students values.
Total: 19 marks	