SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter
Words										

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2001
MATHEMATICAL METHODS (CAS)
PILOT STUDY

Sample written examination 2
(Analysis Task)

For November examination period

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

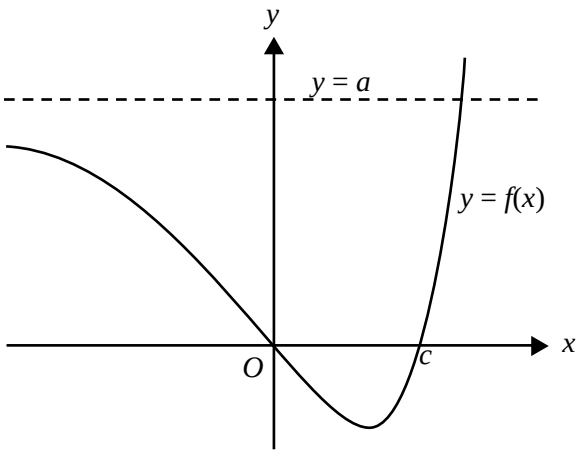
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	57

Materials
- Question and answer book of 13 pages.
- A detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved CAS calculator, ruler, protractor, set-square and aids for curve-sketching.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided on the front of this book.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Question 1

The graph of the function $f: R \rightarrow R, f(x) = e^{2x} - 4e^{x} + 3$ is shown below.



The graph of f has a horizontal asymptote $y = a$

a. Write down the exact value of a.

1 mark

The graph of f passes through the origin and the point $(c, 0)$.

b. Find the exact value of c.

2 marks

c. Find the exact values of the coordinates of the turning point of the graph of f.

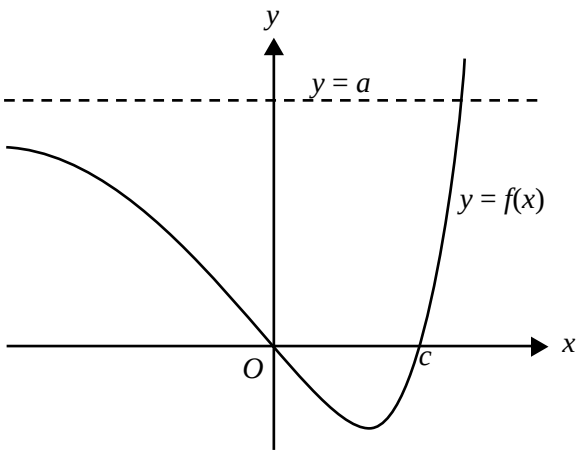
3 marks

d. Write an appropriate definite integral which determines the area of the region bounded by the graph of f and the x-axis and find the exact value of this area.

3 marks

e. Let g be the function whose graph is the reflection of the graph of f in the y-axis.

i. Sketch the graph of g on the axes below.



2 marks

ii. Write down the rule for g.

1 mark

Consider the family of functions $f_k : R \rightarrow R,\ f_k(x) = e^{2x} - 2ke^x + 3$, where k is a real number.

f. **i.** For what values of k will the graph of f_k have a turning point?

3 marks

ii. Find the coordinates of this turning point when it exists.

1 mark

iii. Find the value of k for which the graph of f_k touches the x-axis.

2 marks

Total 18 marks

Working space

Question 2

Pedro fishes in an area where there are large numbers of salmon. It is known that the lengths of salmon in this area are normally distributed with mean 36.0 cm and standard deviation 2.5 cm. Assume that all salmon are equally likely to be caught.

Salmon with lengths in the top 15% of lengths of salmon in the area are called gourmet salmon.

a. What is the minimum length of a gourmet salmon, in centimetres, correct to one decimal place?

2 marks

b. If Pedro catches 20 salmon, what is the probability, correct to three decimal places, that at least one will be a gourmet salmon?

3 marks

c. Pedro knows that if he catches as many salmon as he did last Tuesday, the probability of catching at least one gourmet salmon is approximately 0.68. How many salmon did he catch last Tuesday?

2 marks

Salmon which are shorter than 33 cm are declared to be undersized by the government. If such salmon are caught, they must be returned to the water.

d. What is the probability, correct to three decimal places, that a salmon caught at random will be undersized?

2 marks

e. Pedro's friend Jose returns from fishing with 20 salmon of which 4 are undersized. The government inspector is waiting for him, and takes a random sample of three salmon from Jose's catch of 20 salmon. If more than one of the salmon are undersized, Jose must pay a fine. What is the probability, correct to three decimal places, that Jose will be fined?

3 marks

f. In the same area a particular type of codfish is caught. The weight, measured in kilograms, of an adult codfish is described by the random variable X with probability density function f having rule

$$f(x) = \begin{cases} A\sin\left(\frac{\pi}{6}(x-2)\right) & \text{for } 2 < x < 8 \\ 0 & \text{elsewhere} \end{cases}$$

i. Find the exact value of A.

2 marks

ii. Find the median weight of an adult codfish from the same area.

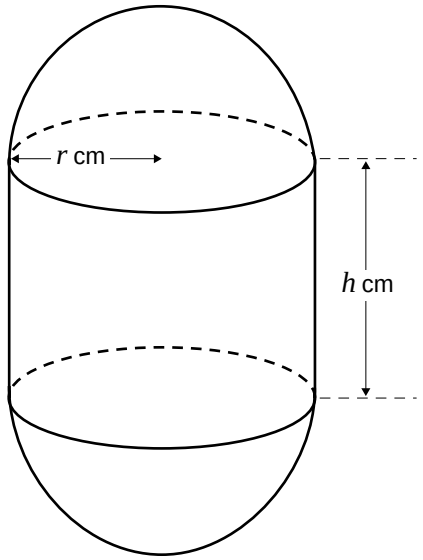
2 marks

Total 16 marks

TURN OVER

Question 3

Victoria Jones wants to construct a time capsule in which to bury some of her treasures. The time capsule will be a right circular cylinder of height *h* cm, and radius *r* cm, with hemispherical caps of radius *r* cm on each end, as shown in the diagram.



Let the total volume of the capsule be V cm^3.

a. Express V in terms of *r* and *h*.

1 mark

b. If the total volume of the capsule is 8000 cm^3, then it can be shown that $h = \frac{8000}{\pi r^2} - \frac{4r}{3}$.

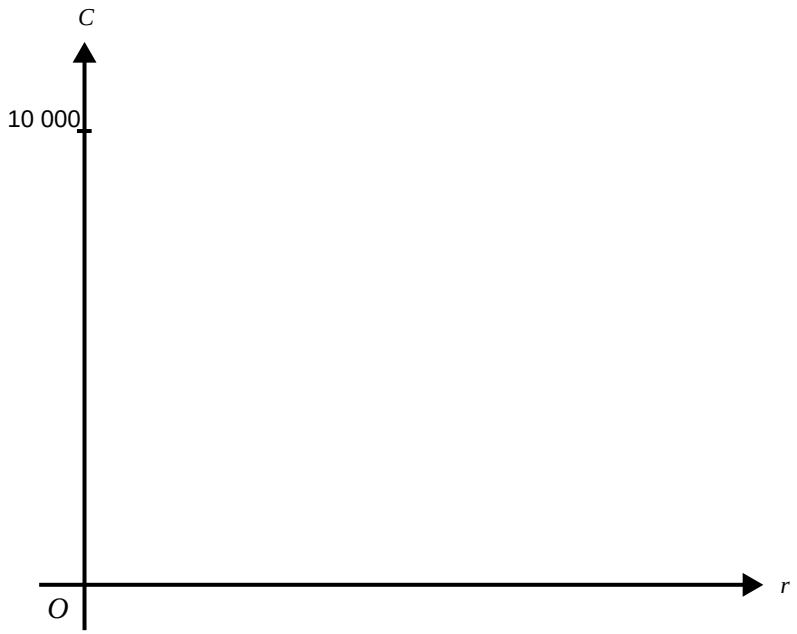
The values which *r* may take lie in an interval. Find the end-points of this interval, correct to two decimal places.

2 marks

c. The material for the cylindrical part of the capsule costs 2 cents per cm^2 of surface. The material for the hemispherical caps costs 3 cents per cm^2 of surface. (The surface area of a sphere of radius r is $4\pi r^2$.)
Find an expression for C cents, the total cost of the materials for the capsule, in terms of r.

2 marks

d. Sketch the graph of C over the appropriate domain on the axes below. Label any horizontal or vertical asymptotes with their equations. (You are not required to show the coordinates of any turning point.)



3 marks

e. **i.** Find $\frac{dC}{dr}$ in terms of r.

1 mark

ii. Hence find the exact value of r for which C is a minimum.
(You do not need to justify that the value you find is a minimum.)

2 marks

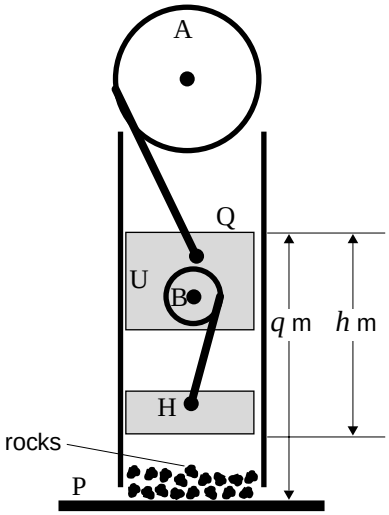
iii. Find the minimum cost to the nearest dollar.

1 mark

Total 12 marks

Question 4

A device for crushing rock is shown in the diagram below. It consists of a steel platform (P) on which the rock is placed and a machine which raises and lowers a heavy 'hammer' (H). The wheel A rotates, causing the upper block U to move up and down. The other wheel B, attached to the block U, rotates independently causing the hammer H to move up and down.



Q is the top of block U. The distance, q m, between Q and the platform P is modelled by the formula

$$q = -2\cos(at) + b,$$

where t is the time in minutes and a and b are constants. When $t = 0$, Q is at its lowest point, 3 m above the platform. Wheel A rotates at a rate of 1 revolution per minute.

a. Show that $a = 2\pi$ and $b = 5$.

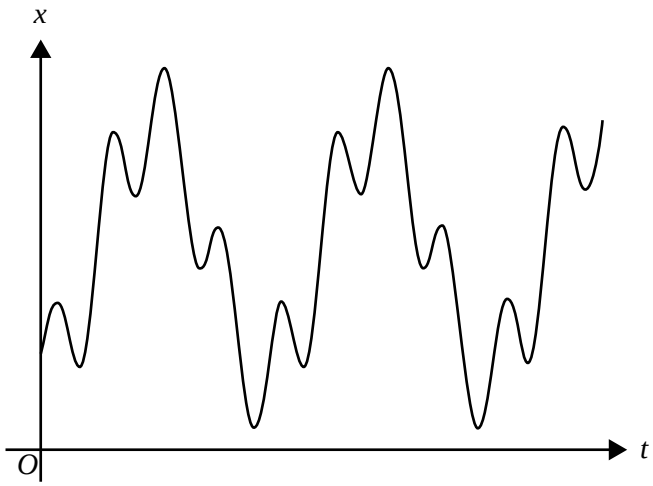
2 marks

Wheel B rotates at 4 revolutions per minute. The distance, h m, between the bottom of the hammer and Q at time t minutes is modelled by the formula $h = -\sin(8\pi t) + 2$.

Let the distance between the bottom of the hammer and the platform at time t minutes be x m.

b. Show that $x = -2\cos(2\pi t) + \sin(8\pi t) + 3$

1 mark



A section of the graph of x as a function of t is shown.

c. **i.** Find $\frac{dx}{dt}$.

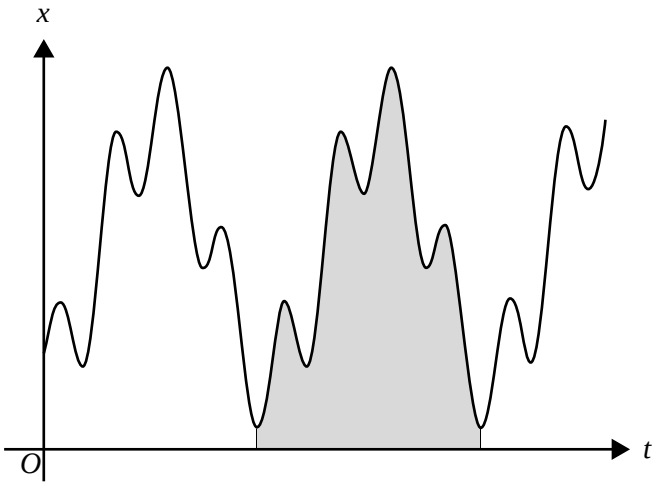
1 mark

ii. Find the exact value of the rate of change of x with respect to t when $t = 2$.

1 mark

d. Find the first time after $t = 0$, correct to the nearest one-hundredth of a minute, when this model predicts that the hammer will be at its least distance from the platform. Find this least distance, correct to the nearest millimetre.

3 marks



e. The width of the shaded region shown is the time taken for one cycle of x. Write down an appropriate definite integral which determines the area of the shaded region and find the exact value of this area.

3 marks

Total 11 marks

END OF QUESTION AND ANSWER BOOK