STUDENT NUMBER

Letter

Figures

Words

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2001

MATHEMATICAL METHODS (CAS) PILOT STUDY

Sample written examination 1
(Facts, skills and applications)

For November examination period

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

PART II

QUESTION AND ANSWER BOOK

Directions to students

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of the Part I question book.

At the end of the examination
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Structure of book

Number of questions	Number of questions to be answered	Number of marks
8	8	23

Materials
- Question and answer book of 10 pages, including two blank pages for rough working.
- Up to four pages (two A4 sheets) of pre-written notes typed or handwritten.
- An approved CAS calculator, ruler, protractor, set-square and aids for curve-sketching.

Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided on the cover of this book.
- A decimal approximation will not be accepted if an exact answer is required to a question.
- Appropriate working should be shown if more than one mark is available.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Working space

Instructions for Part II

Answer **all** questions in the spaces provided.

Question 1

A canteen serves coffee and tea. It is found that 10% of customers who have tea on a particular day choose coffee the next day and 60% of customers who choose coffee on a particular day choose tea on the next day. It is found that 700 people use the canteen each day and they all have tea or coffee but not both. On a particular Monday 350 have tea and 350 have coffee.

a. How many people will have tea and how many will have coffee on the Friday of that week?

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b. In the long term, how many people will have each drink?
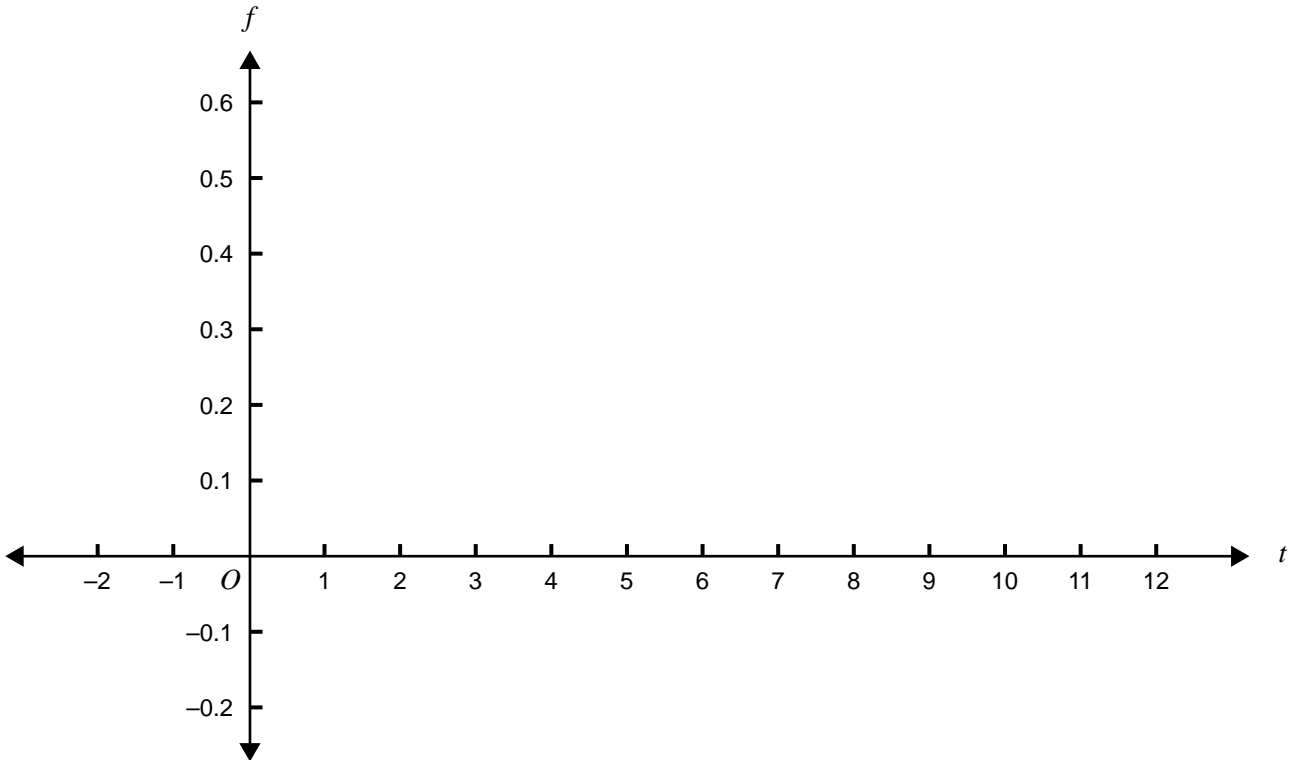
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$2 + 2 = 4$ marks

Question 2

The first symptoms for a certain disease appear between 5 and 10 days after contact. The probability of the first symptoms of this disease appearing at various times, t days after contact, is described by the probability density function

$$f(t) = \begin{cases} \dfrac{6}{125}(t - 5)(10 - t) & \text{for} \quad 5 \le t \le 10 \\ 0 & \text{elsewhere} \end{cases}$$

a. Sketch the graph of f on the set of axes provided below.

b. Find the exact value of the probability that the first symptoms appear between 5 and 7 days after contact.

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$2 + 2 = 4$ marks

TURN OVER

Question 3

Let f and g be functions with domain R where $f(x) = -x^3 + 4x^2 + 9x - 9$ and $g(x) = x + 1$.

a. Write $g(x) - f(x)$ as a product of linear factors over R.

b. Find all values of x for which $g(x) > f(x)$.

$$1 + 1 = 2 \text{ marks}$$
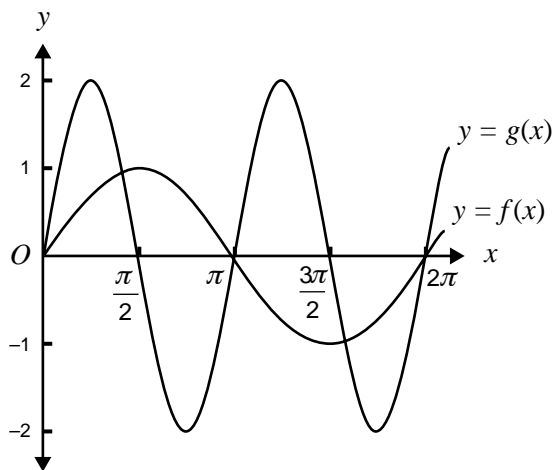
Question 4

The diagram below shows the graphs of two circular functions, f and g.

State the type of transformation, together with any relevant scale factors, distances or directions, required to transform the graph whose equation is $y = f(x)$ into the graph whose equation is $y = g(x)$.

$$2 \text{ marks}$$

Question 5

Consider the family of parabolas that pass through the point $(1, 1)$ with gradient 2 at this point.

a. Find the general rule for this family of parabolas, in the form $y = ax^2 + bx + c$, expressing a and b in terms of c.

b. Find the rule of the particular member of this family of parabolas that also passes through the point $(0,1)$.
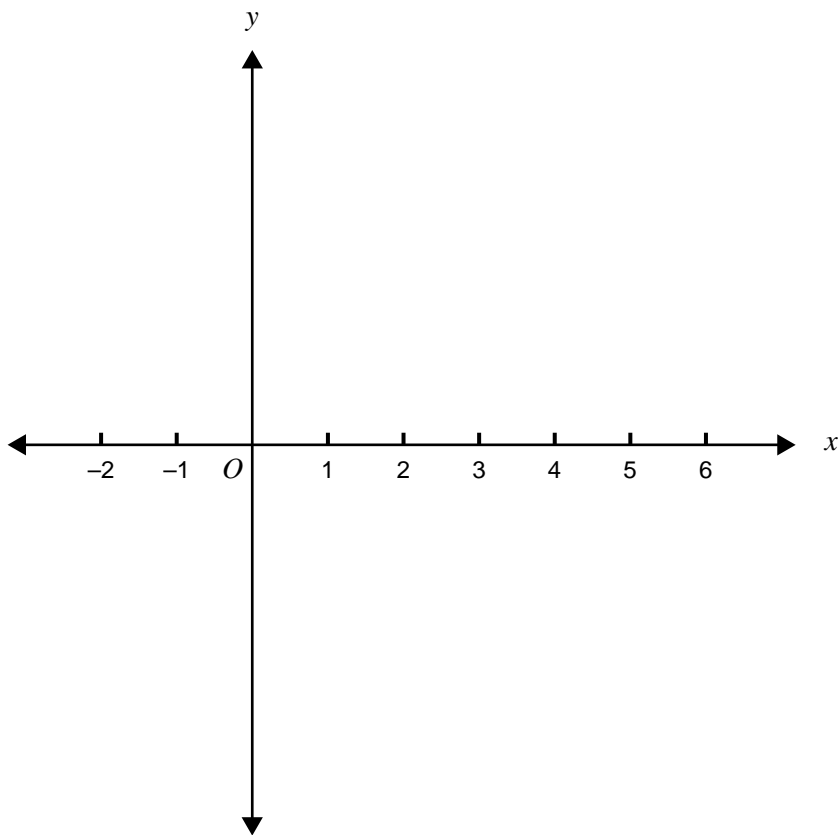
$3 + 1 = 4$ marks

TURN OVER

Question 6

On the set of axes provided below, sketch a continuous curve with equation $y = f(x)$ having the following properties

$f(0) = 0 \qquad f'(0) = 0$

$f(4) = 0 \qquad f'(3) = 0$

$\qquad\qquad f'(x) < 0$ for $\{x : x > 3\}$

$\qquad\qquad f'(x) > 0$ for $\{x : x < 3\}\backslash\{\ 0\ \}$

3 marks

Question 7

Given $f:(0, 200] \rightarrow R, f(x) = (100 - x)\log_{10}(x)$ find

   i.   the maximum value of $f(x)$, correct to three decimal places

   ii. the values of x for which $f(x) = 0$

   iii. the value of $f'(x)$ when $f(x) = 0$ and $x > 1$.

$$1 + 1 + 1 = 3 \text{ marks}$$

Question 8

Let $g(x) = \cos(f(x))$. If $g'(x) = -2x \sin(x^2)$, find the rule for f.

1 mark

TURN OVER

Working space

END OF PART II QUESTION AND ANSWER BOOK