VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2001

MATHEMATICAL METHODS (CAS)
PILOT STUDY

Sample written examination 1
(Facts, skills and applications)

For November examination period

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the examination
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
27	27	27

Materials
- Question book of 15 pages with a detachable sheet of miscellaneous formulas in the centrefold and one blank page for rough working.
- Answer sheet for multiple-choice questions.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved CAS calculator, ruler, protractor, set-square and aids for curve-sketching.
- At least one pencil and an eraser.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Instructions for Part 1

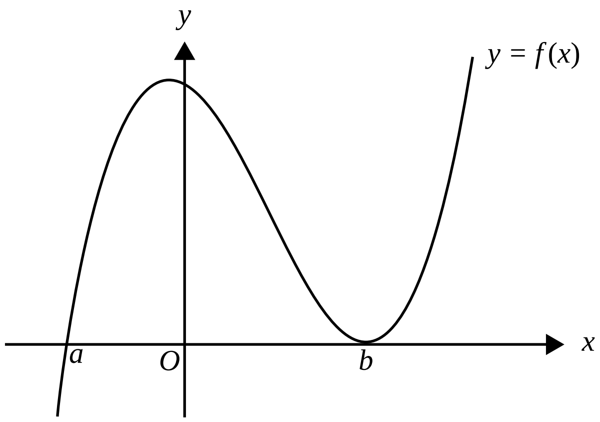
This part consists of 27 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1



The graph shown could be that of a function f whose rule is

A. $f(x) = (x - a)(x - b)^2$

B. $f(x) = (x + a)(x - b)^2$

C. $f(x) = (x - a)(x + b)^2$

D. $f(x) = (x + a)(x + b)^2$

E. $f(x) = (x - a)^2(x - b)$

Question 2

The graph whose equation is $y = \sqrt{x}$ is reflected in the x-axis and then translated 2 units to the right and 1 unit down.

The equation of the new graph is

A. $y = \sqrt{(x - 2)} + 1$

B. $y = -\sqrt{(x - 2)} - 1$

C. $y = -\sqrt{(x + 2)} - 1$

D. $y = -\sqrt{(x - 2)} + 1$

E. $y = \sqrt{(x - 1)} + 2$

Question 3

The equations of the vertical and horizontal asymptotes of the graph of the function with the rule $y = \frac{2}{x-4} + 3$ are, respectively,

A. $x = -4$, $y = 3$
B. $x = 2$, $y = 3$
C. $x = 3$, $y = 4$
D. $x = 4$, $y = -3$
E. $x = 4$, $y = 3$

Question 4

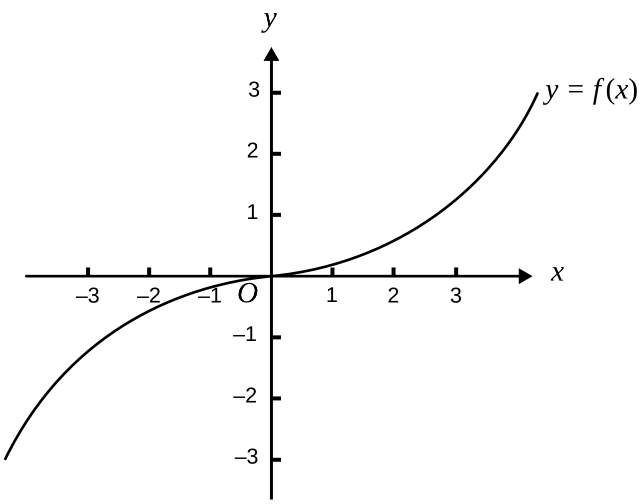
x	*y*
1	1.7
2	3.2
3	1.5
4	0.5
5	1.2
6	2.6
7	3.4
8	2.3

The data in the above table would be best modelled using

A. a linear function.
B. a power function.
C. an exponential function.
D. a circular function.
E. a logarithmic function.

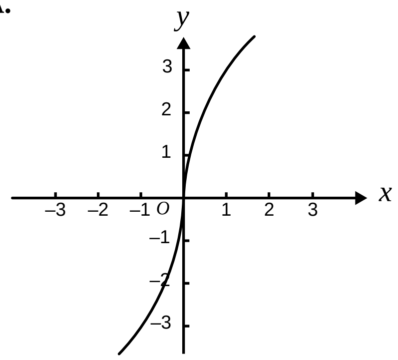
Question 5

The graph of the function with equation $y = f(x)$ is shown below.

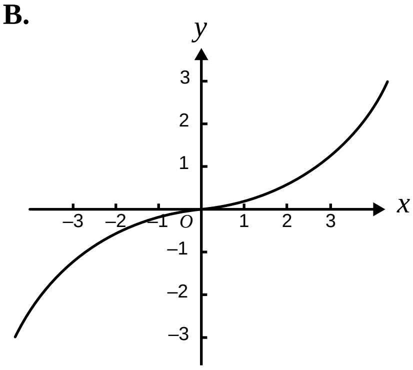


Which one of the following is most likely to be the graph of the inverse function?

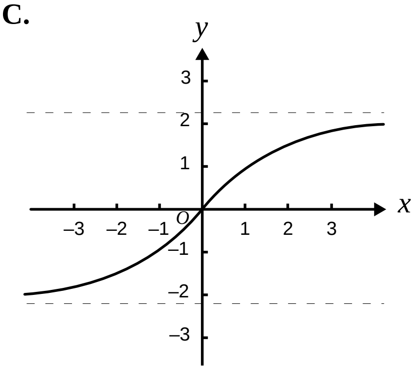
A.



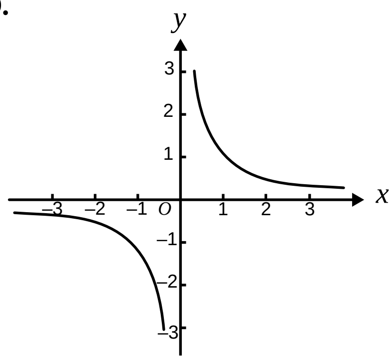
B.



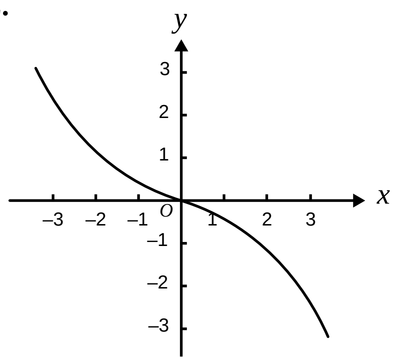
C.



D.



E.



Question 6

Let $f: D_1 \to R,\ f(x) = \dfrac{1}{x+2}$ where D_1 is the maximal domain for f.

Let $g: D_2 \to R,\ g(x) = e^{2x}$ where D_2 is the maximal domain for g.

Let $h: D_3 \to R,\ h(x) = \dfrac{1}{x+2} - e^{2x}$ where D_3 is the maximal domain for h.

Which one of the following is true?

A. $D_1 = D_3$ and Range (f) = Range (h)

B. $D_1 \neq D_3$ and Range (f) = Range (h)

C. $D_2 = D_3$ and Range (g) = Range (h)

D. $D_1 = D_3$ and Range $(g) \neq$ Range (h)

E. $D_1 \neq D_3$ and Range (g) = Range (h)

Question 7

Which one of the following functions does **not** have an inverse function?

A. $f: R \to R,\ f(x) = 2x - 5$

B. $g: [0, \infty) \to R,\ g(x) = x^2$

C. $h: R \to R,\ h(x) = x^3$

D. $k: [-2, 2] \to R,\ k(x) = \sqrt{(4 - x^2)}$

E. $m: R^+ \to R,\ m(x) = 2 - \dfrac{3}{x}$

Question 8

Under the linear transformation of the plane $T: R^2 \to R^2$, defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix},$$

the image of the line with equation $3x + 2y = 1$ is

A. $3x + 4y = 2$

B. $6x + 2y = 1$

C. $3x + 4y = 1$

D. $6x + 2y = 2$

E. $6x + 5y = 1$

Question 9

Let $f : R \to R$, where $f(x) = |\sin(x)|$.

Which one of the following statements is **not** true?

A. $f\left(\frac{3\pi}{2}\right) = 1$

B. $f(-x) = f(x)$

C. $f'(-\frac{\pi}{4}) = -f'(\frac{\pi}{4})$

D. $f'(\pi) = -1$

E. the minimum value of $f(x)$ is 0

Question 10

The decay of radioactive material is described by the formula

$$M = M_0 e^{-kt},$$

where M_0 is the initial mass of material, $k > 0$ is the constant of decay and M is the mass of material remaining after time t years.

The time in years that it takes for the material to decay to half its initial mass is

A. $\dfrac{k}{2}$

B. $k \log_e\left(\frac{1}{2}\right)$

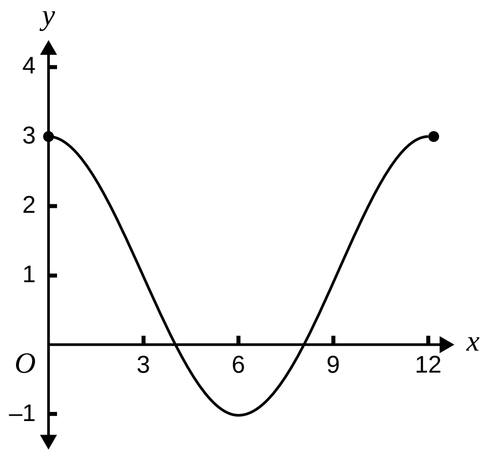
C. $\dfrac{\log_e\left(\frac{1}{2}\right)}{k}$

D. $\dfrac{\log_e(2)}{k}$

E. $k \log_e(2)$

Question 11

The diagram below shows one cycle of the graph of a circular function.



The amplitude, period and range of the function are, respectively,

	amplitude	period	range
A.	2	$\frac{\pi}{6}$	[0, 12]
B.	2	12	[–1, 3]
C.	3	12	[0, 12]
D.	4	$\frac{\pi}{6}$	[0, 12]
E.	4	12	[–1, 3]

Question 12

For the equation $2\sin(3x) = 1$, the **sum** of the solutions in the interval $[0, \pi]$ is equal to

A. $\frac{\pi}{18}$

B. $\frac{\pi}{6}$

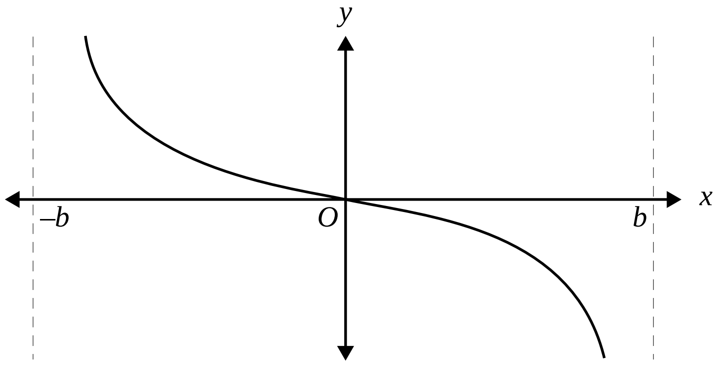
C. 2π

D. $\frac{n\pi}{18}$

E. $12n\pi$

Question 13

The diagram shows one cycle of the graph with equation $y = \tan(ax)$.
Vertical asymptotes have equations $x = b$ and $x = -b$.



Possible values of a and b are

	a	b
A.	–3	$\frac{\pi}{6}$
B.	–3	$\frac{2\pi}{3}$
C.	$-\frac{1}{3}$	$\frac{\pi}{6}$
D.	$-\frac{1}{3}$	$\frac{2\pi}{3}$
E.	3	$\frac{\pi}{6}$

Question 14

Using the approximation formula, $f(x + h) \approx f(x) + h f'(x)$ where $f(x) = \sqrt{x}$ with $x = 16$, an approximate value of $\sqrt{15.96}$ is given by

A. $f(4) + 0.04 f'(4)$
B. $f(16) + 0.04 f'(16)$
C. $f(16)$
D. $f(4) - 0.04 f'(4)$
E. $f(16) - 0.04 f'(16)$

Question 15

For the function with equation $y = -x^3 - x^2 + 2x + 2$, the subset of R for which the function is increasing is

A. $\left(-\infty, \frac{-1-\sqrt{7}}{3}\right)$

B. $\left(\frac{-1-\sqrt{7}}{3}, \frac{-1+\sqrt{7}}{3}\right)$

C. $\left(\frac{-1+\sqrt{7}}{3}, \infty\right)$

D. $(-1, \sqrt{2})$

E. $(2, \infty)$

Question 16

Rainwater is being collected in a water tank. The volume, V m^3, of water in the tank after time, t hours, is given by $V = 2t^2 - 3t + 2$.

The average rate of change of volume over the first ten hours in m^3 per hour is

A. 10

B. 17

C. 19

D. 37

E. 172

Question 17

The tangent is horizontal at one point on the graph of the function $f: R \to R$, where $f(x) = xe^{-ax}$ and $a > 0$.

The x-coordinate of this point is

A. ae

B. $\frac{1}{ae}$

C. $\frac{1}{a}$

D. 0

E. a

Question 18

If $y = \frac{\sin(x)}{x}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{x\cos(x) - \sin(x)}{x^2}$

B. $\cos(x)$

C. $\frac{\sin(x) - x\cos(x)}{x^2}$

D. $\frac{\cos(x) - \sin(x)}{x}$

E. $x\cos(x) - \sin(x)$

Question 19

$\int_1^4 (2f(x)+1)dx$ can be written as

A. $2\int_1^4 (f(x)+1)dx$

B. $2\int_1^4 f(x)dx+1$

C. $\int_1^4 2f(x)dx$

D. $2\int_1^4 f(x)dx+3$

E. $2\int_1^4 f(x)dx+x$

Question 20

Using the left rectangle approximation with rectangles of width 1, the area of the region bounded by the x-axis, the y-axis, the line $x = 3$ and by the curve whose equation is $y = e^x$ is approximated by

A. $1 + e + e^2$

B. $1 + e + e^2 + e^3$

C. $e + e^2 + e^3$

D. $e^3 - e$

E. $\dfrac{\frac{1}{2} + e + e^2 + e^3}{2}$

Question 21

The average value of $\dfrac{1}{3x}+\sin(2x)$ over the interval from $x = \dfrac{\pi}{3}$ to $x = \pi$ is

A. $\dfrac{3}{2\pi}(\dfrac{1}{3}\log_e(3)-\dfrac{3}{4})$

B. $(\dfrac{1}{3}\log_e(3)-\dfrac{3}{4})$

C. $\dfrac{3}{2\pi}(\dfrac{4}{27}\pi^2-\dfrac{3}{4})$

D. $(\dfrac{4}{27}\pi^2-\dfrac{3}{4})$

E. $\dfrac{4}{9}(\sin(2))\pi^2+\dfrac{1}{3}\log_e(3)$

Question 22

If $\dfrac{dy}{dx} = \dfrac{2}{(4x+1)^{\frac{3}{2}}}$ where $4x + 1 > 0$, and c is a real constant, then y is

A. $\dfrac{-12}{(4x+1)^{\frac{5}{2}}} + c$

B. $\dfrac{-1}{8(4x+1)^2} + c$

C. $\dfrac{x^2}{4} + \dfrac{x}{2} + c$

D. $\dfrac{1}{2}\log_e(4x+1) + c$

E. $\dfrac{-1}{(4x+1)^{\frac{1}{2}}} + c$

Question 23

The probability distribution for the discrete random variable X is given by

x	0	1	2	3
$\Pr(X = x)$	k	$2k$	$4k$	$8k$

The value of k is

A. $\dfrac{1}{35}$

B. $\dfrac{1}{34}$

C. $\dfrac{1}{15}$

D. $\dfrac{1}{4}$

E. 15

Question 24

The number, X, of cars waiting in the right-hand turn lane at a set of traffic lights as the lights change, has the following probability distribution.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	0.2	0.3	0.2	0.1

The variance of X, correct to two decimal places, is

A. 1.25
B. 1.56
C. 1.80
D. 2.19
E. 4.80

Question 25

Vandals enter a factory where computer chips are manufactured and mix 24 normal chips with 12 faulty chips in a box. The factory owner discovers the mixed box and selects a sample of k chips for testing, where $k > 3$.

The probability that she selects exactly 3 faulty chips is

A. ${}^{k}C_3\left(\frac{1}{3}\right)^{k-3}\left(\frac{2}{3}\right)^{3}$

B. ${}^{k}C_3\left(\frac{2}{3}\right)^{k-3}\left(\frac{1}{3}\right)^{3}$

C. $\left(\frac{1}{3}\right)^{3}$

D. $\dfrac{{}^{24}C_3 \times {}^{12}C_{k-3}}{{}^{36}C_k}$

E. $\dfrac{{}^{12}C_3 \times {}^{24}C_{k-3}}{{}^{36}C_k}$

Question 26

Andrea throws a netball towards a goal ring. If the ball passes through the ring, she scores a goal. Andrea knows that on average she scores a goal 17 times out of every 20 throws. The result of each throw is independent of the previous throw.

If Andrea were to throw the netball 10 times towards a goal ring, the probability of obtaining more than 8 goals is

A. ${}^{10}C_9(0.15)^1\,(0.85)^9$

B. ${}^{10}C_9(0.15)^1\,(0.85)^9 + (0.85)^{10}$

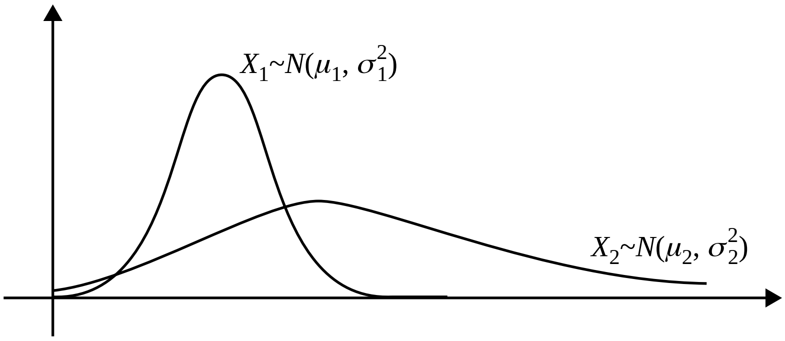
C. ${}^{10}C_8\,(0.15)^2\,(0.85)^8 + {}^{10}C_9(0.15)^1\,(0.85)^9 + (0.85)^{10}$

D. $(0.85)^{10}$

E. $\dfrac{{}^{17}C_8 \times {}^{3}C_2}{{}^{20}C_{10}}$

Question 27

The diagram below shows two normal distribution curves with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively.



Which one of the following sets of statements is true?

A. $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$
B. $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$
C. $\mu_1 < \mu_2$ and $\sigma_1 > \sigma_2$
D. $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$
E. $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK