PART I

Structure of Booklet

Number of questions	Number of questions to be answered	Marks
27	27	27

Directions to students

Materials

Multiple-choice question booklet of 13 pages.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of the answer sheet for multiple-choice questions.

Answer **all** questions.

There is a total of 27 marks available for Part I.

Unless otherwise indicated, the diagrams in this booklet are not drawn to scale.

All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II).

Specific Instructions for Part I

This part consists of 27 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1

The range of the function $f: [0, \infty) \rightarrow R$, where $f(x) = 1 - 2(x - 1)^2$, is

A. $[0, \infty)$

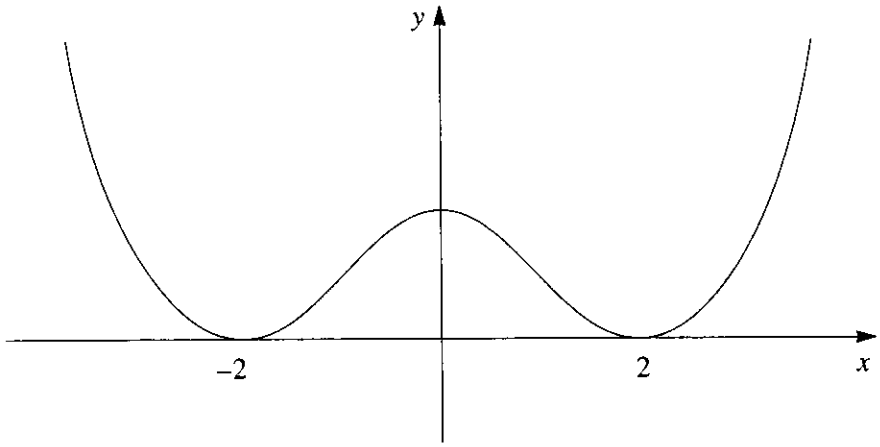
B. $(-\infty, 0]$

C. $(-\infty, -1]$

D. $(-\infty, 1]$

E. $[1, \infty)$

Question 2



The equation of the graph above is

A. $y = (x^2 - 4)(x^2 + 4)$

B. $y = (x + 2)(x - 2)$

C. $y = x^4 - 16$

D. $y = (x^2 + 4)(4 - x^2)$

E. $y = (x^2 - 4)^2$

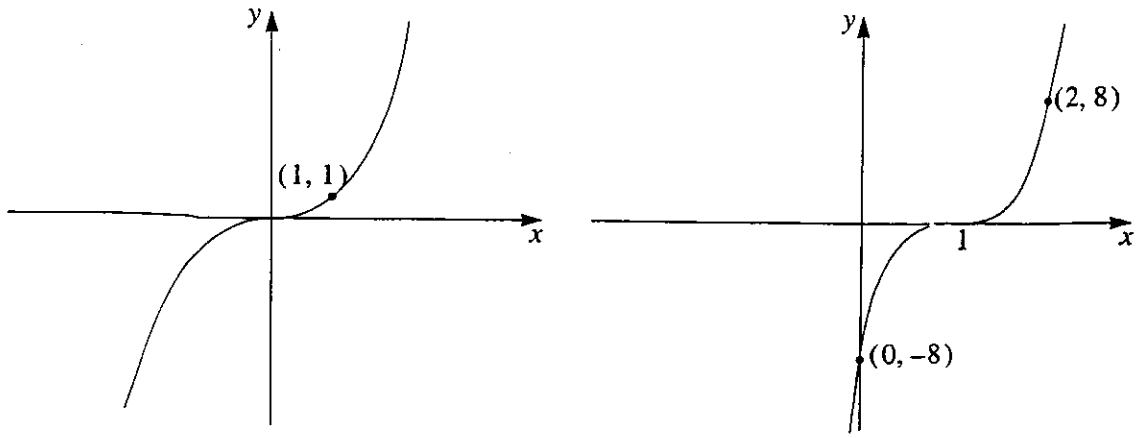
Question 3

The term independent of x in the expansion of $\left(2x - \frac{3}{x}\right)^4$ is

A. 216
B. 36
C. –36
D. –72
E. –216

Question 4

The graph of the polynomial $y = f(x)$ is shown on the left. It is linearly transformed into the graph on the right.

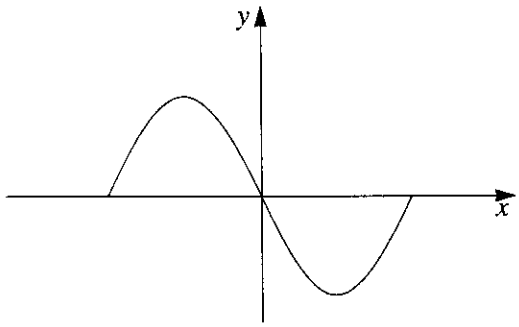


The equation of the polynomial shown on the right would then be

A. $y = 8f(x - 1)$
B. $y = 8f(x - 1) - 8$
C. $y = f(x - 1) - 8$
D. $y = f(x + 1) - 8$
E. $y = 8f(x + 1) - 8$

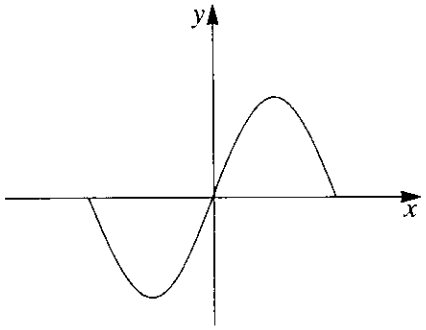
Question 5

The graph of $y = f(x)$ is shown below.

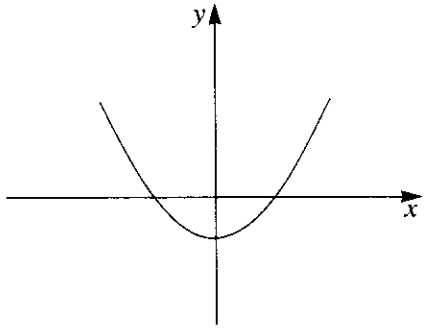


The graph of its inverse would then be

A.



B.



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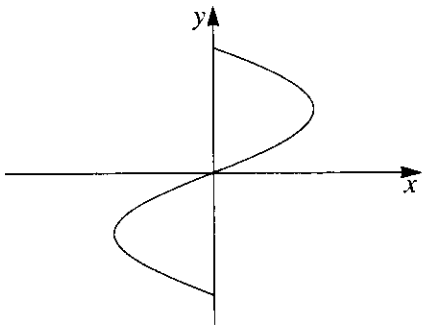

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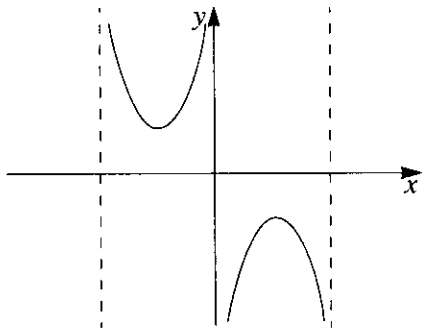
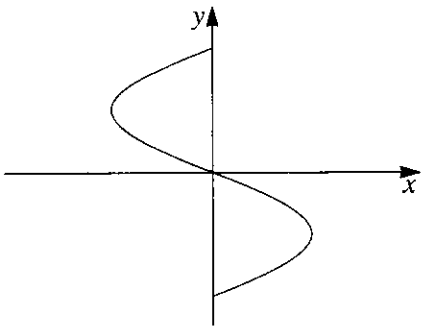


E.



Question 6

The subset of R containing all the solutions to the equation $-x^2 + 1 = 1 - e^{-x}$ is

A. $[-1, 0]$

B. $[0, 1]$

C. $[-1, 0] \cup [1, 10]$

D. $[1, 10]$

E. $(-\infty, 0] \cup [0, 1]$

Question 7

If $\log_{10} x = \log_{10}(by - a) - \log_{10} a$, where $a > 10$, then y is equal to

A. $\log_{10}\frac{a(1+x)}{b}$

B. $\frac{a(1+x)}{b}$

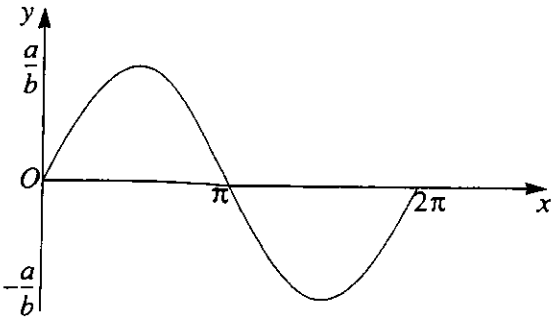
C. $\frac{b(x+a)}{a}$

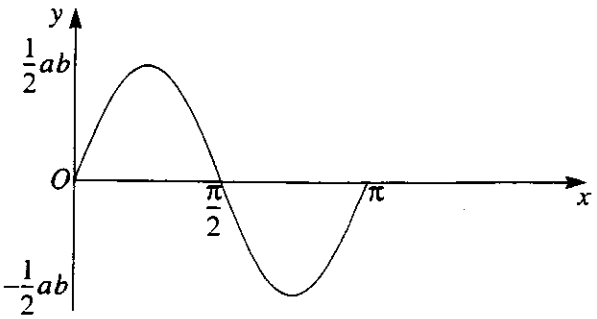
D. $\frac{b(x+1)}{a}$

E. $\frac{a(x-1)}{b}$

Question 8

The graph of $y = f(x)$ is shown on the left.





Which one of the following equations best represents the equation of the graph on the right?

A. $y = \frac{1}{2}f(x)$

B. $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$

C. $y = bf\left(\frac{1}{2}x\right)$

D. $y = \frac{1}{2}b^2 f\left(\frac{1}{2}x\right)$

E. $y = \frac{1}{2}b^2 f(2x)$

Question 9

The minimum value, maximum value and period respectively for the function

$$f: R \to R, f(x) = -a + 5a\sin c\pi x \text{ where } a, c > 0$$

are

A. $-4a, 4a, 2c$

B. $-6a, 4a, \frac{2}{c}$

C. $-5a, 5a, \frac{2}{c}$

D. $-6a, 4a, 2c$

E. $-6a, 4a, \frac{2\pi}{c}$

Question 10

$P(x, f(x))$ and $Q(x+h, f(x+h))$ are two points on the curve with equation $f(x) = x^{-1}$. The gradient of the line joining P and Q is

A. $\lim_{h \to 0} \frac{1}{x+h}$

B. $\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

C. $-\frac{1}{x(x+h)}$

D. $\frac{1}{x+h} - \frac{1}{x}$

E. $\lim_{h \to 0} \frac{1}{x+h} - \frac{1}{x}$

Question 11

The derivative of $e^{\cos 2x}$ with respect to x is

A. $-2e^{\cos x}\sin 2x$

B. $e^{-2\sin 2x}$

C. $-2e^{\cos 2x}\sin 2x$

D. $e^{x}(\cos 2x - 2\sin 2x)$

E. $-\frac{1}{2}e^{\cos 2x}\sin 2x$

Question 12

If $y = \frac{\sqrt{2x-1}}{e^{2x}}$ then $\frac{dy}{dx}$ is equal to

A. $\frac{3-4x}{e^{2x}\sqrt{2x-1}}$

B. $\frac{-4x-1}{e^{2x}\sqrt{2x-1}}$

C. $e^{2x}\left(2\sqrt{2x-1}+\frac{1}{\sqrt{2x-1}}\right)$

D. $e^{2x}\left(\frac{1}{\sqrt{2x-1}}-2\sqrt{2x-1}\right)$

E. $e^{-2x}\left(\frac{1}{2\sqrt{2x-1}}-2\sqrt{2x-1}\right)$

Question 13

The volume $V(t)$ of water in a dam at time t is given by $V(t) = 3 + \frac{3}{2}\sin\frac{t}{10}$, $0 \le t \le 10$. The rate of change of the volume of water at $t = 5$, correct to two decimal places, is

A. 0.13
B. –0.07
C. 0.07
D. –3.07
E. 2.87

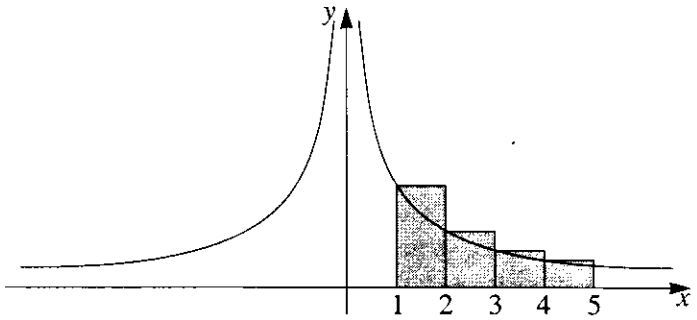
Question 14

Given that $y = x^3 + x$, the approximate change in y as x increases from 2 to 2.01 is closest to

A. 10.01
B. 13.01
C. 0.01
D. 0.13
E. 3.13

Question 15

The total area of the shaded rectangles can be used as an approximation for the area between the curve with equation $y = \frac{2}{x}, x \neq 0$, the x-axis and the lines with equation $x = 1$ and $x = 5$.



This approximation, in square units, is equal to

A. $4\frac{1}{6}$

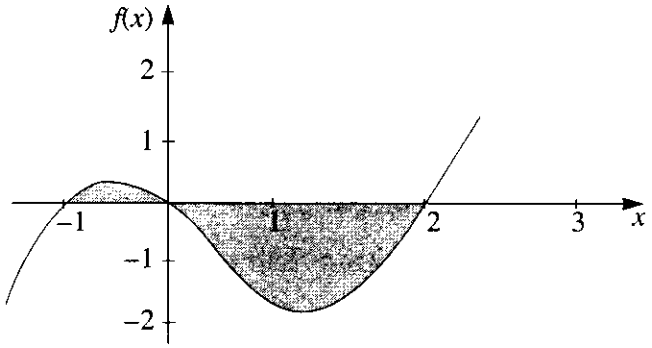
B. 3.33

C. 4

D. 3.22

E. 2

Question 16



The shaded area in the above graph can be found by evaluating

A. $\int_{-1}^{2} f(x)dx$

B. $\int_{-1}^{0} f(x)dx + \int_{0}^{2} f(x)dx$

C. $\int_{-1}^{0} f(x)dx - \int_{0}^{2} f(x)dx$

D. $\int_{-1}^{0} f(x)dx - \int_{2}^{0} f(x)dx$

E. $\int_{0}^{-1} f(x)dx + \int_{0}^{2} f(x)dx$

Question 17

The area bounded by the curve $y = 3\sin 2x$, the x-axis and the lines $x = -\pi$ and $x = \pi$ is

A. $\int_{-\pi}^{\pi} 3\sin 2x \, dx$

B. $2\int_{0}^{\pi} 3\sin 2x \, dx$

C. 0

D. $4\int_{0}^{\frac{\pi}{2}} 3\sin 2x \, dx$

E. $4\int_{-\pi}^{\pi} 3\sin 2x \, dx$

Question 18 and Question 19 refer to the following information.

The number of customers served at a milk bar between 3.00 p.m. and 3.30 p.m. over a period of twenty-five days was summarised in the table below.

Number of customers (x)	0	1	2	3	4	5	6
Number of days on which customers were served (f)	2	0	4	3	2	6	8

Question 18

During this twenty-five day period, the proportion of days on which at least 4 customers were served between 3.00 p.m. and 3.30 p.m. was

A. 0.64
B. 0.44
C. 0.08
D. 0.36
E. 0.08

Question 19

The expected number of customers served per day between 3.00 p.m. and 3.30 p.m. is **closest** to

A. 6
B. 5
C. 4
D. 3
E. 4

Question 20

If X is a binomial random variable with $n = 12$ and a mean of 7.2, then the variance of X is equal to

A. 2.88
B. 8.29
C. 1.70
D. 4.80
E. 2.68

Question 21

The multiple-choice section of a Mathematical Methods trial examination 1 has 27 questions. Each question has 5 alternative answers. The probability of a student randomly guessing 20 correct answers is

A. ${}^{20}C_5\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^{15}$

B. ${}^{27}C_{20}\left(\frac{4}{5}\right)^{20}\left(\frac{1}{5}\right)^7$

C. $\frac{{}^{20}C_{15}\,{}^{7}C_5}{{}^{27}C_5}$

D. $\frac{{}^{20}C_5\,{}^{7}C_0}{{}^{27}C_5}$

E. ${}^{27}C_{20}\left(\frac{1}{5}\right)^{20}\left(\frac{4}{5}\right)^7$

Question 22 and Question 23 refer to the following information.

The training squad for a sports team consists of 8 males and 12 females. A team of 4 athletes is to be randomly chosen.

Question 22

The probability that the team has a majority of females is closest to

A. 0.36
B. 0.47
C. 0.10
D. 0.35
E. 0.46

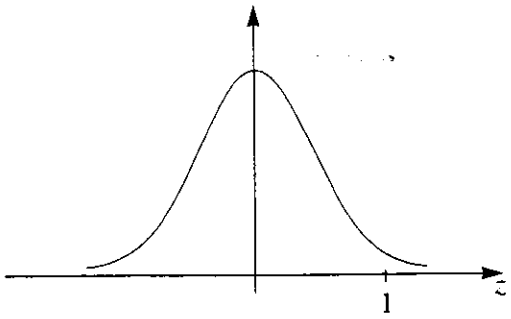
Question 23

The standard deviation of the number of females in the team is closest to

A. 0.81
B. 0.89
C. 0.90
D. 1.55
E. 2.40

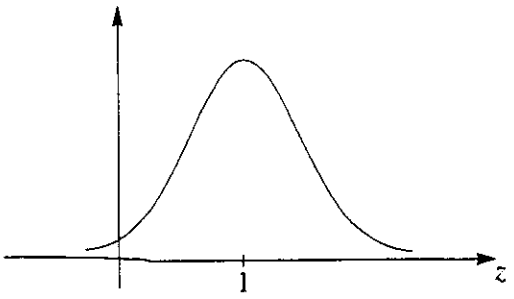
Question 24

The graph below shows the distribution of the random variable Z, which has a normal distribution.

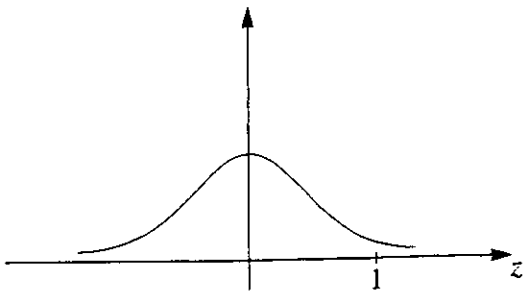


Using the same scale, the random variable $Y = 2Z + 1$ would look most like

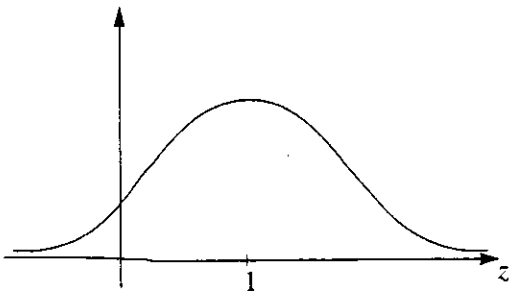
A.



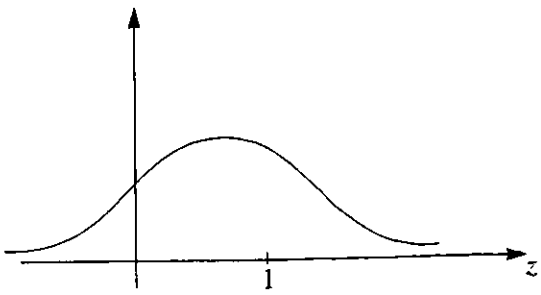
B.



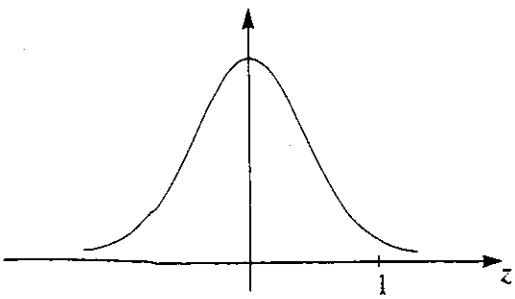
C.



D.



E.



Question 25

For a large sample of boxes of Browns Chocolate Balls, the sample mean of chocolate balls per box is 132 with a variance of 9. Which of the following intervals contains the number of chocolate balls in approximately 95% of boxes?

A. (123, 141)

B. (126, 138)

C. (129, 135)

D. (114, 150)

E. (130, 134)

Question 26 and Question 27 refer to the following information.

The time taken to answer a typical question on "Who wants to be a Billionaire" is normally distributed with a mean of 60 seconds and a standard deviation of 10 seconds.

Question 26

The probability that a randomly chosen person from the audition pool will take less than 50 seconds to answer the question is

A. $\Pr(Z < -1)$

B. $\Pr(Z < 1)$

C. $\Pr(-1 < Z < 1)$

D. $\Pr(Z > -1)$

E. $1 - \Pr(Z < -1)$

Question 27

The probability of a contestant taking less than 50 seconds to answer a question, given that he or she took less than 60 seconds to do so, is closest to

A. 0.1587

B. 0.8413

C. 0.6826

D. 0.3174

E. 0.5000

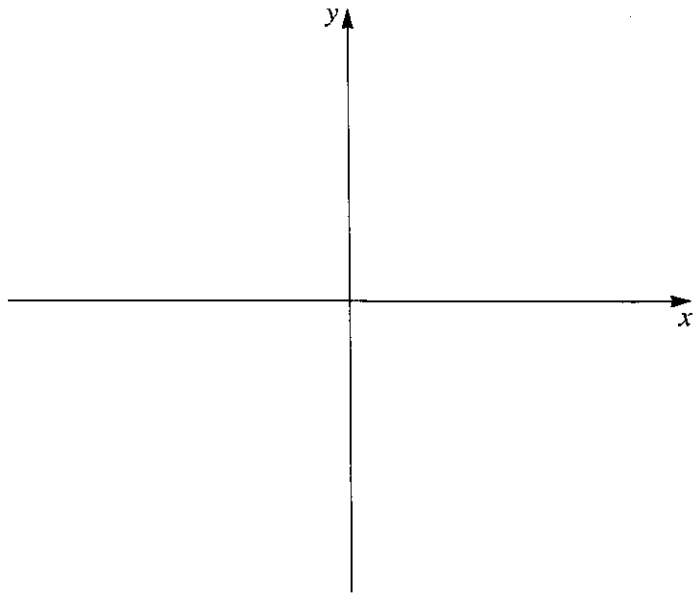
END OF PART I QUESTION BOOKLET

Specific Instructions for Part II

Answer all questions in this part in the spaces provided.

Question 1

a. Sketch the graph of the function defined by $f: [1, \infty) \rightarrow R$ where $f(x) = x^2 - 2x$.



2 marks

b. Find the rule for the inverse, f^{-1} .

2 marks

c. On the same set of axes, sketch the graph of the inverse function, f^{-1}, and clearly label all intercepts with the axes.

2 marks

Total 6 marks

Question 2

a. Differentiate $\sin(2x^2)$.

2 marks

b. Hence find $\int x\cos(2x^2)\,dx$.

2 marks
Total 4 marks

Question 3

Consider the function $f(x) = \log_e x^2$.

a. Find the value of $f(x)$ when $x = e$.

1 mark

b. Determine the exact value of the gradient of the tangent at this point.

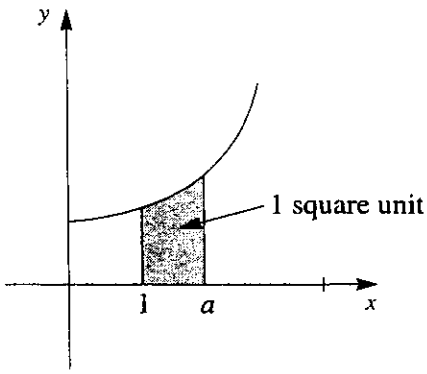
1 mark

c. Hence find the exact equation of the normal at the point where $x = e$.

3 mark
Total 5 marks

Question 4

The area enclosed between the curve $f: [0, \infty) \rightarrow R$, $f(x) = x^2 + a$ (where $a > 1$), and the lines $x = 1$ and $x = a$ is 1 square unit.



Use a calculus approach to find the value of a to 3 decimal places

4 marks

Question 5

An urn contains 3 black and 7 white balls. 5 balls are chosen *without replacement*.

a. Find the probability that 3 of these balls are black. Give your answer correct to 3 decimal places.

2 marks

b. Given that the first 3 balls drawn are white, calculate the probability that at least one of the next 2 balls drawn is black.

2 marks

Total 4 marks

END OF PART II QUESTION AND ANSWER BOOKLET