The Mathematical Association of Victoria

2001

MATHEMATICAL METHODS

Trial Examination 1

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **27 marks** available for this part.

Part II consists of seven questions. Answer all questions in **Part II** in the spaces provided. There are **23 marks** available for this part.

There are **50 marks** available for this task.

A formula sheet is attached.

These questions have been produced to assist students in their preparation for the 2001 Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority (VCAA) Assessing Panels. The Association gratefully acknowledges the permission of the VCAA to reproduce the formula sheet.

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Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \qquad \int \sin ax\ dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \qquad \int \cos ax\ dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}\left(\tan ax\right) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2 p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Multiple-Choice Answer Sheet

Student's Name _____

Circle the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**

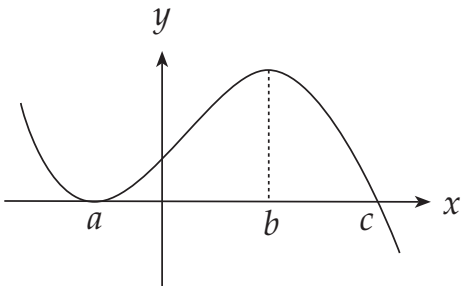
Part I (Multiple-choice questions)

Question 1

The range of the function $f(x) = \dfrac{-2}{(x+3)^2} + 3$ is

A. $(-\infty, 3]$
B. $(3, \infty)$
C. $R \setminus \{-3\}$
D. $(-\infty, -3)$
E. $(-\infty, 3)$

Question 2



The equation of the graph shown could be

A. $y = (x - a)(x - c)^2 - b$
B. $y = (x + a)^2(x - c)$
C. $y = -(x - a)(x - c)(x - b)$
D. $y = (c - x)(x - a)^2$
E. $y = (x - a)(x - c)^2$

Question 3

x	0.1	0.5	1	2	3	4	5	10
y	– 0.61	2.6	4	5.4	6.2	6.8	7.2	8.6

The rule connecting y and x is closest to

A. $y = \sqrt{x} + 4$

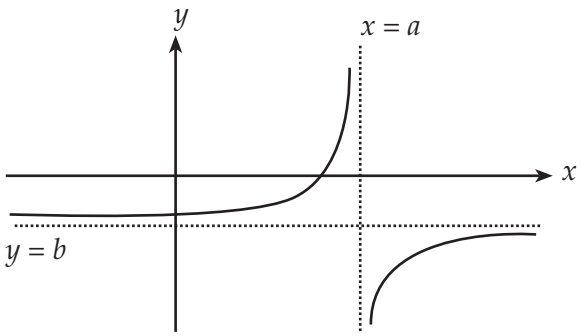
B. $y = 2\sqrt{x} + 4$

C. $y = 2e^x + 4$

D. $y = 2\log_e(x) + 4$

E. $y = \log_e(x) + 4$

Question 4



The equation of the graph shown could be

A. $y = \dfrac{1}{x - a} + b$

B. $y = \dfrac{1}{a - x} + b$

C. $y = -\dfrac{1}{x - b} + a$

D. $y = \dfrac{1}{x + a} + b$

E. $y = -\dfrac{1}{x - a} - b$

Question 5

Which of the following functions does not have an inverse function?

A. $f:(-\infty,-4] \to R, f(x) = 5(x-4)^2 + 7$

B. $f:R \to R, f(x) = -2(x+1)^3 + 3$

C. $f:[2,\infty) \to R, f(x) = 9(x-3)^4 - 6$

D. $f:[-3,0] \to R, f(x) = \sqrt{9-x^2}$

E. $f(x) = \begin{cases} 2x & \text{for } x \in (-\infty,3] \\ x^2 + 3 & \text{for } x \in (3,\infty] \end{cases}$

Question 6

If $f(x) = e^{(x-1)} + 5$ then $f^{-1}(x) =$

A. $\log_e x - 4$

B. $\log_e (x-5) + 1$

C. $\log_e x - \log_e 5 + 1$

D. $e^{(1-x)} + 0.2$

E. $\dfrac{1}{e^{(x-1)}} + 5$

Question 7

Consider the equation $(x^2 + a)(x^3 + b)(x + c)^2 = 0$, where $a > 0$, $b > 0$ and $c > 0$, and $a \neq b \neq c$. The number of distinct real solution(s) this equation has is

A. 1

B. 2

C. 3

D. 4

E. 5

Question 8

The term independent of a in the expansion of $\left(a^3b - \frac{1}{ab^2}\right)^{16}$ is

A. ${}^{16}C_{12}\,\frac{1}{b^{20}}$

B. b^{20}

C. $\frac{1}{b^{20}}$

D. $-{}^{16}C_{12}\,\frac{1}{b^{20}}$

E. $-{}^{16}C_{12}\,\frac{1}{b^{24}}$

Question 9

The exact solution of the equation $25^x = 5^x + 2$ is

A. $x = \frac{\log 2}{\log 5}$

B. $x = \frac{\log 5}{\log 2}$

C. $x = -1$

D. $x = 0$

E. $x = 2$

Question 10

If $\log_a(3r)^6 - 3\log_a(9r) - \log_a(r^4) = 2$, where $a > 0$, then r equals

A. $\frac{3}{a^2}$

B. $\frac{1}{3a^2}$

C. a^2

D. $\frac{1}{a^2}$

E. 0.5

Question 11

The height of the tide, h metres, at the entrance to a port is given by

$$h = 3\sin(\frac{\pi a}{4} t) + 8$$

where t is the number of hours after midnight and a is a positive integer.
The number of times the height is 10 m within the first day is

A. 3
B. 6
C. $\frac{6}{a}$
D. $3a$
E. $6a$

Question 12

If m is the smallest solution and n the largest solution to $\cos 2x - \sqrt{3}\sin 2x = 0$ for $\{x: -\pi \leq x \leq \pi\}$ then $m + n$ is

A. $-\frac{\pi}{3}$
B. $\frac{\pi}{6}$
C. $\frac{\pi}{3}$
D. $\frac{4\pi}{3}$
E. $\frac{3\pi}{2}$

Question 13

Which of the following statements is **false** for the graph of

$$y = \tan 0.5x$$

A. The graph repeats itself every 2π units.
B. The gradient at $(\frac{\pi}{2},1)$ is 1.
C. The vertical asymptotes have equation $x = (2k + 1)\pi$, $k \in J$.
D. The graph has a stationary point at $x = 2k\pi$, $k \in J$.
E. The range of y is R.

Question 14

The average rate of change of the function $f(x) = 2x^2 - 3x + 4$ between $x = 1$ and $x = 3$ is

A. 1
B. 3
C. 5
D. 9
E. 13

Question 15

The equation of the tangent to the curve $y = ae^{-x}$ at the point $(0, a)$ is

A. $y = ax + a$
B. $y = ax - a$
C. $y = a - x$
D. $y = a - ax$
E. $y = a + x$

Question 16

If $f(x) = \dfrac{x}{\sqrt{36 - x^2}}$ then $f'(x)$ equals

A. $\dfrac{-36}{36 - x^2}$

B. $\dfrac{36 - 2x^2}{\left(36 - x^2\right)^{\frac{3}{2}}}$

C. $\dfrac{36}{(36 - x^2)^{\frac{3}{2}}}$

D. $\dfrac{-x^2}{36 - x^2}$

E. $\dfrac{36}{36 - x^2}$

Question 17

The derivative of $\tan(x^2+3)$ is

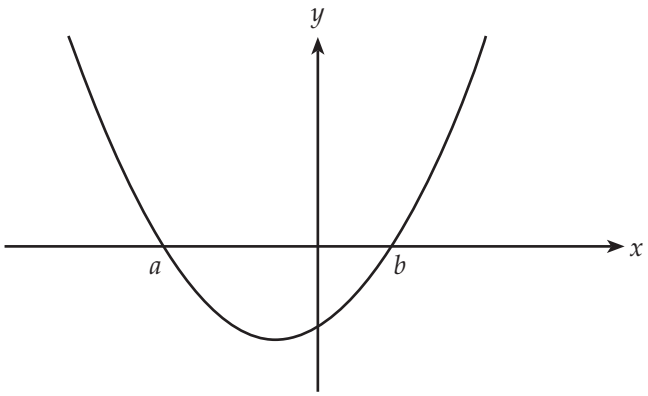
A. $\sec^2(x^2+3)$
B. $\tan(2x)$
C. $\sec^2(x^2+3) + \tan(2x)$
D. $2x\sec^2(x^2+3)$
E. $2x + \tan(x^2+3)$

Question 18

For the function $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x$, $f'(x) > 0$ when x is

A. $(-\infty,-2] \cup [0,\infty)$
B. $[-1,1] \cup [2,\infty)$
C. $[1,2]$
D. $(-\infty,-2] \cup [2,\infty)$
E. $(-1,1) \cup (2,\infty)$

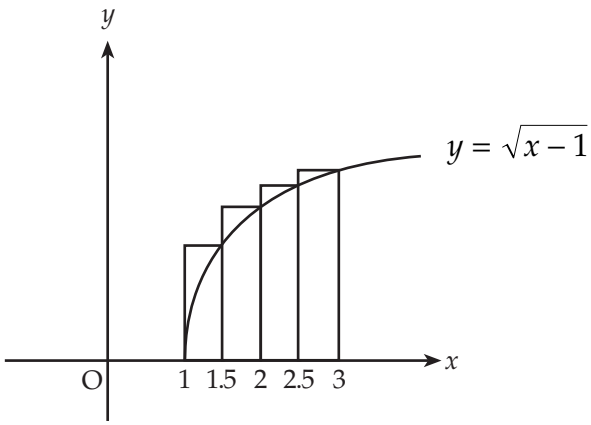
Question 19



The graph of $f'(x)$ is shown above. Which of the following equations could be the graph of $y = f(x)$?

A. $y = (x-a)(x-b)$
B. $y = 0.5\,(x^2-ax)(x^2-bx)$
C. $y = 0.5\,(x^2-ax)(x^2-bx) + c$, where c is a constant
D. $y = x^3 - (a+b)x^2 + abx + c$, where c is a constant
E. $y = k\left[\frac{1}{3}x^3 - \frac{1}{2}(a+b)x^2 + abx + c\right]$, where k and c are constants

Question 20



The approximate area, in square units, bounded by the curve $y = \sqrt{x-1}$, the x–axis and the lines $x = 1$ and $x = 3$ using the four rectangles shown above is

A. $0.5\left(\sqrt{0.5}+1+\sqrt{1.5}+\sqrt{2}\right)$

B. $0.5\left(\sqrt{0.5}+1+\sqrt{1.5}\right)$

C. $\sqrt{0.5}+1+\sqrt{1.5}+\sqrt{2}$

D. $\int_1^3 \sqrt{x-1}\,dx$

E. $3\sqrt{2}$

Question 21

The shaded area for the graph may be determined by evaluating

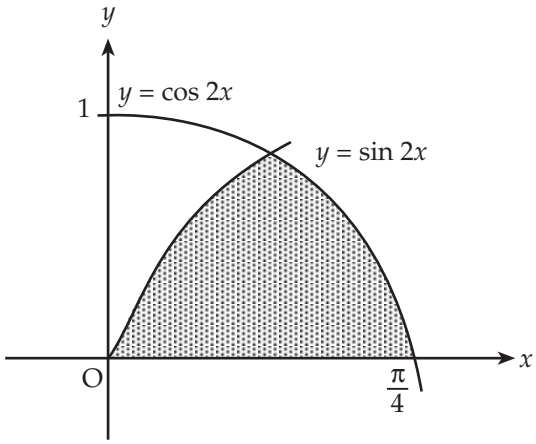
A. $\int_{\frac{1}{\sqrt{2}}}^{\frac{\pi}{4}} \cos 2x dx + \int_0^{\frac{1}{\sqrt{2}}} \sin 2x dx$

B. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos 2x dx + \int_0^{\frac{\pi}{8}} \sin 2x dx$

C. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 2x dx + \int_0^{\frac{\pi}{8}} \cos 2x dx$

D. $\int_0^{\frac{\pi}{4}} (\cos 2x - \sin 2x) dx$

E. $\int_0^{\frac{\pi}{4}} (\sin 2x - \cos 2x) dx$



Question 22

Two dice are rolled. The probability distribution for X, the difference between the numbers on the dice, is given by

x	0	1	2	3	4	5
$\Pr(X = x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	a	b	$\frac{2}{36}$

The probability that X is less than 2 given that X is less than 4 is

A. $\frac{8}{18}$

B. $\frac{8}{15}$

C. $\frac{12}{17}$

D. $\frac{17}{18}$

E. $\frac{2}{3}$

Question 23

ITP is an autoimmune disease where the body destroys its own platelets. Long term studies have shown that 80 per cent of adult ITP sufferers become chronic. If five adult ITP sufferers are randomly selected, then the probability that most (i.e. more than half) of them will become chronic is

A. ${}^5C_4(0.8)^4\,0.2 + (0.8)^5$

B. $(0.8)^5$

C. $\frac{{}^4C_3}{{}^5C_3} + \frac{{}^4C_4}{{}^5C_4}$

D. $1 - [(0.2)^5 + {}^5C_1\,0.8\,(0.2)^4]$

E. $1 - [(0.2)^5 + {}^5C_1\,0.8\,(0.2)^4 + {}^5C_2\,(0.8)^2(0.2)^3]$

Question 24

Carlin decided to participate in a lucky dip at his local street festival. The box contained p CDs: q of them were recent CDs and the rest were older. The CDs were wrapped separately in brown paper. If Carlin was allowed to randomly select 3 CDs for his prize then the probability he would get exactly 3 recent CDs, where $q \geq 3$ is

A. $\dfrac{q(q-1)(q-2)}{p(p-1)(p-2)}$

B. $\dfrac{{}^{9}C_{3}}{{}^{p-q}C_{3}}$

C. ${}^{p}C_{3}\left(\dfrac{q}{p}\right)^{3}\left(\dfrac{p-q}{p}\right)^{0}$

D. ${}^{p}C_{3}\left(\dfrac{q}{p}\right)^{0}\left(\dfrac{p-q}{p}\right)^{3}$

E. $\dfrac{{}^{p-q}C_{3}}{{}^{p}C_{3}}$

Question 25

A committee of five is to be selected at random from r female and 500 male students, where $r > 5$. If X represents the number of female students on the committee. Which one of the following is true?

A. The E(X) using the binomial distribution is greater than the E(X) using the hypergeometric distribution.

B. X can only be approximated using the binomial distribution if r is large.

C. X is a hypergeometric continuous random variable.

D. The Var(X) using the binomial distribution is the same as the Var(X) using the hypergeometric distribution.

E. $\Pr(X = 2) \approx {}^{5}C_{2}\left(\dfrac{r}{500+r}\right)^{2}\left(\dfrac{500}{500+r}\right)^{3}$

Question 26

If 40% of teachers in a given country are over the age of 50 years, what is the probability that for five randomly selected teachers at least three of them will be over 50 years of age?

A. 0.2304
B. 0.3174
C. 0.0230
D. 0.6400
E. 0.3072

Question 27

The heights of women are normally distributed with $\mu = 162$ cm and variance $\sigma^2 = 64$, what is the probability that a woman selected at random has a height greater than 180 cm?

A. 3.408×10^{-6}
B. 0.0122
C. 0.9878
D. 0.5865
E. 0.3893

Part II (Short Answer Questions)

Question 1

If $y = \log_e(\cos 2x)$, find $\frac{dy}{dx}$ and hence find $\int \tan 2x \ dx$.

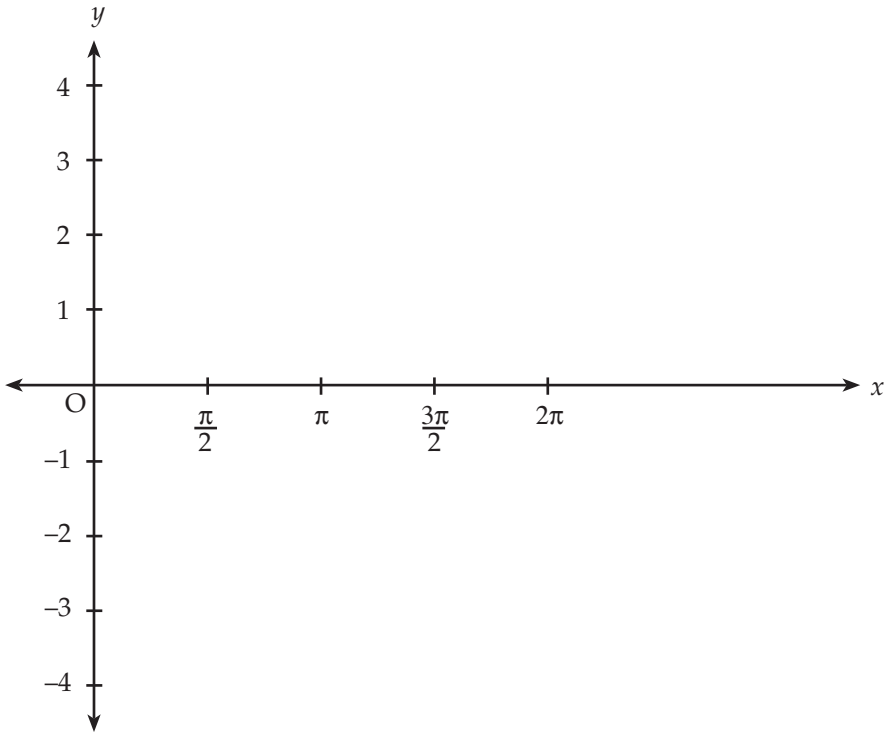
2 marks

Question 2

Given $f(x) = \sqrt{x}$, use calculus to find an approximate value for $\sqrt{15.9}$.

2 marks

Question 3



a. Sketch the graph of $y = \sin(2x)$ on the above set of axes, for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

b. Sketch the graph of y, for $0 \leq x \leq \pi$ on the same set of axes, after a translation of $\frac{\pi}{2}$ units to the left and 1 unit up, followed by a dilation of a factor of 2 parallel to the y-axis.

c. Write down the equation of your new graph.

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1 + 1 +1 = 3 marks

Question 4

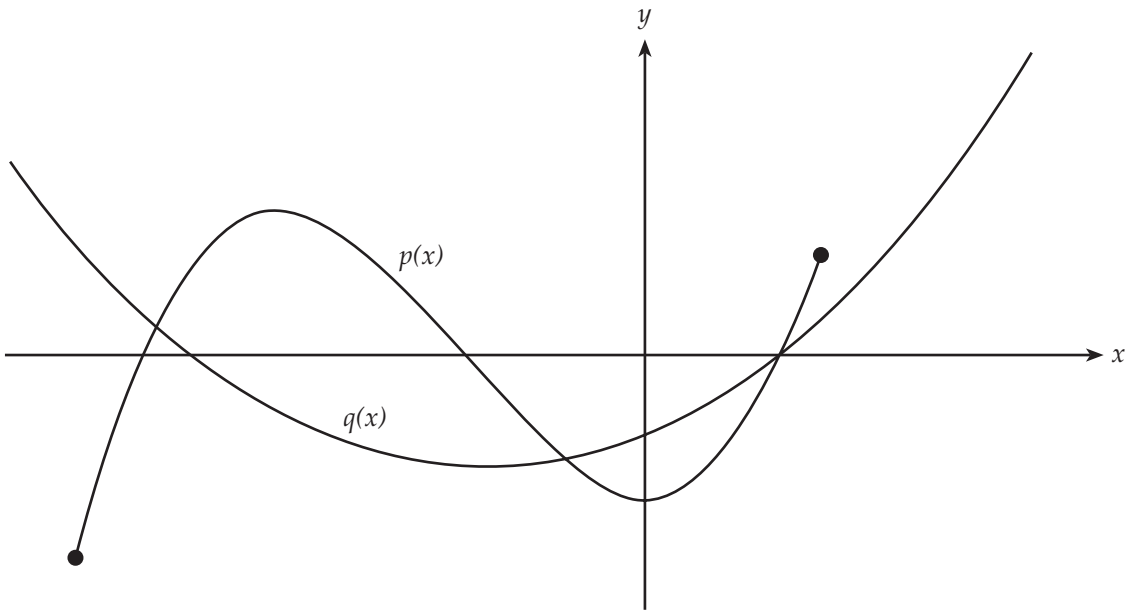
a. Find the coordinates of the points of intersection of the graphs of $f(x) = x^2 + 5x - 6$ and $g(x) = 3x + 2$.

b. Use calculus to find the area enclosed by $f(x)$ and $g(x)$.

2 + 4 = 6 marks

Question 5

The graphs of $y = p(x)$ and $y = q(x)$ are shown on the axes below.
On the same set of axes, sketch $m(x) = p(x) + q(x)$.



2 marks

Question 6

A water tank is being emptied and the volume of water (V litres) remaining in the tank after t minutes is given by

$$V(t) = 3\,(20 - t\,)^3$$

a. What is the domain of this function?

b. Find an expression for the rate at which the volume of water in the tank is changing at t minutes.

c. **i.** At what rate is the tank emptying initially?

ii. When is the tank emptying at a rate of 900 L/min?

1 + 1 + 2 = 4 marks

Question 7

The probability distribution for a discrete random variable X is

x	0	1	2	3	k
$\Pr(X = x)$	0.1	0.2	0.3	0.2	0.2

where $k \in J^+$.

a. Find $\text{Var}(X)$ in terms of k.

b. If $\text{Var}(X) = 7$, determine the value of k.

1 + 1 = 2 marks

Question 8

The heights of women are normally distributed. It is known that 6 per cent of women are taller than 176 cm and 12 per cent shorter than 148 cm. Find the mean, in centimetres correct to one decimal place, of this distribution.

3 marks