Question 1

a. Recently a small chemical plant had been discharging waste into a nearby lake at a rate of 12 m^3/day. Once this was discovered, the chemical plant immediately ordered the installation of filters to slow and eventually stop the rate of flow of waste into the lake.

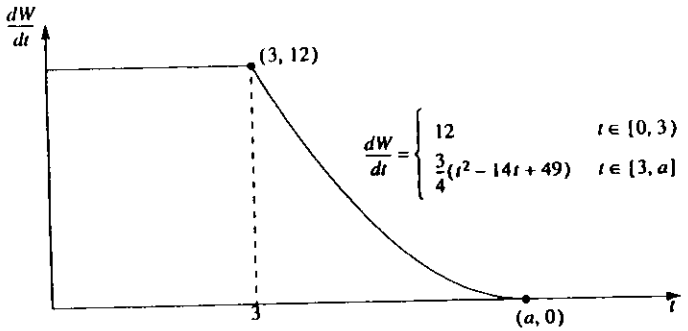
For 3 days the rate of flow of waste was steady at 12 m^3/day.

From the time the filters were installed until the time the flow of waste stopped, the rate of flow (in m^3/day) was approximated by

$$\frac{dW}{dt} = \frac{3}{4}(t^2 - 14t + 49)$$

where t is the time measured in days since the chemical plant learned of the situation.

The graph for the rate of flow of waste $\left(\frac{dW}{dt}\right)$ against time (t) is sketched on the set of axes below.



i. Show that 7 days had elapsed between the time the chemical plant learned of the situation and the time that the waste flow stopped entirely.

2 marks

ii. Calculate the total amount of waste, W, that entered the lake.

3 marks

b. The pressure (P) near the top of an organ pipe t seconds after the measuring device is turned on is modelled by the function

$$P = 760 + 45\cos(512\pi t)$$

i. Find the period and amplitude of the function.

2 marks

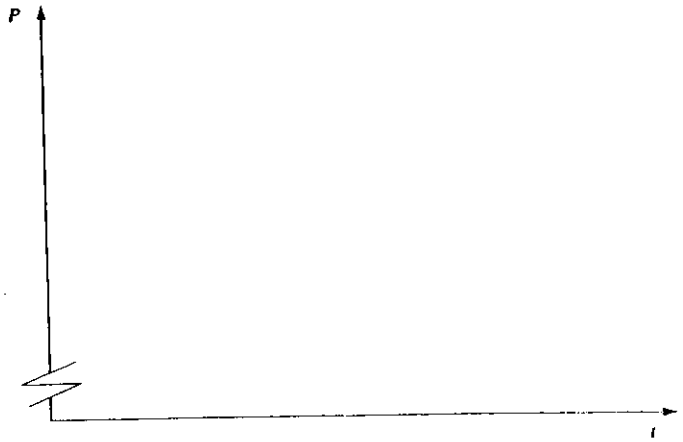
ii. The frequency of the note is defined as **the number of complete periods that occur per second**. What is the frequency of the note produced by this organ pipe?

1 mark

iii. Find, **to the nearest ten thousandth of a second**, the first occasion on which the measuring device records a pressure of 750 units.

3 marks

iv. Sketch the graph of the function.



3 marks

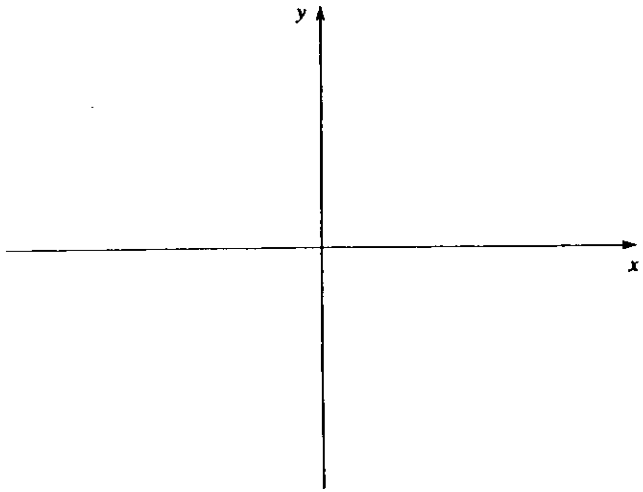
c. Sketch the graphs of the functions

$$f_1(x) = 2\sin 3x,\ x \in [-3, 3]$$

$$f_2(x) = 1 - 2x,\ x \in [-3, 3]$$

on the set of axes below. Hence sketch the graph of the function

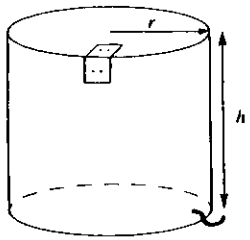
$$f_3(x) = 2\sin 3x + 1 - 2x,\ x \in [-3, 3]$$



4 marks
Total 18 marks

Question 2

a. A cylindrical compost bin of radius r m and height h m has a circular lid and an open base so worms can access the compost. It is required to make such a bin so that its volume is held constant at $\frac{2\pi}{3}$ cubic metres.



i. By using an appropriate formula, express h in terms of r.

2 marks

ii. Express the external surface area (A m^2) of the bin in terms of the radius r.

3 marks

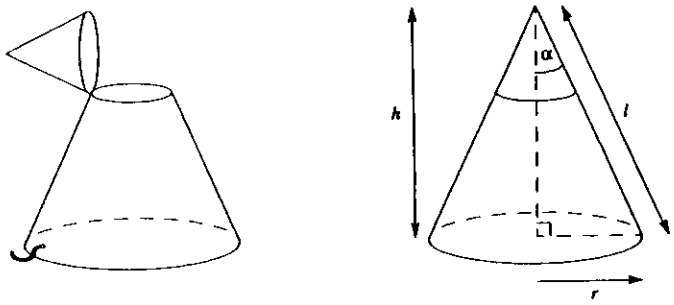
iii. It is required to mass produce these bins at a minimum cost. Use calculus to calculate the radius of the mass produced bins for minimum cost of production (to the nearest centimetre) (Hint: assume the cost is proportional to the surface area).

4 marks

iv. Hence or otherwise calculate the surface area (in square metres to 2 decimal places) of each bin for minimum cost of production.

1 mark

b. A new flip-top conical bin was modelled by the company as shown.



Its surface area (A) is found from the formula

$$A = \pi r l$$

where

l = the slant edge length (m)

r = the radius of the base (m)

h = the height (m)

α = the semivertex angle (radians)

i. Write an equation for r in terms of h and α.

1 mark

ii. Write an equation for l in terms of h and r.

1 mark

iii. Show that $A = \pi h^2 \tan\alpha \sqrt{1 + \tan^2\alpha}$.

2 marks

iv. If the angle α was fixed at $\frac{\pi}{6}$ radians, express A in terms of h.

2 marks

v. Calculate the approximate change in A (δA) when h changes from 1 m to 1.01 m.

2 marks

Total 18 marks

Question 3

A newly formed company has two branches. The management estimates that the weekly revenue for branch 1 will be modelled by the function

$$R_1 = 15 + 24\log_e\left(1 + \frac{t}{4}\right)$$

and for branch 2 by the function

$$R_2 = 11 + 45\log_e\left(1 + \frac{t}{9}\right).$$

Both functions are defined over the domain $t \in [0, 150]$, where t is the number of weeks after the company opens and the weekly revenues are given in hundreds of dollars.

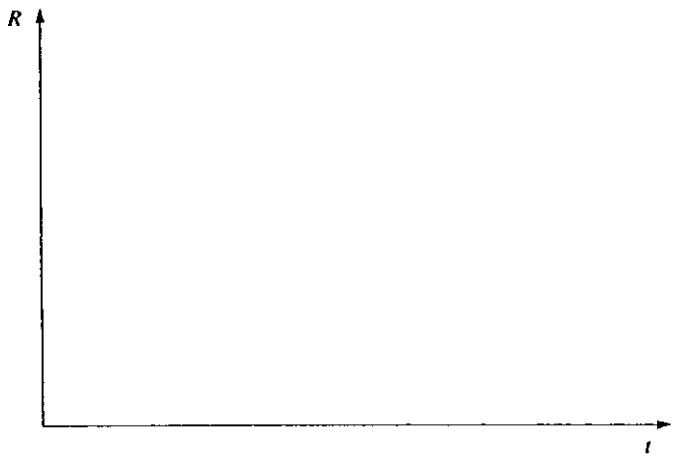
a. i. Find the initial revenues of each branch.

2 marks

ii. Find the revenues of each branch during the 50th week.

2 marks

iii. Sketch the graphs of the functions R_1 and R_2 on the axes below.



3 marks

b. i. Find the inverse of the function R_1 stating its domain and range.

4 marks

ii. Use the inverse function to find (correct to one decimal place) the value of t at which the revenue of branch 1 first reaches $9000 per week.

2 marks

c. i. Find an expression for the total revenue of the company during week t in terms of t.

2 marks

ii. Find the total revenue when $t = 36$ weeks in the form $R = A + \log_e B$, where A and B are constants.

3 marks

Total 18 marks

Question 4

A new casino game of chance called "Red Ball" consists of the random selection of 3 balls from a barrel **replacing each ball after it is selected before the next ball is withdrawn.** The 3 balls are selected from 3 red and 4 white balls (all identical except for their colour).

It costs \$10 to play one game. If 3 white balls are selected, the player loses \$10. If 1 red ball and 2 white balls are selected in any order, the player wins \$10 (that is, they receive their money back for no gain). If 2 red balls and 1 white ball are selected in any order, the player wins \$12 (a \$2 gain). If 3 red balls are selected, the player wins \$20 (a \$10 gain).

a. If X = "the number of red balls in the sample of 3", complete the probability distribution for X (round $\Pr(X = x)$ values to 4 decimal places).

x	0	1	2	3
$\Pr(X = x)$	0.1866	0.4198		

2 marks

b. Calculate the mean number of red balls in each sample of 3. (Write your answer as a fraction.)

1 mark

c. Calculate the variance of X (to 4 decimal places).

1 mark

d. If Y is the random variable "amount gained when 3 balls are selected", complete the probability distribution for Y.

y(\$)				
$\Pr(Y = y)$				

2 marks

e. Calculate the expected amount gained, $E(Y)$, to the nearest cent.

1 mark

The players of this game began complaining and convinced the casino to change the rules by having each ball selected **not replaced** before the next is drawn. All other rules remain the same.

f. Complete the new probability distribution for X (the number of red balls in the sample of 3). Write your answers to 3 decimal places.

x	0	1	2	3
$\Pr(X = x)$	0.114	0.514		

2 marks

g. Calculate the new expected value of X. (Write your answer as a fraction.)

1 mark

h. Calculate the new variance of X (to 4 decimal places).

1 mark

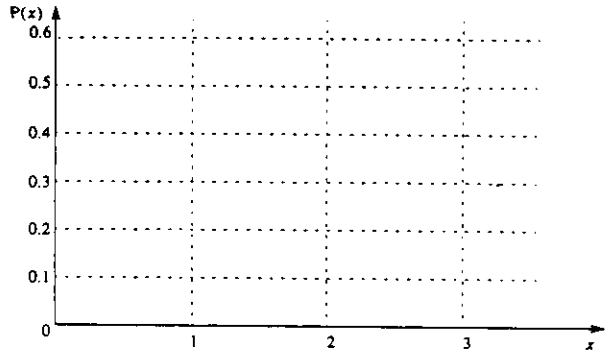
i. If Y represents the gain made (as before), calculate $E(Y)$ for this new game.

2 marks

j. Which game (balls replaced or balls not replaced) is the better game for the player? (Give a reason for your answer.)

1 mark

k. On the axes below sketch graphs of the two distributions for X. Mark the first distribution points with □ and connect with a solid line (——). Mark the second distribution points with * and connect with a dashed line (----).



2 marks
Total 16 marks

END OF QUESTION AND ANSWER BOOKLET