Question 1

a. **i.** We must show that $a = 7$, i.e. find t such that $\frac{dW}{dt} = 0$.

$\frac{3}{4}(t^2 - 14t + 49) = 0$ M1

$(t-7)^2 = 0$

$\therefore t = 7$

$\therefore (7, 0)$ corresponds to $(a, 0)$

$\therefore a = 7$

i.e. 7 days elapsed since the chemical plant learned of the situation. A1

ii. $W = \int_0^3 12\ dt + \frac{3}{4}\int_3^7 (t-7)^2\ dt$ A1

$W = \left[12t\right]_0^3 + \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)\left[(t-7)^3\right]_3^7$ M1

$W = 36 + \frac{1}{4}(0 + 64)$

$= 52\ \text{m}^3$ A1

$\therefore$ Total waste over the 7 day period was 52 m^3.

b. **i.** Amplitude = 45 (no units given). A1

Period $\tau = \frac{2\pi}{n} = \frac{2\pi}{512\pi} = \frac{2}{512} = \frac{1}{256}$ sec A1

ii. Frequency $= \frac{1}{\tau} = 256$ cycles per sec A1

iii. Solving $750 = 760 + 45\cos(512\pi t)$ M1

$750 = 760 + 45\cos(512\pi t)$

$45\cos(512\pi t) = -10$

$\cos(512\pi t) = \frac{-10}{45}$

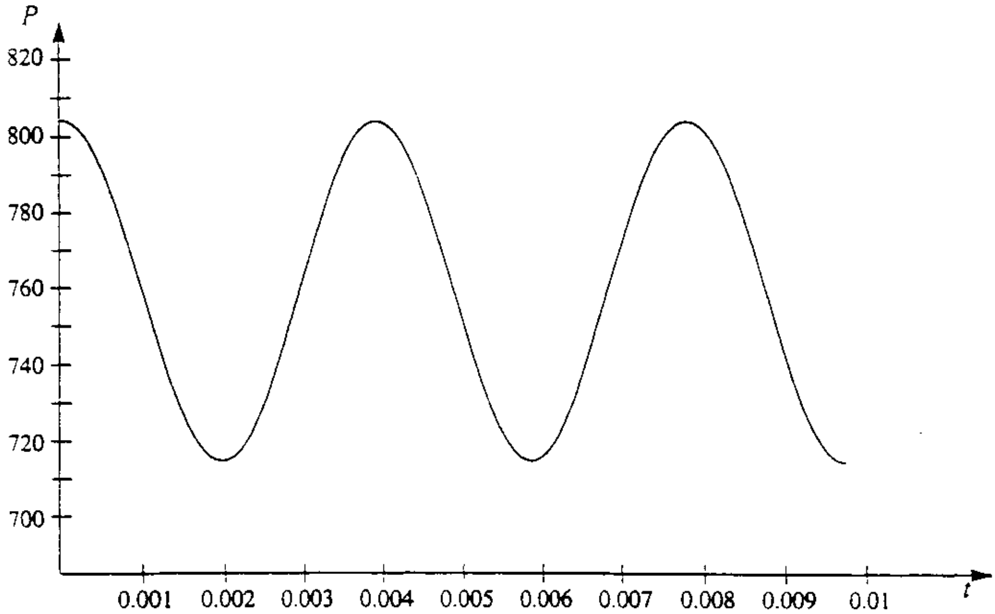
$512\pi t = \text{Cos}^{-1}\left(\frac{-10}{45}\right)$

$= 1.7948894$

$= \frac{1.7948894}{512\pi}$

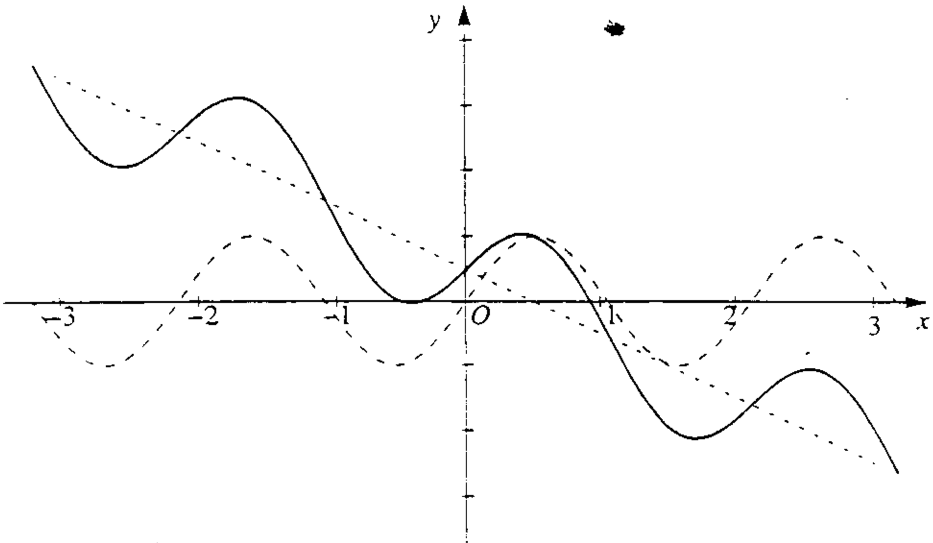
$= 0.00111588$ M1

to the nearest ten thousandth of a second this is: 0.0011sec. A1 iv.



Range correct: [715,805]	A1
Correct shape and phase (i.e. starting at (0,805)	A1
Correct period (i.e. on cycle in about 0.04 sec.).	A1

c.



Correct graphs of $y = 2\sin 3x$ and $y = 1 - 2x$: A1 A1

Principle of addition of ordinates: M1

Correct graph of $y = 2\sin 3x + 1 - 2x$: A1

Total 18 marks

Question 2

a. **i.** $V = \pi r^2 h$ M1

$\frac{2\pi}{3} = \pi r^2 h$

$\therefore h = \frac{2}{3r^2}$ A1

ii. $A = (2\pi rh + \pi r^2)$ A1

$= 2\pi r\left(\frac{2}{3r^2}\right) + \pi r^2$ M1

$\therefore A = \left(\frac{4\pi}{3r} + \pi r^2\right)$ A1

iii. As cost $\propto A$, $\frac{dA}{dr} = 0$ for minimum cost. A1

$\frac{dA}{dr} = -\frac{4\pi}{3}r^{-2} + 2\pi r$ A1

$\therefore \left(-\frac{4\pi}{3}\right) + 2\pi r^3 = 0$ M1

$\therefore 6r^3 = 4$

$\therefore r = \sqrt[3]{\frac{2}{3}}$

$= 0.87$ m or 87 cm A1

iv. Substituting $r = \sqrt[3]{\frac{2}{3}}$ into the area formula from **a. ii.**:

$A = \frac{4\pi}{3\sqrt[3]{\frac{2}{3}}} + \pi\left(\sqrt[3]{\frac{2}{3}}\right)^2$

$= 7.19\ \text{m}^2$ A1

Students could also use graphics calculators to find the minimum value when A vs r is plotted.

b. **i.** As $\tan\alpha = \frac{r}{h}$, $r = h\tan\alpha$ A1

ii. As $l^2 = h^2 + r^2$, $l = \sqrt{h^2 + r^2}$ A1

iii. $A = \pi rl$

$= \pi(h\tan\alpha)\sqrt{h^2 + r^2}$

$= \pi(h\tan\alpha)\sqrt{h^2 + h^2\tan^2\alpha}$ M1

$\therefore A = \pi h^2 \tan\alpha\sqrt{1 + \tan^2\alpha}$ A1

iv. when $\alpha = \frac{\pi}{6}$, $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore A = \pi h^2\left(\frac{1}{\sqrt{3}}\right)\sqrt{1+\frac{1}{3}}$ M1

$= \pi h^2 \frac{1}{\sqrt{3}}\sqrt{\frac{4}{3}}$

$\therefore A = \frac{2}{3}\pi h^2$ A1

v. using $\frac{\delta A}{\delta h} \approx \frac{dA}{dh}$

$\delta A \approx \frac{dA}{dh} \times \delta h$

$\approx \frac{4\pi}{3} \times h \times \delta h$ M1

When $h = 1$ and $\delta h = 0.01$, $\delta A = \frac{4\pi}{3} \times 1 \times 0.01$

$= \frac{0.04\pi}{3}$ or $0.042\ \text{m}^2$ A1

Total 18 marks

Question 3

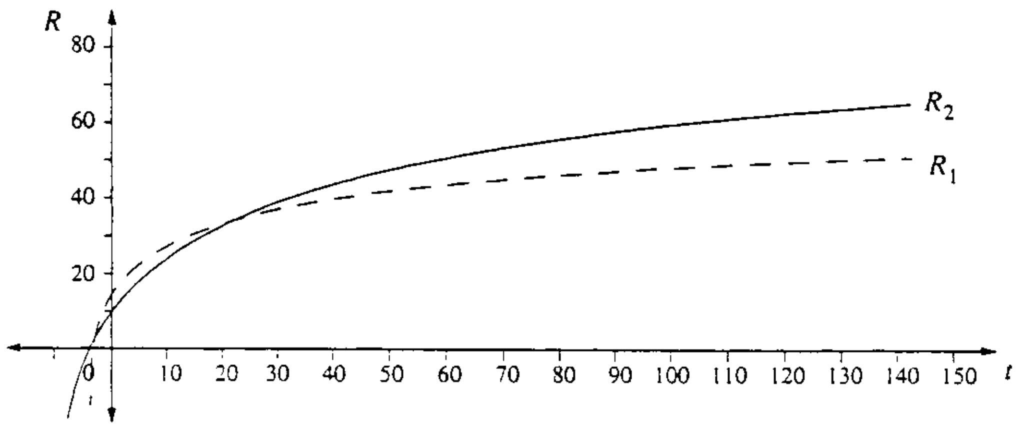
a. i. $R_1(0) = 15 + 24\log_e\left(1+\frac{0}{4}\right) = 15 + 24\log_e(1) = 15$ or \$1500. A1

$R_2(0) = 11 + 45\log_e\left(1+\frac{0}{9}\right) = 11 + 45\log_e(1) = 11$ or \$1100. A1

ii. $R_1(50) = 15 + 24\log_e\left(1+\frac{50}{4}\right) = 15 + 24\log_e(13.5) = 77.464552$ or \$7746. A1

$R_2(50) = 11 + 45\log_e\left(1+\frac{50}{9}\right) = 11 + 45\log_e(6.5555556) = 95.614079$ or \$9561. A1

iii.



Correct shapes for the graphs. A1
Graphs the correct way around and properly labelled. A1
Graphs cross each other as indicated by earlier parts. A1

b. **i.** $R_1 = 15 + 24\log_e\left(1 + \frac{t}{4}\right)$

The inverse (using 'neutral variables') is $x = 15 + 24\log_e\left(1 + \frac{y}{4}\right)$

Correct method to invert the function using exponentials. M1

$$x = 15 + 24\log_e\left(1 + \frac{y}{4}\right)$$

$$24\log_e\left(1 + \frac{y}{4}\right) = x - 15$$

$$\log_e\left(1 + \frac{y}{4}\right) = \frac{x-15}{24}$$

$$1 + \frac{y}{4} = e^{\left(\frac{x-15}{24}\right)}$$

$$\frac{y}{4} = e^{\left(\frac{x-15}{24}\right)} - 1$$

$$y = 4\left(e^{\left(\frac{x-15}{24}\right)} - 1\right)$$ A1

Domain = range of original function = $\left[15, 15 + 24\log_e\left(1 + \frac{150}{4}\right)\right] = [15, 102.6158]$. A1

Range = domain of original function = $[0,150]$. A1

ii. $y = 4\left(e^{\left(\frac{x-15}{24}\right)} - 1\right)$ using the variables of the problem is: $t = 4\left(e^{\left(\frac{R-15}{24}\right)} - 1\right)$.

Substituting $R = 90$ into the inverse. M1

$$t = 4\left(e^{\left(\frac{90-15}{24}\right)} - 1\right) = 4(e^{(3.125)} - 1) = 87.03958$$ A1

c. **i.** Total revenue $R_T = R_1 + R_2$ M1

$$R_T = 15 + 24\log_e\left(1 + \frac{t}{4}\right) + 11 + 45\log_e\left(1 + \frac{t}{9}\right)$$

$$= 26 + 24\log_e\left(1 + \frac{t}{4}\right) + 45\log_e\left(1 + \frac{t}{9}\right)$$ A1

ii. $R_T(36) = 26 + 24\log_e\left(1 + \frac{36}{4}\right) + 45\log_e\left(1 + \frac{36}{9}\right)$ M1

Use of log laws to simplify the expression as specified. M1

$$R_T(36) = 26 + 24\log_e\left(1 + \frac{36}{4}\right) + 45\log_e\left(1 + \frac{36}{9}\right)$$

$$= 26 + 24\log_e(10) + 45\log_e(5)$$

$$= 26 + \log_e(10^{24}) + \log_e(5^{45})$$

$$= 26 + \log_e((10^{24})(5^{45}))$$

So $A = 26$, $B = (10^{24})(5^{45}) = 2.8421709 \times 10^{55}$ A1

Question 4

a.

x	0	1	2	3
$\Pr(X = x)$	0.1866	0.4198	0.3149	0.0787

X is binomial

$\Pr(X = 2) = {}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^1 = 0.3149$ A1

$\Pr(X = 3) = {}^3C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^0 = 0.0787$ A1

b. The mean of this binomial $\mu = E(X) = np$

$\therefore\ E(X) = 3 \times \frac{3}{7}$

$= \frac{9}{7}$ or $1\frac{2}{7}$ A1

c. The variance $\text{VAR}(X) = np(1-p)$

$= 3 \times \frac{3}{7} \times \frac{4}{7}$

$= 0.7347$ A1

d.

$y(\$)$	−10	0	+2	+10
$\Pr(Y = y)$	0.1866	0.4198	0.3149	0.0787

A2
(deduct 1 for each error)

e. $E(X) = \Sigma(y \cdot \Pr(Y = y))$

$= -10 \times 0.1866 + 2 \times 0.3149 + 10 \times 0.0787$

$= -\$0.4492$ (i.e. 45 cents lost for every \$10 spent) A1

f.

x	0	1	2	3
$\Pr(X = x)$	0.114	0.514	0.343	0.029

X is hypergeometric

$\Pr(X = 2) = \dfrac{{}^3C_2 \times {}^4C_1}{{}^7C_3} = 0.343$ A1

$\Pr(X = 3) = \dfrac{{}^3C_3 \times {}^4C_0}{{}^7C_3} = 0.029$ A1

g. $E(X) = n\frac{D}{N}$ $n = 3$ $D = 3$ $N = 7$

$= 3 \times \frac{3}{7}$

$= \frac{9}{7}$ or $1\frac{2}{7}$ A1

h. $\mathrm{VAR}(X) = \frac{nD(n-D)(N-n)}{N^2(N-1)}$

$= \frac{3 \times 3 \times 4 \times 4}{49 \times 6}$

$= 0.4898$ A1

i.

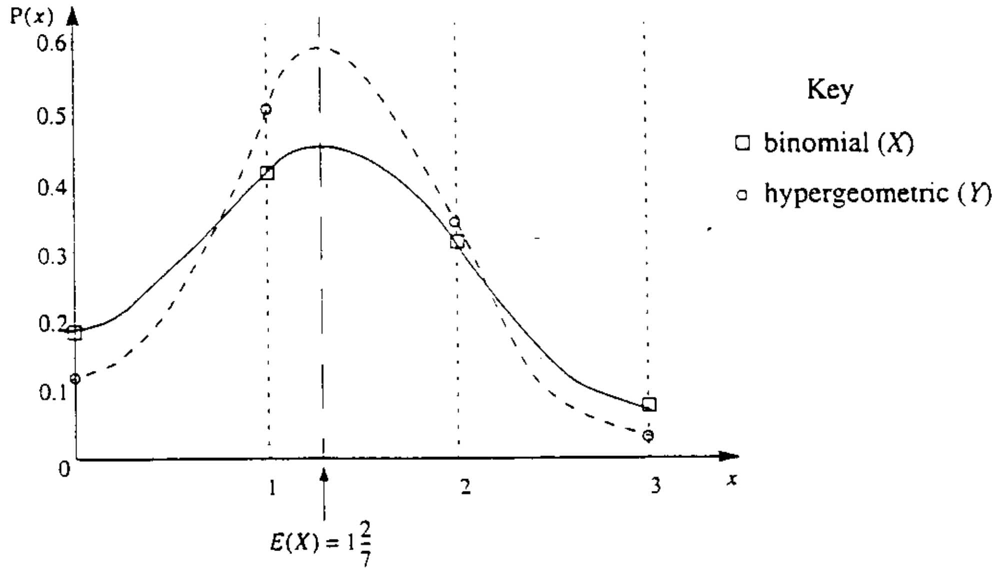
$y(\$)$	−10	0	+2	+10
$\Pr(Y = y)$	0.114	0.514	0.343	0.029

$E(Y) = -10 \times 0.114 + 2 \times 0.343 + 10 \times 0.029$ M1

$= \$\text{-}0.164$ A1

j. The second game loses 16.4 cents for each $10 spent, whereas the first game loses 45 cents for each $10 wagered. Hence the latter game (NOT REPLACING THE BALL) is better for the player. A1

k.



A2