The Mathematical Association of Victoria

2000

MATHEMATICAL METHODS

Trial Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

Directions to students

This examination consists of four questions.

Answer all questions.

All working and answers should be written in the spaces provided.

The marks allotted to each part of each question appear at the end of each part.

There are **61 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the 2000 Mathematical Methods Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

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Question 1

Oil spills in oceans and seas can do untold damage to wildlife and their habitats. On December 25, 1996, a large oil tanker transporting its cargo of heavy crude oil past France's Mediterranean coastline sank due to cracks opening up in the rusting hull. After sinking, the tanker continued to leak its cargo into the sea creating an oil slick that started to spread evenly in a circle. The oil slick started to spread at 9am. The radius, R kilometres of the oil slick, can be determined at any time, t hours, after 9am to be

$$R = \frac{t}{2}, \text{ where } t \geq 0$$

a. **i.** What was the radius of the oil slick at 10am, December 25, 1996? Give your answer in kilometres.

[1 mark]

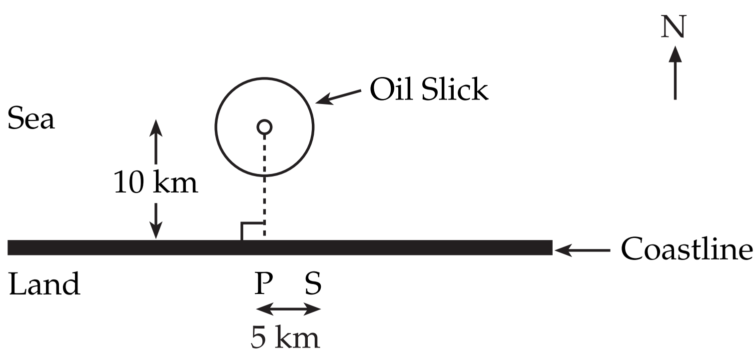
ii. What was the exact area of the oil slick at 10am, December 25, 1996? Give your answer in square kilometres.

[1 mark]

b. What is the rate of increase of the radius with respect to time of the oil slick?

[1 mark]

The tanker sank exactly 10 kilometres from the small town of Pinot (P) on the French coast. Pinot is one of many small towns that exist along this part of the coast and its neighbouring town, Swino (S) is only 5 kilometres away. A diagram of this area is shown below.



c. Assuming the same weather conditions and the same rate of increase of the radius of the oil slick, at what time, and on which day, did the oil reach the beach at Pinot?

[2 marks]

d. How far is Swino from the centre of the oil slick? Give your answer in kilometres, correct to one decimal place.

[1 mark]

e. Assuming the same weather conditions and the same rate of increase of the radius, at what time and which day did the oil reach the beach at Swino? Give your answer to the nearest minute.

[2 marks]

A similar accident happened approximately 3 years later, at 1pm on April 25. This spill was 5 kilometres directly north of Swino. Swino has oil spill protection equipment. The equipment consists of large booms that soak up crude oil and a boat for placing these booms in a circle around the oil slick. The total length of the oil slick booms is 4 kilometres. The radius, R kilometres, of the new oil slick can be determined, at any time t hours after 1pm on Anzac Day, to be

$$R = kt, \text{ where } k \text{ is a real constant}$$

f. At 3 pm on April 25 the circumference of this slick had grown to $\frac{2\pi}{5}$ kms. Find the value of k.

[2 marks]

g. Travelling in a circle, the boat was able to spread the booms around the oil slick immediately at 1pm on April 25. The distance, x km travelled by the boat in spreading the booms, t hours after 1pm on April 25, is given by

$$x = \frac{t^2}{10}, \text{ where } t \geq 0$$

How far from the centre of the slick would the boat start spreading the booms if the slick is to be encircled? Give an exact answer.

[3 marks]

h. What is the total length of booms that remain on the boat once the slick is encircled? Give an exact answer.

[2 marks]

Total 15 Marks

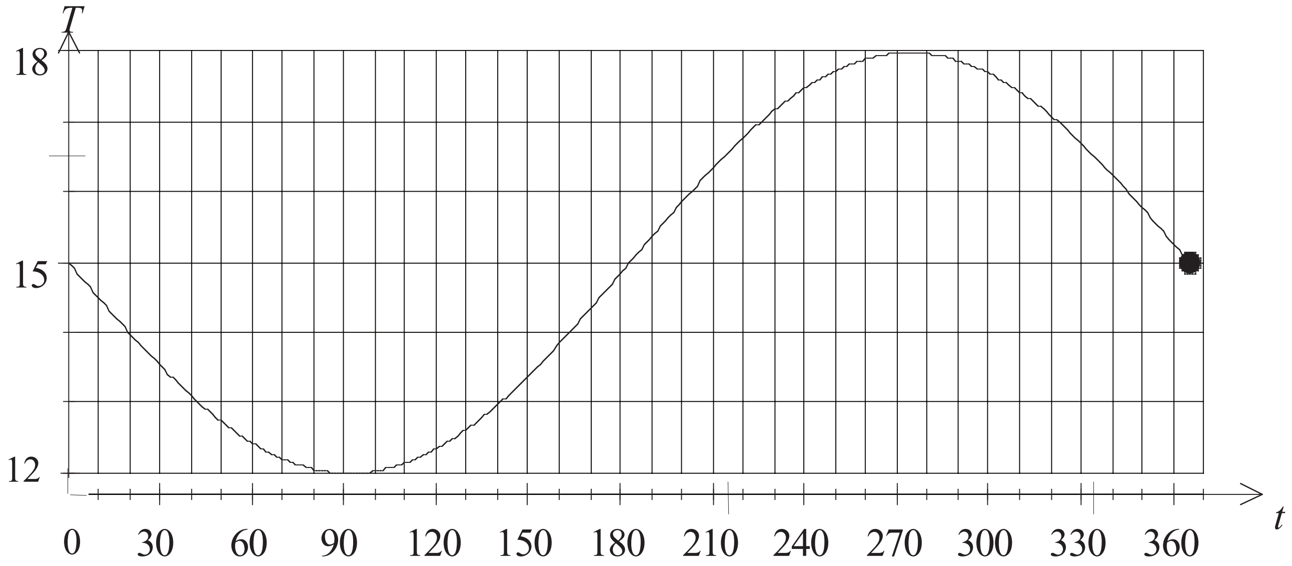
Question 2

Port Phillip Bay's water temperature varies throughout the year. Its maximum temperature is during summer and its minimum temperature is during winter. The water temperature can be modelled by

$T(t) = A\sin nt + B$, where $0 \le t \le 365$, A, n and B are real constants,

T is the temperature of the water in degrees Celsius and t is the time in days.

The graph of this relationship is shown below, where the 1st day is taken as $t = 0$. The temperature of the water at $t = 0$ is 15 °C



a. **i.** From the graph, determine the value of B.

[1 mark]

ii. From the graph, determine the value of A.

[1 mark]

iii. From the graph, determine the value of n and hence the full equation for the graph.

[2 marks]

b. Calculate the maximum water temperature and the value of t for which it occurs? Give the value of t to the nearest day.

[2 marks]

Gus Dragnet owns a fleet of scalloping boats that operate in Port Phillip Bay. Gus knows the best temperature for scalloping is equal to or above 16.5 °C.

c. From the graph determine the optimum domain of days in which Gus should use his boats for scalloping.

[2 marks]

In Port Phillip Bay there are two types of fish — Snapper and Flathead. These increase and decrease their population during different times of the year, depending on the water temperature.

The percentage increase in the Snapper population can be modelled by

$$P(t) = k\cos\frac{2\pi t}{365}, \text{ where } 0 \leq t \leq 365, \ k \text{ is a real constant}$$

and P is the population increase in terms of per cent.

When $t = 182.5$, Snapper decreased its population by 3%.

d. Find the value of k.

[1 mark]

The percentage increase in the Flathead population can be modelled by

$$F(t) = \sin\frac{2\pi t}{365}, \quad \text{where } 0 \le t \le 365, \; F \text{ is the percentage population increase}$$

e. What is the value of t when both fish species have an equal population increase? Give your answer to the nearest day.

[2 marks]

f. What is the value of t when both fish species have an equal population decrease? Give your answer to the nearest day.

[1 mark]

g. If $t = 0$ is the 1st of May 1998, determine the actual date for your answers to **e** and **f**.

[2 marks]

Total 14 marks

Question 3

Dr. Appleton developed a tablet for the treatment of a red body rash in children under 2 years of age. The tablet had a 100% success rate in curing the children, though some children needed a longer treatment period than others before the rash vanished.

The probability that Dr. Appleton's tablet will cure the children of the rash in a certain number of days, where k is a constant is given in the table below.

x, Number of Days	1	2	3	4
$\Pr(X = x)$	$3k^2$	$\frac{7k}{6}$	$\frac{7k}{12}$	$\frac{k}{4}$

a. Find the exact value of k.

[3 marks]

b. Find the exact probability of a child being cured in 2 days or less.

[2 marks]

The active ingredient in the tablet is petaline. The percentage concentration of petaline absorbed into the child's blood stream via the stomach wall can be measured by the formula:

$$C(t)=\frac{1}{2}\left(-t^3+\frac{8t^2}{3}+11t\right),\quad 0\leq t\leq 4.9$$

C is the percentage concentration of petaline in the child's blood stream and *t* is the time in hours after the tablet was swallowed by the child.

c. Using Calculus find the maximum concentration of petaline in the child's body and when this concentration occurs.

[4 marks]

d. What is the rate of increase of the concentration of petaline at $t=\frac{8}{9}$ hours? Give your answer to the nearest percentage.

[1 mark]

Dr. Appleton sells his tablets in boxes of 20. He claims on the side of the box that the concentration of petaline in each tablet follows a normal distribution with a mean of p% and a standard deviation of d %.

To satisfy the requirements of the Australian Drug Authority, he submitted his tablets to tests. These tests found the probability of a tablet having a concentration of petaline less than $(p-4)$% is 0.1587 and the probability of a tablet having a concentration of petaline greater than $(\frac{3}{2}p-10)$% is 0.4013.

e. Find the value of d.

[3 marks]

f. Find the value of p.

[3 marks]

Total 16 marks

Question 4

The Securities Commission, the board that oversees the Australian Stock Exchange, has recently been worried about the rapid increase in the share prices of some stocks listed on the Australian Stock Exchange. Stocks are only traded on working days (i.e., Monday to Friday). The commission is concerned that some people have inside information on a stock, use it to their advantage and buy the stock before the general public knows the good news.

One stock being investigated is "Eloop," an Internet company that produces its own search engine for the Internet. Eloop's stock price over the last 5 days has been modelled by the equation

$P(t) = Ae^{t} + B$, where $0 \le t \le 5$, t is the number of trading days,
P is the price of the stock in cents and A and B are real constants.

$t = 0$ is defined as the start of trading on Monday 14th of August and continues until $t = 5$ which is defined as the start of trading on Monday 21st of August.

At the start of trading on Monday 14th of August Eloop's price was 20 cents and its price at the start of trading on Thursday 17th of August was 25 cents.

a. **i.** Using the price and t-value for Monday 14th of August, write an equation that involves A and B.

[1 mark]

ii. Using the price and t-value write for Thursday 17th of August, write an equation that involves A and B.

[1 mark]

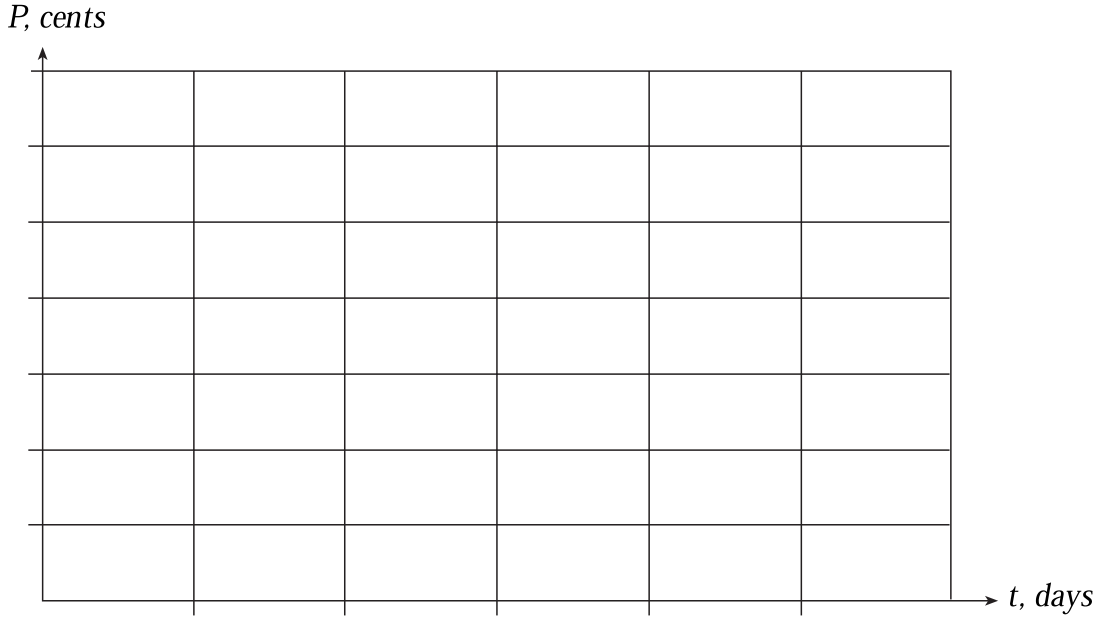
iii. Using these two equations, or otherwise, find the values of A and B. Give the values of A and B correct to 3 decimal places.

[3 marks]

b. According to the model, what price would Eloop's stock be at the start of trading on Monday 21st August? Give your answer correct to one decimal place.

[1 mark]

c. Sketch the graph of Eloop's price below, labelling the axes and any axial intercepts.



[2 marks]

The Securities Commission uses the area under the price curve to determine whether they will investigate the company for inside information trading.

d. **i.** Find the area under the price curve between the start of trading on Tuesday 15th and the start of trading on Wednesday 16th of August. Give your answer correct to two decimal places.

[2 marks]

ii. Find the area under the price curve between the start of trading on Wednesday 16th and the start of trading on Thursday 17th of August. Give your answer correct to two decimal places.

[2 marks]

iii. What is the percentage increase in the area under the price curve between the Tuesday - Wednesday area and the Wednesday -Thursday area? Give your answer to the nearest per cent.

[1 mark]

If the increase in the area under the price curve is greater than 50% over a day compared to the previous day's area, an investigation will be undertaken.

e. **i.** At the end of which day will the securities commission launch its investigation?

[2 marks]

ii. What was the value of the percentage increase that caused this investigation? Give your answer to the nearest percent.

[1 mark]

Total 16 marks

END OF TRIAL EXAMINATION 2