The Mathematical Association of Victoria

2000

MATHEMATICAL METHODS

Trial Examination 1

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **30 marks** available for this part.

Part II consists of seven questions. Answers all questions in **Part II** in the spaces provided. There are **20 marks** available for this part.

There are **50 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the 2000 Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

Published by The Mathematical Association of Victoria
"Cliveden", 61 Blyth Street, Brunswick, 3056
Phone: (03) 9380 2399 Fax: (03) 9389 0399
E-mail: office@mav.vic.edu.au website: http://www.mav.vic.edu.au

Multiple-Choice Answer Sheet

Student's Name _____

Cross through the letter that corresponds to each answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E

Part I (Multiple-choice questions)

Question 1

The gradient of a line joining the two coordinates A (–3, 4) and B (1, –3) is

A. $-\frac{7}{4}$

B. $-\frac{4}{7}$

C. –1

D. $-\frac{1}{4}$

E. 0

Question 2

The graph of $y = 3\cos 2(\frac{3}{2}x + \frac{\pi}{4}) + 2$ has a period of

A. $\frac{\pi}{8}$

B. $\frac{\pi}{4}$

C. $\frac{2\pi}{3}$

D. π

E. 2π

Question 3

Given $f(x) = \frac{4}{3}x^3 - 2x^2 - kx + 5$ and $f'(3) = 2$ the value of k must be

A. $\frac{11}{3}$

B. 5

C. 22

D. 24

E. 28

Question 4

The graph shown in figure 1 has a formula $y = a \sin b(x - c)$.

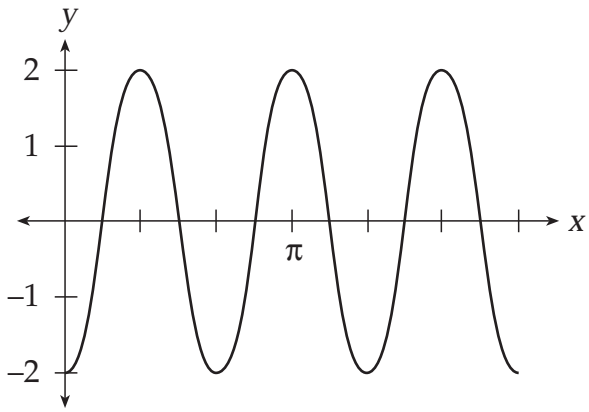


Figure 1

The values of a, b and c respectively are

A. 3, 2, $\frac{\pi}{2}$

B. 3, 2, $-\frac{\pi}{2}$

C. 2, 2, $\frac{\pi}{2}$

D. 2, 3, $-\frac{\pi}{6}$

E. 2, 3, $\frac{\pi}{6}$

Question 5

The coefficient of the x^8 term in the expansion of $(3x^2 - 4)^5$ is

A. –1620

B. –405

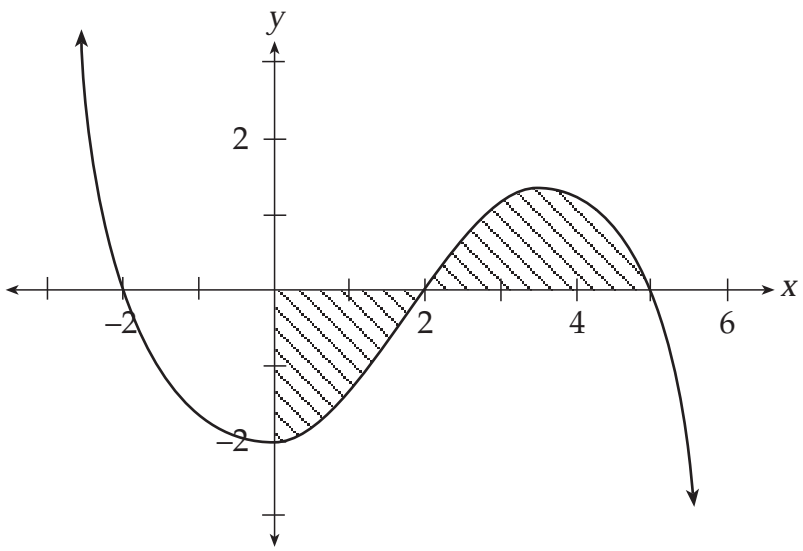
C. 60

D. 720

E. 2160

Question 6

The graph of the function $y = \frac{1}{10}(-x^3 + 5x^2 + 4x - 20)$ is shown below.



Using a graphics calculator, or otherwise, the value of the shaded area can be found to be closest to

A. 0.21

B. 2.475

C. 3.51

D. 4.74

E. 5.25

Question 7

The distance Samantha Welsh travelled on her pushbike was given by

$$x = 2t^2 + 3t \text{ , } t > 0,$$

where t is the time in hours and x is the distance in kilometres. The average rate of change of the distance Samantha travelled between $t = 3$ and $t = 7$ is

A. 16 km/hr

B. 23 km/hr

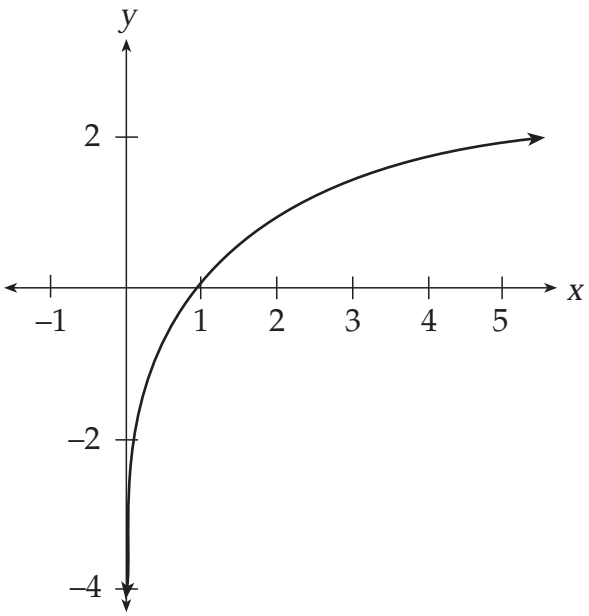
C. 27 km/hr

D. 92 km/hr

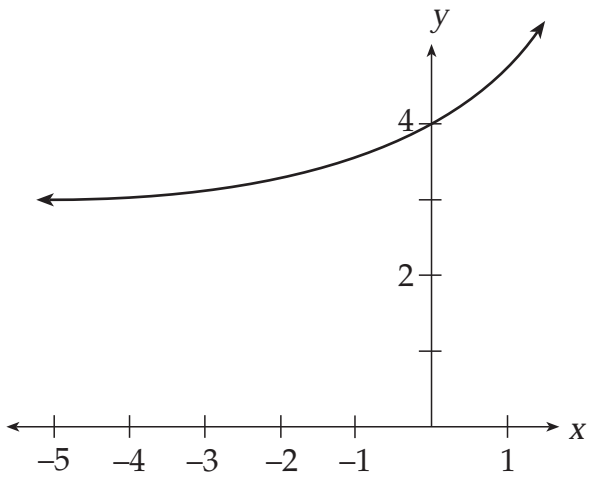
E. 119 km/hr

Question 8

The graph A can be transformed into graph B by



Graph A



Graph B

A. A reflection in the line y = *x* and a translation of 3 units along the *x*-axis in a positive direction.

B. A reflection in the line y = *x* and a positive translation of 3 units parallel to the *y*-axis

C. A reflection in the *x*-axis and a translation of 3 units along the *x*-axis in a negative direction

D. A reflection in the *x*-axis and a negative translation of 3 units parallel to the *y*-axis

E. A reflection in the *y*-axis and a positive translation of 3 units parallel to the *y*-axis

Question 9

Simsang, a microwave oven maker, packed its ovens into boxes of twelve. Steve performed a test on these boxes before they were sent to the retail shops. Steve selected 4 ovens at random without replacement from a box and, if all 4 worked, the box was passed and sent to the shops. If (one or more) of the ovens failed the test, the box was failed and all the microwaves were checked. What is the probability that a box with 4 defective microwaves will pass Steve's test?

A. $\frac{224}{495}$

B. $\frac{3}{5}$

C. $\frac{16}{165}$

D. $\frac{56}{165}$

E. $\frac{14}{99}$

Question 10

Tyler Hood, a descendent of Robin Hood, shoots arrows at a target. The percentage of Tyler's bullseyes is normally distributed with a mean of 34 and a standard deviation of 7. What is the approximate probability that from a bag of 50 arrows, Tyler shoots between 30% and 39% of his arrows in the bullseye?

A. 0.1745

B. 0.3271

C. 0.4786

D. 0.5214

E. 0.7436

Question 11

The area of the region bound between the two graphs $f(x)$ and $g(x)$, where

$$f(x) = -2x^2 + 4x + 5 \text{ , } x \in \mathrm{R}$$

$$g(x) = \frac{1}{2}(3x^2 - 13x + 24) \text{ , } x \in \mathrm{R}$$

can be calculated by solving which of the following integrals?

A. $\int_0^2 [f(x) - g(x)]\,dx$

B. $\int_1^2 [f(x) - g(x)]\,dx$

C. $\int_1^2 [g(x) - f(x)]\,dx$

D. $\int_0^2 [g(x) - f(x)]\,dx$

E. $\int_0^2 [f(x) + g(x)]\,dx$

Question 12

The value of the definite integral $\int_1^4 \left(x^2 - \frac{1}{x}\right)dx$ is

A. $21 - \log_e 4$

B. $-21 + \log_e 4$

C. 15

D. $15\frac{3}{4}$

E. $\frac{64}{3} - \log_e 5$

Question 13

The value of x that is the solution to the equation $5e^{3x} = 6$ is?

A. 0

B. $\frac{1}{3}\log_e 30$

C. $\log_e 10$

D. $\frac{1}{3}\log_e \frac{6}{5}$

E. $\frac{6}{5e^3}$

Question 14

The value for x in the equation $\frac{1}{2}\log_{10} x + \frac{2}{3}\log_{10} 125 - \frac{1}{3}\log_{10} 27 = 2$ is

A. 0.9208

B. 2

C. 2.176

D. 144

E. 150

Question 15

The function, $f: R \to R,$ where $f(x) = e^{2x}\cos x$ has a derivative $f'(x)$, the value of $f'(x)$ is equal to

A. $e^{2x}\cos x$

B. $e^{2x}(2\sin x - \cos x)$

C. $e^{2x}(2\sin x + \cos x)$

D. $e^{2x}(2\cos x - \sin x)$

E. $2e^{2x}\cos x$

Question 16

From the graph in figure 2, the rate of change of $f(x)$ is positive in which interval(s)? Choose the best answer from the available information.

A. $\{x < -2\} \cup \{2 < x < 5\}$

B. $\{x < -2\} \cup \{2 \leq x \leq 5\}$

C. $\{-0.4 < x < 3.8\}$

D. $\{x < -0.4\} \cup \{x > 3.8\}$

E. $\{x \leq -0.4\} \cup \{x \geq 3.8\}$

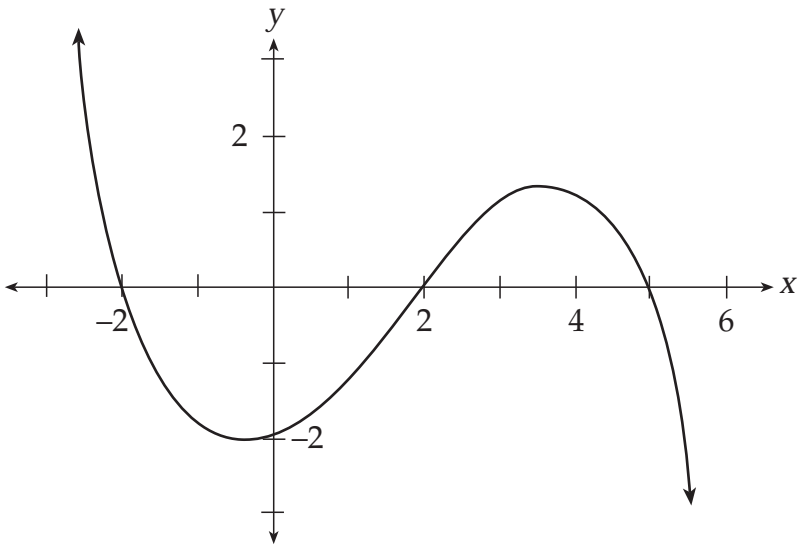


Figure 2

Question 17

The sum of the solutions of $2\cos(x) = 1$ where $0 \leq x \leq 2\pi$ is

A. 0

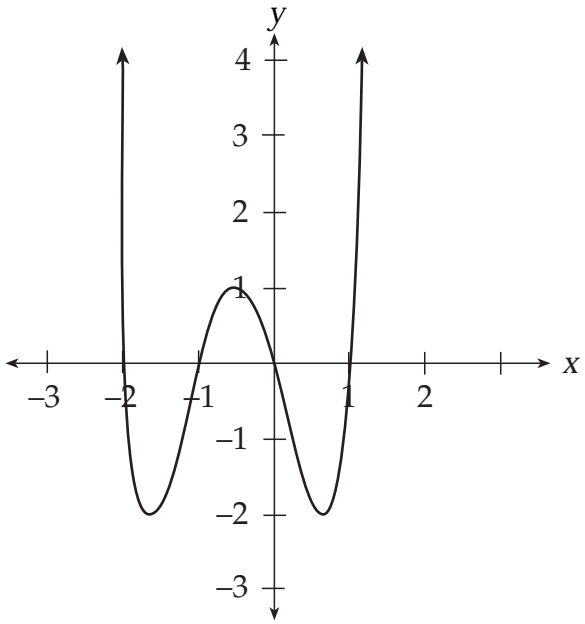
B. $\frac{\pi}{3}$

C. π

D. $\frac{5\pi}{3}$

E. 2π

Question 18



The factorised form of the equation for the above graph is closest to

A. $2x(x + 2)(x + 1)(x - 1)$

B. $x(x + 2)(x - 1)(x + 1)$

C. $x(x - 2)(x - 1)(x + 1)$

D. $2x(x - 2)(x - 1)(x + 1)$

E. $x(x + 2)(x - 1)2$

Question 19

The graph of the equation $y = 1 - \frac{3}{x - 2}$ has asymptotes with the equations

A. $x = 2, \quad y = 3$

B. $x = -2, \quad y = 2$

C. $x = 2, \quad y = 3$

D. $x = 2, \quad y = 1$

E. $x = -2, \quad y = 1$

Question 20

The solution(s) to the equation $\sin(2x) = -\cos(2x)$ $0 \le x \le 2\pi$ are

A. $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

B. $-\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

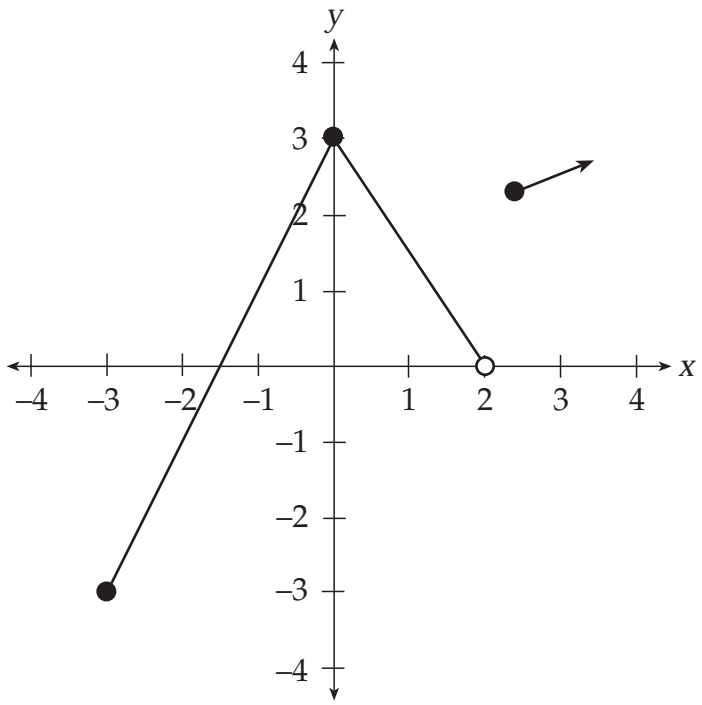
C. $-\frac{\pi}{8}, \frac{3\pi}{8}$

D. $\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

E. $\frac{3\pi}{4}, \frac{7\pi}{4}$

Question 21

The graph of the function f is shown below.



In order for the inverse f^{-1} to exist, a possible restricted domain of f is

A. $x \in [0, 3]$

B. $x \in [-3, 0] \cup [0, 3]$

C. $x \in [0, 2) \cup [3, \infty)$

D. $x \in [-3, 0) \cup [3, \infty)$

E. $x \in [-3, 2]$

Question 22

For equations of the type $y = A(x + b)^2 + B$ their graphs will always have two x-intercepts when

A. $A + B > 0$

B. $b^2 > 0$

C. $2Ab > 0$

D. $-4AB > 0$

E. $-AbB > 0$

Question 23

The following data would most likely be representative of what type of function?

X	–5	–4	–3	–2	–1	0	1	2	3
Y	6	2	0	0	2	6	12	20	30

A. Power

B. Polynomial

C. Circular

D. Exponential

E. Linear

Question 24

The NASA Star Wars defence system protects America from ballistic missile attack by shooting down the ballistic missile with another missile. In recent trials NASA has found that the probability of hitting a missile is 0.38. If four missiles were launched at America, the probability of three or more of the ballistic missiles being shot down is

A. ${}^4C_3(0.62)^3(0.38)^1 + (0.62)^4$

B ${}^4C_3(0.38)^3(0.62)^1 + (0.38)^4$

C. $12 \times (0.38)^3(0.62)^4$

D. ${}^4C_3(0.38)^3 + (0.38)^4$

E. $(0.38)^3(0.62)^1 + (0.38)^4$

Question 25

A graph of the form $y = A\sin(Bx)$, where A and B are constants, passes through the point $\left(\frac{\pi}{12}, \frac{3}{2}\right)$ and has a period of π. Its full equation is

A. $y = 2\sin\left(\frac{\pi x}{12}\right)$

B. $y = \frac{3}{2}\sin(2x)$

C. $y = 3\sin\left(\frac{3}{2}x\right)$

D. $y = 3\sin(2x)$

E. $y = 2\sin(3x)$

The following refers to question 26 and 27

The following is a probability distribution for a biased die

X	1	2	3	4	5	6
$Pr(X=x)$	$0.05 + k$	$0.5 - 3k$	0.1	$4k$	$k + 0.1$	$2k$

Question 26

The probability of rolling an even number is

A. $3k + 0.5$

B. $2k + 0.25$

C. 0.5

D. $4k$

E. $2k$

Question 27

The probability of rolling a total of eleven or more for two rolls of this die is

A. $k(4k + 0.4)$

B. $3k + 0.1$

C. $k + 0.1$

D. $k(8k + 0.4)$

E. $4k2$

Question 28

If $f'(x) = -2\cos(4x)$, then $f(x)$ is equal to

A. $\cos(4x) + c$

B. $-8\sin(4x) + c$

C. $-\frac{1}{2}\sin(4x) + c$

D. $(\sin 4x + \cos 4x) + c$

E. $-\frac{1}{4}\cos(4x) + c$

Question 29

The heights of a sample of 305 women are normally distributed, with a mean of 1.67 metres and a standard deviation of 4cm. How many women from this sample, to the nearest one person, would be shorter than 1.65m?

A. 67

B. 94

C. 102

D. 152

E. 210

Question 30

Joanne Hingis, the younger sister of Martina has a probability of a successful first serve in tennis of 0.65. In Joanne's first round match at the Australian Open, she served 60 individual points. Assuming her serves are independent, what is Joanne's expected mean and standard deviation for her first serve, correct to 2 decimal places.

A. $\overline{X} = 39 \quad \sigma_X = 13.65$

B. $\overline{X} = 39 \quad \sigma_X = 3.69$

C. $\overline{X} = 21 \quad \sigma_X = 13.65$

D. $\overline{X} = 21 \quad \sigma_X = 3.69$

E. $\overline{X} = 60 \quad \sigma_X = 0.65$

Part II (Short Answer Questions)

Question 1

In the binomial expansion of $(ax + b)^5$ the x^3 term has a coefficient of 1080 and the x^1 term has a coefficient of 240. Find the value of a in the binomial expression.

[3 marks]

Question 2

Find $f'(\frac{\pi}{3})$ as an exact value, given $f(x) = 2\tan(x)\sin(x)$.

[3 marks]

Question 3

Note: For this question give each answer correct to 4 decimal places

In one pack of a dozen oysters it is known that three of the oysters are infected with a poison that affects the nervous system of humans. A test has been developed to detect this poison. From the pack a sample of three oysters is selected at random without replacement and tested for the poison.

a. If X represents the number of oysters detected with the poison, fill in the probability distribution table below.

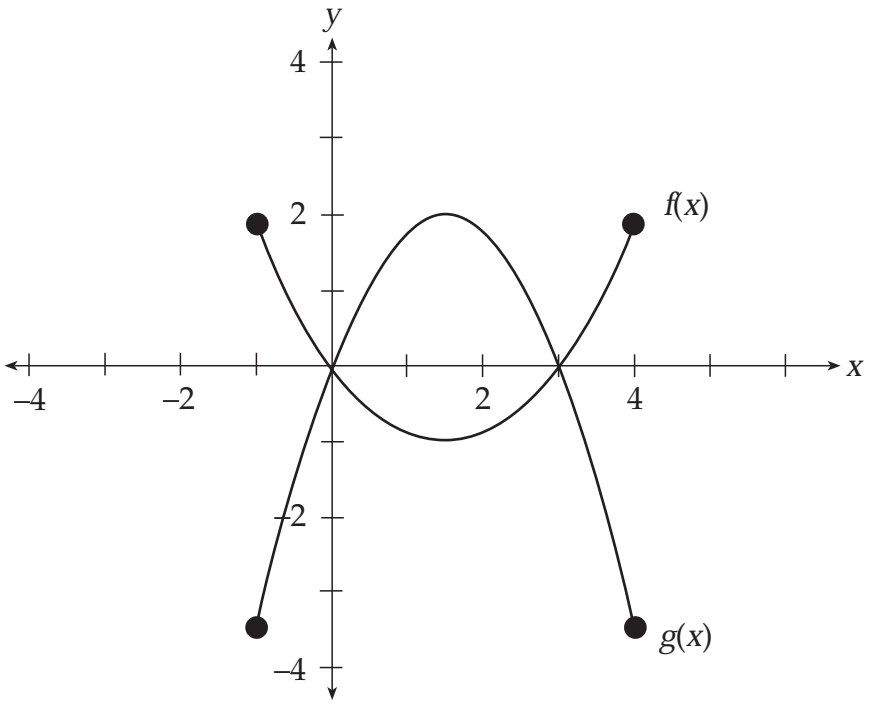
X	0	1	2	3
$\Pr(X = x)$				

b. What is the probability of at least one of the infected oysters being detected?

[3 marks]

Question 4

Two graphs of the functions $f(x)$ and $g(x)$ are shown below. Sketch $h(x)$, $x \in \{-1 \leq x \leq 4\}$ on the same set of axes, given $h(x) = g(x) + f(x)$.



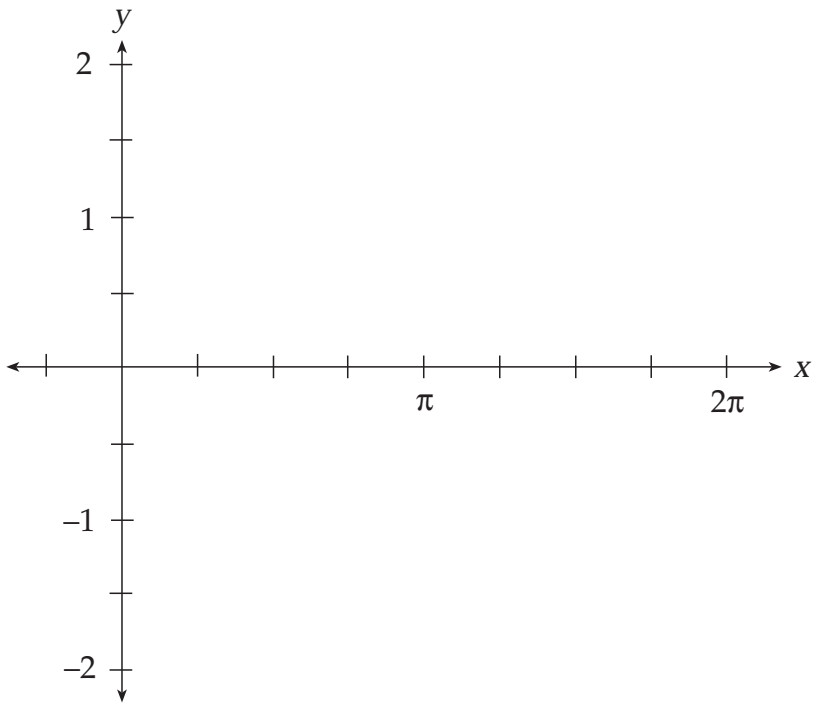
[2 marks]

Question 5

The function $y = \frac{1}{2}\cos 2x,\ x \in \{0 \le x \le 2\pi\}$ undergoes the following transformations:

1. Reflection in the *x*-axis
2. Dilation by +2 parallel to the *y*-axis
3. Translation of 1 unit upwards parallel to the *y*-axis

a. Draw the resultant graph on the axes provided.



b. What is the resultant equation for this new graph?

[3 marks]

Question 6

The graph $y = x^2$ has a tangent at the point (2, 4).

a. Given the tangent's gradient is 4, find the equation of this tangent.

[1 mark]

b. Use integration, or some other suitable method, to determine the area between the tangent, the curve $y = x^2$ and the y-axis.

[2 marks]

Question 7

The equation $5y + 6x = 7$ crosses the curve $y = \frac{1}{5}(7x^2 + x - 35)$ at the points P and Q respectively. Find the distance between the points P and Q. Give your answer as an exact value.

[3 marks]

TOTAL 20 marks

END OF TRIAL EXAMINATION 1

