YEAR 12

IARTV TEST — OCTOBER 2000

MATHEMATICAL METHODS

EXAMINATION 2 (ANALYSIS TASK)

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Time: 1 hour 30 minutes

Directions to students

Materials
Question and answer booklet of 10 pages.
An approved calculator or graphics calculator may be used.
You may bring to this examination two A4 sheets of notes which can be written on both sides.
A protractor, set square and aids for curve-sketching may be used.

The task
Time allowed: 1 hour 30 minutes.
Answer all questions in the spaces provided in this booklet.
There is a total of 64 marks available.

At the end of the task
Hand in this booklet.

No. 25

QUESTION ONE (18 marks)

Two musical groups provide sound waves which have the same frequency but amplitudes of 1 and $\sqrt{3}$ units respectively. A sound engineer measures their outputs as

$$a(t) = \sin(t) \quad \text{and} \quad b(t) = \sqrt{3}\cos(t).$$

The combined sound from both groups can be measured by:

$$c(t) = \sin(t) + \sqrt{3}\cos(t) \quad \text{for} \quad 0 \le t \le 2\pi.$$

You are required to do some calculations to determine the shape of this combined sound.

(a) Find $c'(t)$, i.e. $\dfrac{dc}{dt}$.

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(1 mark)

(b) Use an algebraic method to find the ***exact*** coordinates of the turning points of $y = c(t)$ on the domain $0 \le t \le 2\pi$.

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(4 marks)

(c) State the ***exact*** coordinates of the left endpoint of the function $y = c(t)$ on the domain $0 \le t \le 2\pi$.

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(1 marks)

(d) Use an algebraic method to find the ***exact*** coordinates of the t -intercepts of $y = c(t)$ on the domain $0 \le t \le 2\pi$.

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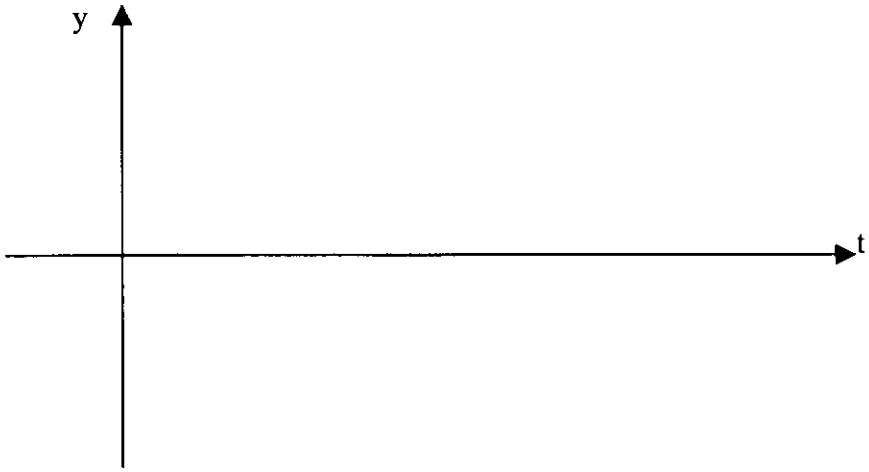
(2 marks)

(e) Sketch the graph of $y = c(t)$. Clearly show endpoints, intercepts, turning points and shape.

(2 marks)



The engineer notices that the curve of the combined sound could also be written in the form
$y = A\cos(B(t - C))$ where A, B, C are constants and $0 \le t \le 2\pi$.

(f) Give the values of the constants.

A=.....

B=.....

C=.....

(2 marks)

(g) Using the result from (f), or a graphics calculator, find the solutions to the equation
$\sin(t) + \sqrt{3}\cos(t) = 1$ for $0 \le t \le 2\pi$.
(Give your answers as either exact values or correct to two decimal places.)

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(2 marks)

No. 25

In order to meet noise regulations, the engineer needs to ensure that the combined wave is restricted to $-\sqrt{3} \leq c(t) \leq \sqrt{3}$ for at least 70% of the time.

(h) What proportion of the time is *c(t)* within the required interval?

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(2 marks)

The engineer is also required to provide a measurement of the average height of the combined wave.

$$\text{Average height} = \frac{\text{Total area under the combined wave}}{2\pi} \text{ on } 0 \leq t \leq 2\pi.$$

(i) Find the average height for the combined wave.

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(2 marks)

QUESTION TWO (18 MARKS)

A Compact Disc (CD) Company estimates that its profit, P, (in thousands of dollars per month) from a new hit recording is given by

$$P = 12 - 15\log_{10}(1+t),$$

where t is the number of months after release.

(a) Find the profit two months after release.

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(1 mark)

(b) Rearrange the formula to make t the subject.

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(2 marks)

(c) Determine the number of months after release when the profit is \$3000 per month.

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(2 marks)

(d) Let

$10^y = x$

i) Show that $y = \dfrac{\log_e x}{\log_e 10}$.

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(2 marks)

No. 25

ii) Hence find $\frac{dy}{dx}$

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(1 mark)

(e) Using the result above, or otherwise, give an expression for the rate of change of profit, i.e. $\frac{dP}{dt}$.

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(2 marks)

(f) State the rate of change of profit after two months and briefly explain its meaning.

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(2 marks)

(g) At what time is the profit reducing at a rate of \$3000 per month^2.

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(2 marks)

(h) When does the CD become unprofitable?

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(2 marks)

(i) If this recording is sold for 7 months, what would be the **total profit** for the company for this recording?

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(2 marks)

QUESTION THREE (13 MARKS)

A Bus Company is planning a holiday trip for a charity group. This holiday trip can take up to 50 passengers. It offers the following charges for the holiday:

i) a total of $8000 when there are 20 passengers or less.
ii) $400 per passenger if there are between 20 and 30 passengers,
iii) $400 per passenger less $10 per passenger for each additional passenger above the 30 passengers (so 32 passengers would pay $380 each).

(a) What will be the cost per passenger if there are:

i) 15 passengers .....

ii) 40 passengers? .....

(2 marks)

(b) Give a hybrid function for the cost per passenger, C, as a function of the number of passengers(n).

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(3 marks)

Let x be the number of additional passengers above the 30 passengers.

(c) Give an expression for the total number of passengers.

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(1 mark)

No. 25

The Bus company finds that there are more than 30 passengers who join and pay for the holiday:

(d) Show that the total receipts, R, for the trip can be given by

$R = 12000 + 100x - 10x^2$ (Note receipts are the amount collected by the company)

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(3 marks)

(e) State the domain for the function R.

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(1 mark)

(f) Use a Calculus approach to find the number of passengers that will maximise the receipts, i.e. R.

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(2 marks)

(g) Briefly justify that the receipts will be a maximum.

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(1mark)

QUESTION FOUR (15 MARKS)

Twelve units are taken from a production run of calculator parts and 4 out of the 12 units are found to be defective. A sample of three is taken randomly from these 12.

(a) Give the probability of selecting one defective part if this sample is made without replacement.

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(2 marks)

An electronics engineer estimates that 25% of the components in a certain batch will fail when tested. One of their models requires three of these components. Two of the three components are backup units (that is, only one component need work properly for the device to work properly).

(b) Assuming independence, find the probability that this model of calculator will work properly when tested.

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(2 marks)

The manufacturer buys "Longlife" batteries for these calculators. The number of hours of operational use of these batteries is normally distributed with a mean of 100 hours and a standard deviation of 10 hours.

(c) State the probability, to three decimal places, that a randomly chosen battery will have an operational life of:

i) less than 85 hours

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ii) between 75 and 85 hours

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iii) more than 75 hours given that it is less than 100 hours.

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(6 marks)

No. 25

(d) Find the number of operational hours that are achieved by 80% of batteries.

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(2 marks)

Each calculator requires two batteries and both must be operating for the calculator to work.

(e) Determine the probability that the calculator will:

i) still be operational after 85 hours,

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ii) fail before 85 hours of operation.

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(3 marks)

END OF BOOKLET