Question 1

Lead contamination of grass is often attributed to leaded petrol used in motor vehicles. Lee's farm is next to a country road. The grass near the roadside is found to have a lead content which can be modelled by the formula

$$L = \frac{A}{x} + B$$

where L milligram per kilogram is the lead concentration in the grass,
x metres is the distance of the grass from the roadside
and A and B are constants.

a. The further away from the roadside the lead content of the grass is measured, the closer its value gets to 2 milligram per kilogram. State the value of B.

1 mark

b. At a distance of 10 metres from the roadside, the lead content in the grass is measured at 50 milligram per kilogram. Calculate the value of A.

1 mark

c. The maximum allowable content of lead in grass used for grazing sheep is 10 milligram per kilogram. What is the least distance from the roadside that Lee can graze sheep in order to meet this requirement?

1 mark

d. A paddock on Lee's farm is rectangular measuring 100 m by 150 m, with the road going along two adjacent sides. What percentage of the paddock is suitable for grazing sheep while meeting the requirement in part **c.**?

Give your answer correct to the nearest per cent.

2 marks

The pollution from the traffic also affects the rate at which the grass is growing. Lee finds that the density of the grass can be modelled by the formula

$$G = e^{0.01x}$$

where x metres is the distance of the grass from the roadside

and G kilogram per square metre is the density of grass.

Lee decides that the overall concentration of lead in the grass, T milligram per square metre, can be modelled by

$$T = L \times G$$

e. Write down an expression for T in terms of x.

1 mark

f. What is the overall concentration of lead (T milligram per square metre) in grass which is 50 metres from the roadside?

Give your answer correct to one decimal place.

1 mark

g. At what rate is the overall concentration of lead in the grass changing, with respect to distance from the roadside, in grass which is 50 metres from the roadside?

Give your answer correct to three decimal places.

3 marks

Total 10 marks

Question 2

Let $f: D \rightarrow R$ where $f(x) = 2 - \log_e(x + 1)$, where D is the largest possible domain for which f is defined.

a. Find D.

1 mark

b. Describe fully a set of transformations which, when applied to the graph of $y = \log_e x$, produces the graph of f. Specify the order in which these transformations are to be applied.

2 marks

c. Find the exact coordinates of the points where the graph of f cuts the axes.

2 marks

d. Use calculus to find the exact value of the gradient of the graph of f when $x = 4$.

2 marks

e. **i.** Find the rule for f^{-1}, the inverse of f.

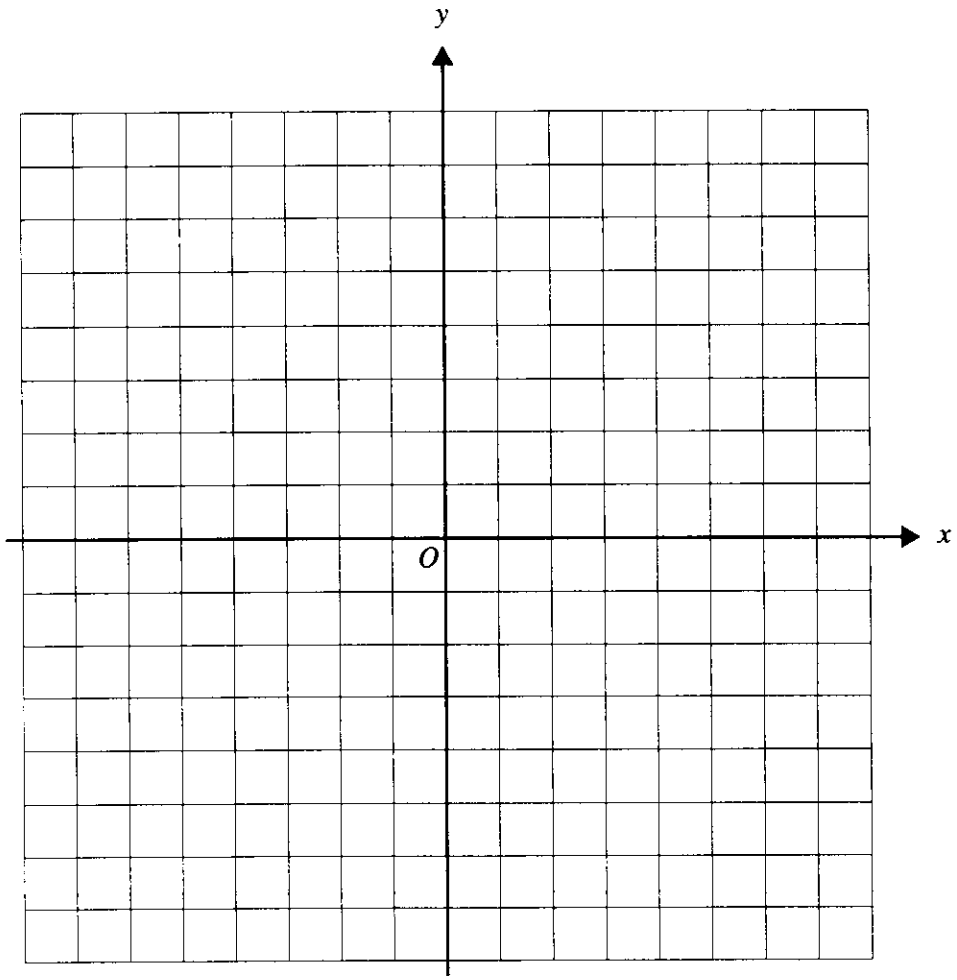
2 marks

ii. Write down the domain of f^{-1}.

1 mark

Working space

f. On the axes below, draw a sketch graph of the inverse function.



2 marks

Working space

g. By considering an appropriate region bounded by the graph of the inverse function, find the exact area of the region enclosed by the graph of f and the coordinate axes.

3 marks

Total 15 marks

Working space

Question 3

A certain piece of software has a millennium bug. The bug will cause data to be damaged, but only when the year is 2000 and only if the user tries to use a particular feature of the software. It is assumed that each use of the software is independent of any other use.

Company Q is a small company which has 5 computers, each loaded with the software.

a. To minimise the damage caused, Company Q decides to copy the data being used with this software on to floppy disks. This will be done on 31 December 1999.

For any one computer, the amount of storage space required is known to be normally distributed with mean 1.2 megabytes and standard deviation 0.3 megabytes.

i. Find the probability, correct to three decimal places, that the data on one computer can be copied on to a disk with a maximum storage space of 1.44 megabytes.

1 mark

ii. A director of Company Q demands that disks with larger storage space be used. She insists that the probability that data from 1 computer can be stored on such a disk is at least 0.85. Find, correct to two decimal places, the minimum amount of storage space required on such a disk.

2 marks

iii. Find the probability, correct to four decimal places, that all of Company Q's 5 computers will individually require more than 1.44 megabytes of storage space.

2 marks

b. It is reliably estimated that the probability that any person using the software package will attempt to use that particular feature (and hence cause the data to be damaged) during the year 2000 is 0.2.

Find the probability, correct to four decimal places, that exactly 4 computers in Company Q will be affected by this millennium bug.

2 marks

c. For each copy of the software package sold, the software company receives \$20. For each of the buyer's computers affected by the millennium bug, the software company will need to spend \$120 to fix the problem. Company Q buys one copy of the software for each of its 5 computers so, for example, if 2 computers in this company are affected by the millennium bug, the **nett loss** for the software company is

$$\$(2 \times 120 - 5 \times 20) = \$140.$$

i. Complete the table below.

Number of computers encountering the bug	0	1	2	3	4	5
Nett loss made by software company			140			

1 mark

ii. Find the probability, correct to four decimal places, that the software company will make a loss of more than \$400 given that at least 4 of company Q's computers are affected by this bug.

2 marks

d. There is another software package for which the probability of failure, p, is also 0.2. In order to estimate this probability more precisely, a random sample will be taken. What sample size is required to estimate the value of p in this case to an accuracy of ± 0.05 with 95% confidence?

3 marks

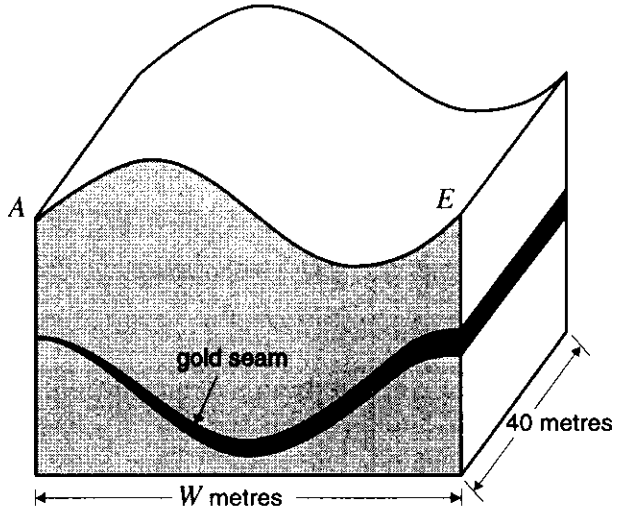
Total 13 marks

Working space

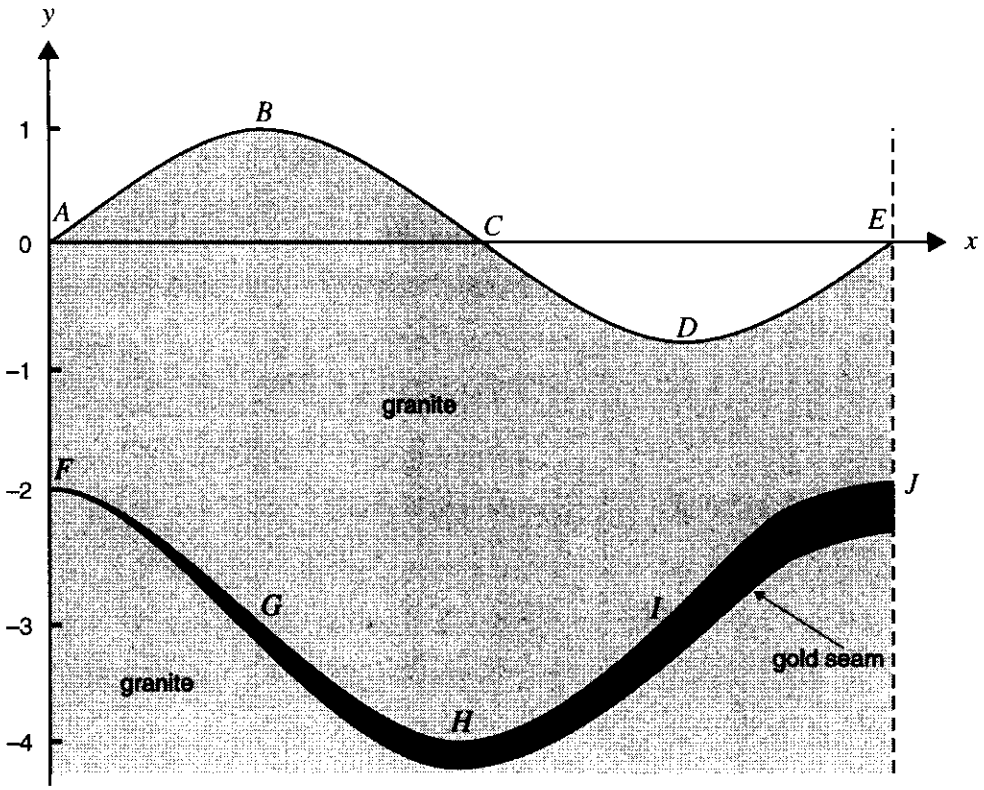
Question 4

Tasmania Jones, prospector and adventurer, is working a small mining site shown below. The site is 40 metres long and W metres wide.

Tasmania has surveyed the site completely so he knows that it has a constant cross-section all along its 40 metre length. He also knows that a gold seam (layer of gold) runs through the site and that it is underneath some granite rock.



The diagram below shows the cross-section, AE, of the site and shows the depths and locations of the rock and the gold seam, where x is the horizontal distance (in metres) from A and y is the vertical distance (in metres) above the line AE.



The equation $y = \sin\left(\frac{\pi x}{10}\right)$, $0 \le x \le W$ represents the surface of the rock ($ABCDE$).

The equation $y = \cos\left(\frac{\pi x}{10}\right) - 3$, $0 \le x \le W$ represents the top (upper surface) of the gold seam ($FGHIJ$).

Question 4 – continued

a. Show that $W = 20$.

1 mark

b. Write down an expression in terms of x which represents the vertical distance from the surface to the top of the gold seam.

1 mark

c. Use calculus to determine the exact value of the minimum vertical distance from the surface to the top of the gold seam. State the exact value of x for which this occurs.

(You may assume that the required value of x is in the interval $(0, W)$. You do not need to justify that the distance you find is a minimum.)

5 marks

d. Tasmania decides to remove all the granite above the gold seam. Use calculus to determine the exact cross-sectional area of granite that he will remove.

4 marks

e. The vertical thickness of the gold seam is given by

$$T = 0.2 - 0.002(20 - x)^{1.5},\ 0 \leq x \leq W$$

Use calculus to find the total volume of gold, V m^3, which can be removed from the site, given that the seam contains 0.2% gold by volume. Give your answer correct to three decimal places.

4 marks

f. Tasmania Jones decides that it is too expensive to remove all the granite from above the gold seam at his site. He decides to excavate all the granite vertically over the 5 metres between D and E. Calculate the percentage of the total amount of gold which he is now able to mine.

Give your answer correct to the nearest per cent.

2 marks

Total 17 marks

END OF QUESTION AND ANSWER BOOK