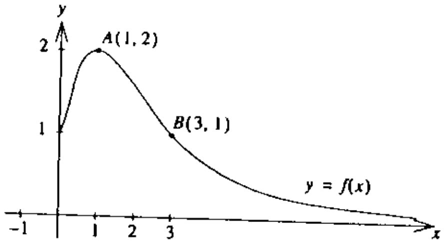
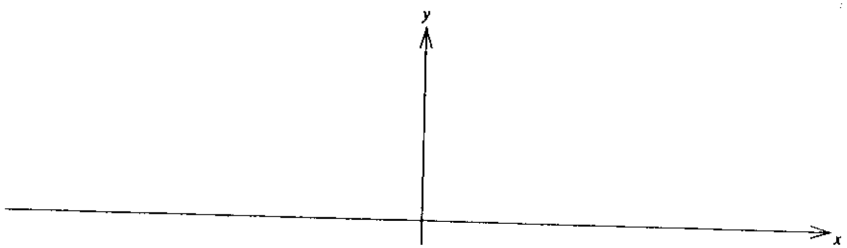
Question 1

The function $y = f(x)$ is defined by the graph sketched below, where the x-axis is an asymptote to the curve. The maximum point A has coordinates (1, 2).



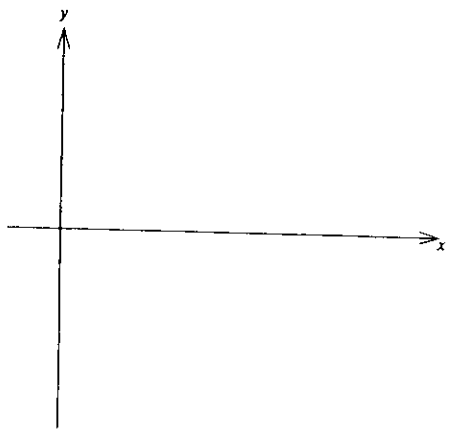
a. Sketch the graphs of (including the coordinates of the points of intersection with the axes and the equation of all asymptotes):

i. $y = f(-x)$



2 marks

ii. $y = -f(x)$



2 marks

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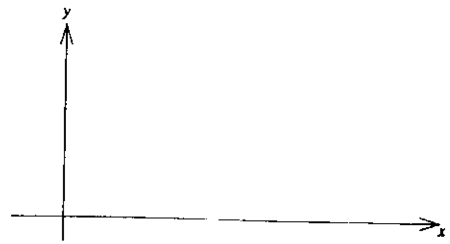
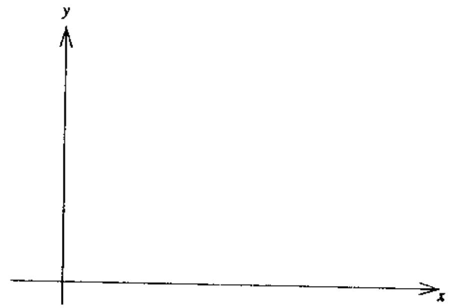
iii. $y = f(x + 1)$



2 marks

iv. $y = f(x) + 1$

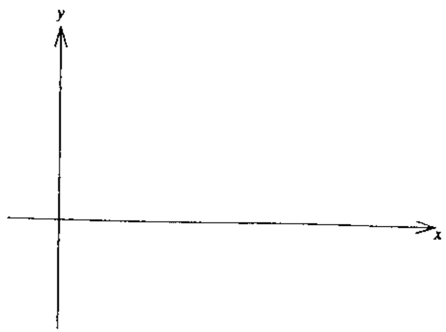


3 marks

v. $y = f^{-1}(x), 0 < x \leq 1$

2 marks

vi. $y = f'(x)$



3 marks

b. Given that $\int_0^3 f(x)dx = k$, find in terms of k

i. $\int_0^3 (f(x)+1)dx$

2 marks

ii. $\int_{-1}^2 f(x+1)dx$

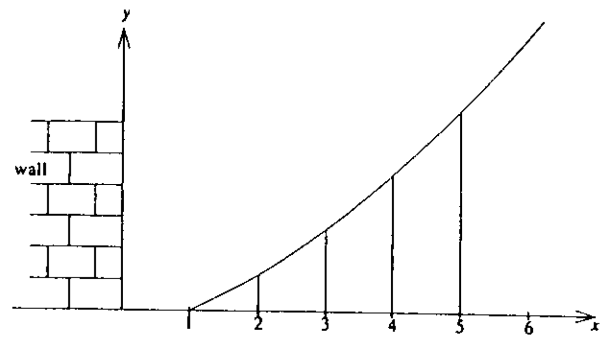
1 mark

iii. $\int_0^3 (k - f(x))dx$

3 marks
Total: 20 marks

Question 2

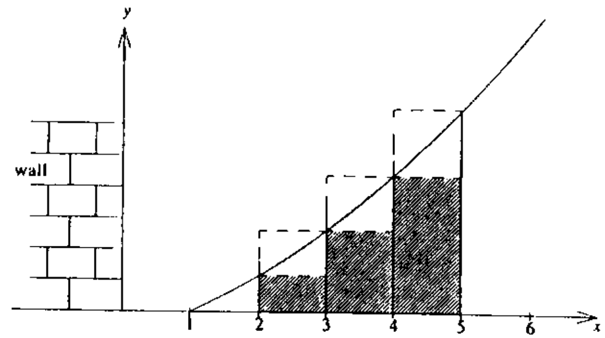
An architect is designing a shelter to be built near a vertical retaining wall. The diagram below shows a side view of the wall (the y-axis) and the shelter roof, which touches the horizontal ground (the x-axis) 1 metre from the wall.



The roof line is modelled by the equation $y = x\log_e x$ for $x \geq 1$.

Vertical roof supports are to be placed at 1 m intervals from the point of contact of the roof with the ground (0, 1).

To roughly calculate the area enclosed between the first four supports and the roof line, the architect tried to find the area of rectangles formed below and above the roof line at points of contact with each support as shown below.



a. Complete the following table with values written to 4 decimal places.

x	2	3	4	5
y				

2 marks

b. Using the values from the table above, calculate the sum of the rectangle areas between $x = 2$ and $x = 5$ existing

i. **below** the roof line.

1 mark

ii. **above** the roof line.

1 mark

c. Using your answers from **b.**, calculate to 3 decimal places an estimate for the area under the curve between $x = 2$ and $x = 5$.

2 marks

d. Differentiate $x^2 \log_e x$ with respect to x and **hence** show $\int x \log_e x\,dx = \frac{x^2}{2}\left(\log_e x - \frac{1}{2}\right) + c$.

4 marks

e. If a and b represent the lower and upper limits of x respectively between which the area under the curve is to be found, show that this area is equivalent to

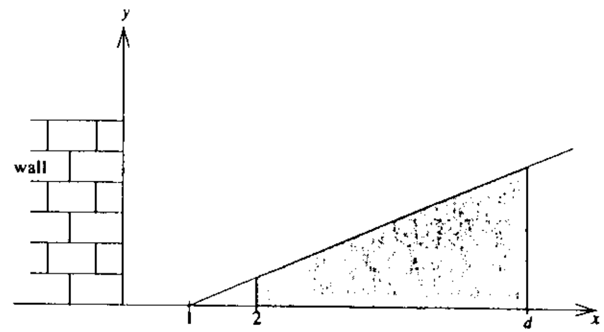
$$\frac{1}{4}(2b^2 \log_e b - 2a^2 \log_e a + a^2 - b^2)$$

3 marks

f. If $a = 2$ and $b = 5$ evaluate the area concerned accurately to 4 decimal places.

1 mark

g. For ease of construction, the architect decided to remodel the entire roof line (from the ground up) with the linear equation $4000y - 8047x + 8047 = 0$.



i. Write a definite integral which will evaluate the area enclosed below the roof line between the limits $x = 2$ and $x = d$ (where d represents the distance from the wall to the end of the shelter).

1 mark

ii. Evaluate this integral (from **g. i.**) to obtain an equation for the area enclosed under the roof line between $x = 2$ and $x = d$.

2 marks

iii. Calculate the distance from the retaining wall to the end of the shelter, d, if the area to be enclosed between the first vertical support and the end of the shelter is 20.0 m^2. (Express your answer to 3 decimal places.)

2 marks

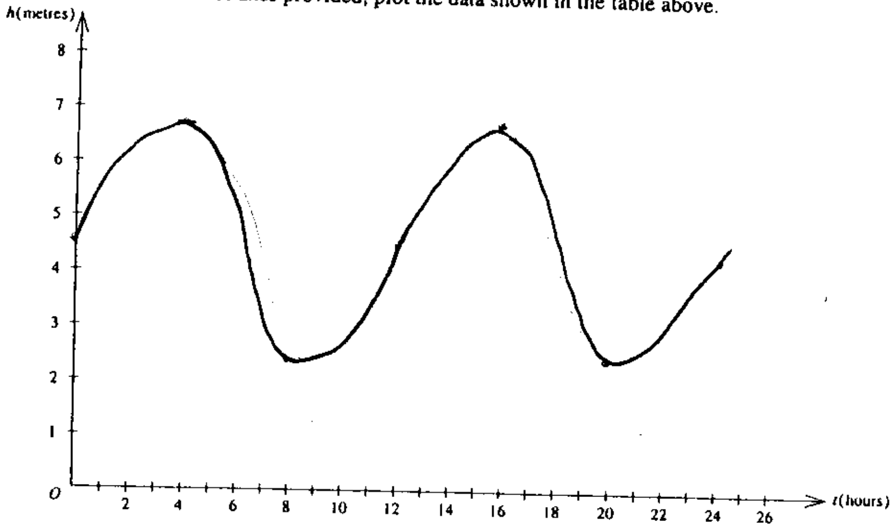
Total: 19 marks

Question 3

The table below shows the height, h metres, of the tide at time t hours after 12 noon on Monday.

t hours	0	4	8	12	16	20	24
h metres	4.5	6.7	2.3	4.5	6.7	2.3	4.5

a. i. On the set of axes provided, plot the data shown in the table above.



2 marks

ii. Which one of the following models best represent the data shown in part i.?

A. $h(t) = a\tan(bt)° + c$

B. $h(t) = a\cos(bt)° + c$

C. $h(t) = a\sin(bt)° + c$

1 mark

iii. Show that $a = 2.5$, $b = 30$ and $c = 4.5$ satisfy the given data set.

1 mark

iv. On the set of axes in i., sketch the graph which models the data.

3 marks

v. State the maximum height that the tide will reach.

1 mark

b. What will the height of the tide be at 5.00 pm on Wednesday?

2 marks

c. Show that the times in the first 24 hours when the height of the tide is 6.0 metres are 1.23, 4.77, 13.23, 16.77 (in hours, to 2 decimal places).

3 marks

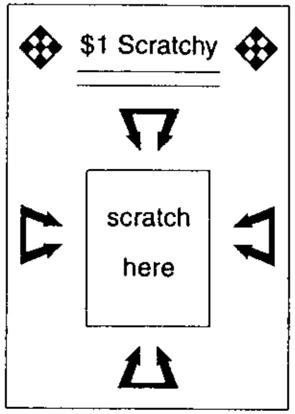
d. Fishing boats will only be allowed in this region if the tide is at least 5.75 metres. How many hours are available to these boats in a 24 hour period?

4 marks

Total: 17 marks

Question 4

A scratch and win game involves buying a ticket for \$1, scratching the coating off a panel to reveal either an amount won or "try again".



There are 1000 tickets made in a batch.

If the random variable, X, represents **the amount won** in dollars from the purchase of one ticket, the number of winning tickets in a particular batch is shown below.

Amount won (x)	Number/batch
\$49	5
\$9	30
\$2	100

a. Complete the probability table.

x	49	9	2	−1
$\Pr(X = x)$				

2 marks

b. Calculate the probability of winning a prize with any one ticket purchased.

1 mark

c. Calculate the expected amount won or lost on the purchase of one ticket.

2 marks

d. A gambler thought she would have a better chance of winning if she purchased more tickets. What would be her expected amount won or lost if she purchased 50 tickets?

1 mark

e. Calculate the variance of the amount won per ticket in dollars (to the fourth decimal place).

2 marks

f. Calculate the standard deviation of the amount won per ticket (to the nearest cent).

1 mark

g. Calculate the probability (to 3 decimal places) that the amount won per ticket lies within two standard deviations from the mean $\Pr(\mu - 2\sigma < X < \mu + 2\sigma)$.

2 marks

h. Another gambler purchased 10 tickets.

i. Calculate the probability (to 4 decimal places) that this gambler wins a prize on at least one ticket.

2 marks

ii. Calculate the probability (to 4 decimal places) that this gambler wins a prize with any two of the ten tickets.

2 marks

i. Batches of tickets were produced which had a proportion of prizes which were normally distributed with a mean of 0.2 and a variance of 0.0036.

i. Calculate the probability (to 4 decimal places) that the proportion of prizes in batches produced was less than 0.16.

1 mark

ii. Calculate the interval of proportions of prizes within which you would expect 95% of the batches to lie.

1 mark

iii. A new government regulation requires that 90% of all batches of tickets contain proportions of prizes of at least 0.15.

Given that the standard deviation remains unchanged, calculate the mean of the normal distribution which satisfies this new regulation.

2 marks

Total: 19 marks