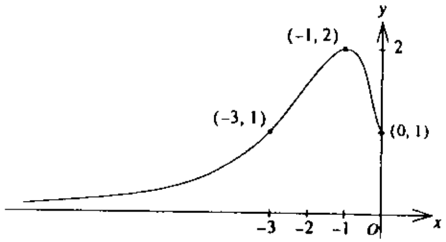
**Question 1**

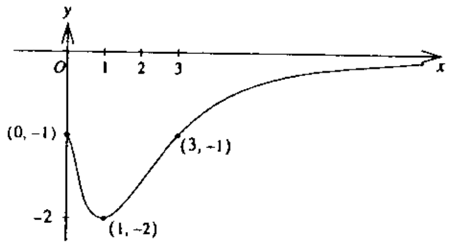
**a.** **i.**




shape (reflection): A1

coordinates: A1

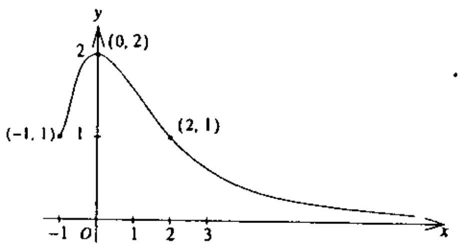
**ii.**




shape (reflection): A1

coordinates: A1

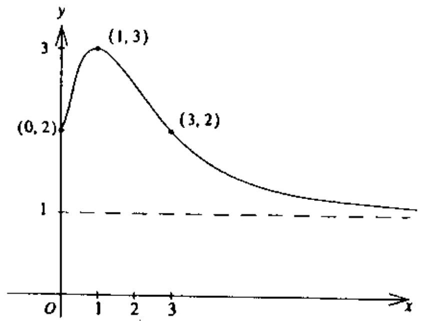
**iii.**




shape (translation): A1

coordinates: A1

**iv.**




Asymptote: A1

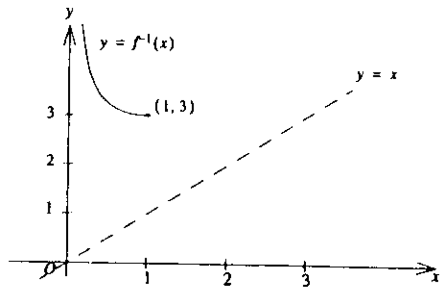
shape (translation): A1

coordinates: A1





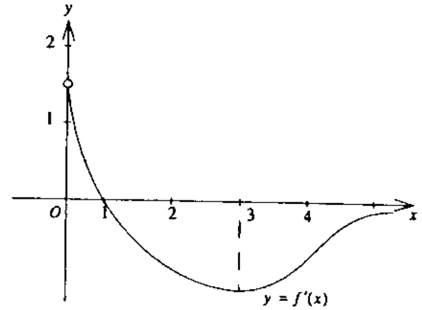
**v.**




shape: A1

range/domain: A1

**vi.**




shape: A1

turning point: A1

Asymptote ($y = 0$): A1

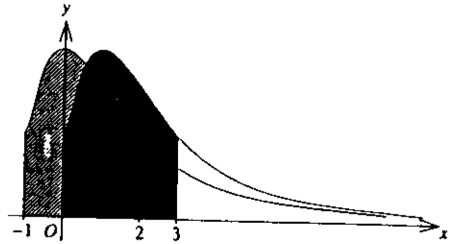
**b.** **i.** $\int_0^3 (f(x)+1)dx = \int_0^3 f(x)dx + \int_0^3 1\,dx$

$= k + \left[x\right]_0^3$ M1

$= k + (3-0)$

$= 3 + k$ A1

**ii.**




Using symmetry: $\int_{-1}^{2} f(x+1)dx = \int_0^3 f(x)dx$

$= k$ A1

**iii.** $\int_0^3 (k - f(x))dx = \int_0^3 k\,dx - \int_0^3 f(x)dx$

$= \left[kx\right]_0^3 - k$ M1, A1

$= 3k - k$

$= 2k$ A1





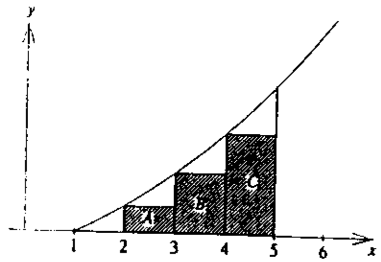
**Question 2**

**a.**

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y$ | 1.3863 | 3.2958 | 5.5452 | 8.0472 |

A2

**b.** **i.** Area of rectangles below with width 1 m = $f(x)$



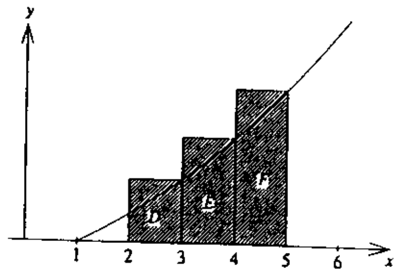

Area $A = f(2) \times 1 = 1.3863$

$B = f(3) \times 1 = 3.2958$

$C = f(4) \times 1 = 5.5452$

Total area: $10.2273\ m^2$ A1

**ii.** Area of rectangles above with width 1 m.




Area $D = f(3) \times 1 = 3.2958$

$E = f(4) \times 1 = 5.5452$

$F = f(5) \times 1 = 8.0472$

Total area: $16.8882\ m^2$ A1

**c.** Approximate area under curve will be the average of above rectangle areas. M1

$A \approx \dfrac{10.2273 + 16.8882}{2}$

$= 13.558 \text{ m}^2$ A1

**d.** $\dfrac{d(x^2\log_e x)}{dx} = x^2 \times \dfrac{1}{x} + 2x \times \log_e x$ use of product rule M1

$= x + 2x\log_e x$ A1

$\therefore \int x\,dx + \int 2x\log_e x\,dx = x^2\log_e x + c$ M1

$2\int x\log_e x\,dx = x^2\log_e x - \int x\,dx$

$\therefore \int x\log_e x\,dx = \dfrac{1}{2}\left(x^2\log_e x - \dfrac{1}{2}x^2\right) + c$

$= \dfrac{x^2}{2}\left(\log_e x - \dfrac{1}{2}\right) + c$ A1

**e.** Area $= \left[\dfrac{x^2}{2}(\log_e x) - \dfrac{x^2}{4}\right]_a^b$ A1

$= \left(\dfrac{b^2}{2}\log_e b - \dfrac{b^2}{4}\right) - \left(\dfrac{a^2}{2}\log_e a - \dfrac{a^2}{4}\right)$ M1

$= \dfrac{1}{4}(2b^2\log_e b - 2a^2\log_e a + a^2 - b^2)$ A1

**f.** Area $= \dfrac{1}{4}(2.5^2\log_e 5 - 2.2^2\log_e 2 + 2^2 - 5^2)$

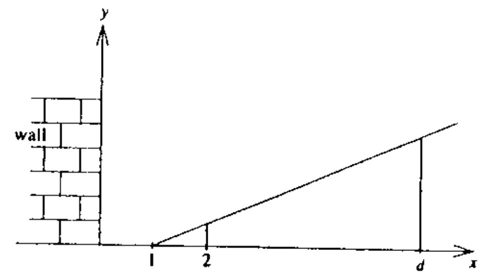
$= 13.4817 \text{ m}^2$ A1

(OR by using graphic calculator $\int_2^5 (x\log_e x)dx$)





**g. i.**




$y = \dfrac{8047}{4000}x - \dfrac{8.47}{4000}$

So Area $= \int_2^d \left(\dfrac{8047}{4000}x - \dfrac{8047}{4000}\right)dx$ A1

**ii.** Area $= \dfrac{8047}{4000}\left[\left(\dfrac{x^2}{2} - x\right)\right]_2^d$ A1

$\therefore A = \dfrac{8047}{4000}\left(\dfrac{d^2}{2} - d\right)$ A1

**iii.** When $A = 20$

$20 \times \dfrac{4000}{8047} = \dfrac{d^2}{2} - d$

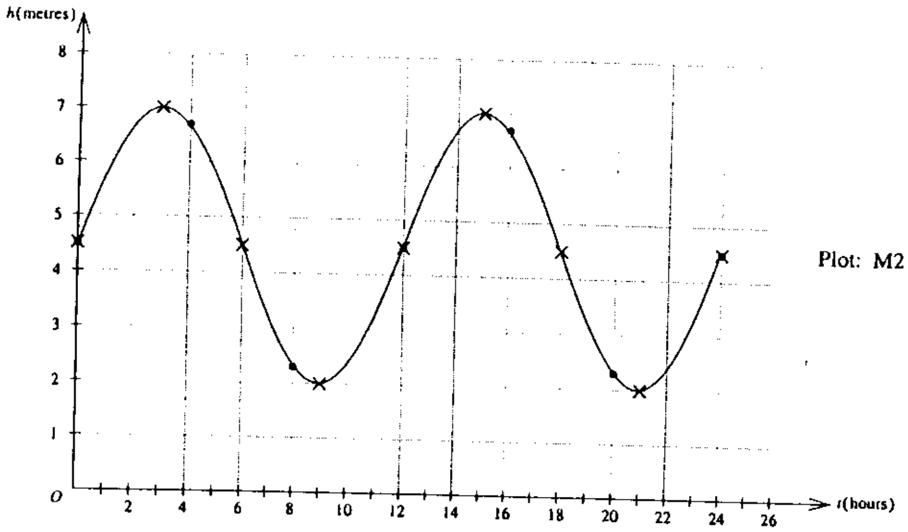
$\therefore \dfrac{d^2}{2} - d - \dfrac{80000}{8047} = 0$ A1

Using graphics calculator $d = 5.570$ m A1

(OR (Heaven forbid) using the quadratic formula)

## Question 3

**a. i.**




Plot: M2

**ii.** C. — A1

**iii.** Using $h(t) = 2.5\sin(30t)° + 4.5$, all that is required is to check one of the coordinates.

e.g. $h(8) = 2.5\sin(30 \times 8)° + 4.5$ — M1

$\approx 2.3349$

$= 2.3$ (to 1 decimal place)

Note: There is no need to solve simultaneous equations.

**iv.** See graph above.

Shape: A1

Max–Min points (i.e. amplitude): A1

Period: A1

**v.** $h_{max} = 7.0$ — A1

**b.** $t = 0$ occurs at 12 noon on Monday $\therefore$ 5.00 pm on Wednesday corresponds to $t = 24 + 24 + 5 = 53$ — A1

$\therefore h(54) = 2.5\sin(30 \times 53)° + 4.5 = 5.75$ — A1

**c.** $h(t) = 6 \Leftrightarrow 2.5\sin(30t)° + 4.5 = 6$

$\Leftrightarrow \quad \sin(30t) = 0.6$ — M1

$30t = 36.8698,\ 180 - 36.8698,\ 360 + 36.8698,\ 540 - 36.8698$ — A1

$30t = 36.8698,\ 143.1302,\ 396.8698,\ 503.1302$

$t = 1.23,\ 4.77,\ 13.23,\ 16.77$ — A1





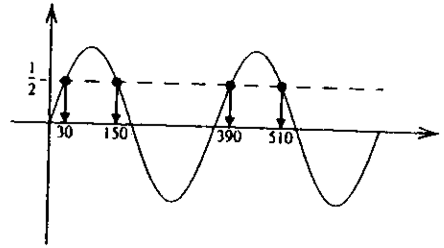

**d.** $h(t) \geq 5.75 \Leftrightarrow 2.5\sin(30t)° + 4.5 \geq 5.75$

$\Leftrightarrow \quad \sin(30t)° \geq \frac{1}{2}$ — M1

$30 \leq 30t \leq 150,\ 390 \leq 30t \leq 510$ — M1

$1 \leq t \leq 5,\ 13 \leq t \leq 17$ — A1

i.e. there are 4 + 4 = 8 hours available. — A1




## Question 4

**a.**

| $x$ | 49 | 9 | 2 | −1 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0.005 | 0.03 | 0.1 | 0.865 |

A2

**b.** $\Pr(\text{win}) = 0.135$ — A1

**c.** $E(X) = \Sigma xp(x)$

$= 49 \times 0.005 + 9 \times 0.03 + 2 \times 0.1 - 1 \times 0.865$

$= -0.15$ — A1

$\therefore$ Expected amount LOST per ticket is 15¢ — A1

**d.** Expected amount LOST $= 50 \times 0.15$

$= \$7.50$ — A1

**e.** $VAR(X) = \Sigma x^2 p(x) - (E(X))^2$

$= (49^2 \times 0.005 + 9^2 \times 0.03 + 2^2 \times 0.1 + (-1)^2 0.865) - (-0.15)^2$

$= 15.7000 - 0.0225$ — M1

$= \$15.6775$ — A1

**f.** $\sigma_x = \sqrt{VAR(X)}$

$= \sqrt{15.6775}$

$= \$3.96$ — A1

**g.** $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) = \Pr(-7.77 < X < 8.07)$

from answers of probabilities in **a.**

Probability required $= \Pr(X = 2) + \Pr(X = -1)$ — M1

$= 0.1 + 0.865$

$= 0.965$ — A1

**h.** **i.** Let $Y$ = winning a prize on a ticket

$Y = \text{Bi}(10, 0.135)$ (i.e. Pr(win) = 0.135, Pr(not win) = 0.865) — M1

Pr(at least one) = 1 − Pr(none)

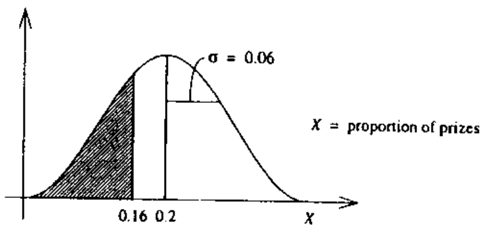
$= 1 - (0.865)^{10}$

$= 0.7655$ — A1

**ii.** $\Pr(Y = 2) = \binom{10}{2}(0.135)^2(0.865)^8$ — M1

$= 0.2570$ — A1

**i.** **i.**




$\Pr(X < 0.16) = \Pr\left(Z < \frac{0.16 - 0.2}{0.06}\right)$

$= \Pr(Z < -0.\dot{6})$

$= 1 - \Pr(Z < 0.\dot{6})$

$= 1 - 0.7475$

$= 0.2525$

OR by use of graphic calculator. — A1

**ii.** $0.95 = \Pr(\mu - 2\sigma < X < \mu + 2\sigma)$

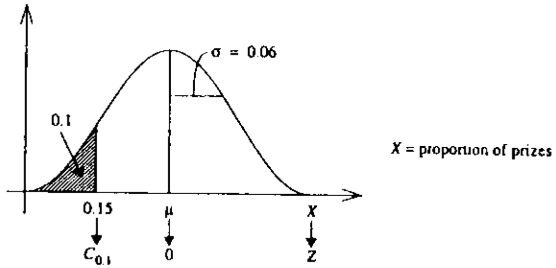
$(0.2 - 2 \times 0.06) < X < (0.2 + 2 \times 0.06)$

$0.08 < X < 0.32$ — A1





**iii.**




10% of the area must be less than $X = 0.15$.

The 0.1 quantile $(C_{0.1}) = -1.2815$ $(C_{0.1} = -C_{0.9})$

Using $Z = \frac{X - \mu}{\sigma}$

$-1.2815 = \frac{0.15 - \mu}{0.06}$ — M1

$\therefore \mu = 0.15 + (0.06)(1.2815)$

$= 0.227$ — A1