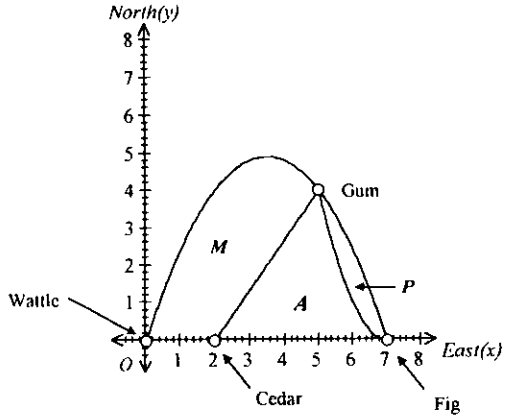
Question 1

Three brothers Adam, Mark and Paul Smith inherit a family farm. The farm has four trees on its boundary. The brothers decide to use the trees to divide the land into three separate farms. The northernmost tree is a large gum tree. The other three trees are a wattle, a cedar and a fig. These three trees all stand on an East-West line. The Wattle is taken as the reference point for the proposed division of the land.

The farm can be represented by a set of cartesian axes with the Wattle situated at the origin, (0, 0) as shown below.



M is defined as the area inherited by Mark.
A is defined as the area inherited by Adam.
P is defined as the area inherited by Paul.
Distances are all measured in kilometres.

a. Write down the coordinates of the cedar tree and the gum tree.

2 marks

b. What is the equation of the boundary line linking the cedar tree and the gum tree?

2 marks

c. What is the distance between the cedar tree and the gum tree?

1 mark

d. Which tree is further away from the cedar tree, the gum tree or the fig tree?

1 mark

e. The equation of the boundary line linking the wattle, gum and fig trees can be expressed as a quadratic equation, $y = ax^2 + bx + c$, where a, b and c are constants. Find the exact values for a, b and c.

4 marks

f. Using the method of integration, find the area of Mark's farm correct to two decimal places.

4 marks

Total 14 Marks

[working space]

Question 2

A new ferris wheel at a theme park revolves at a constant speed in an anti-clockwise direction, requiring passengers to enter and exit the wheel as it is in motion. For safety, passengers enter the ferris wheel at its lowest point.

The height, in metres, of a passenger above the ground t minutes after they have entered the ferris wheel is given by

$$h(t) = 12 - 10\cos\frac{\pi}{2}t, \quad t \geq 0$$

a. A "ride" on this ferris wheel finishes after two revolutions of the wheel. How many whole minutes does a ride take?

2 marks

b. What is the maximum height passengers can reach on the wheel?

1 mark

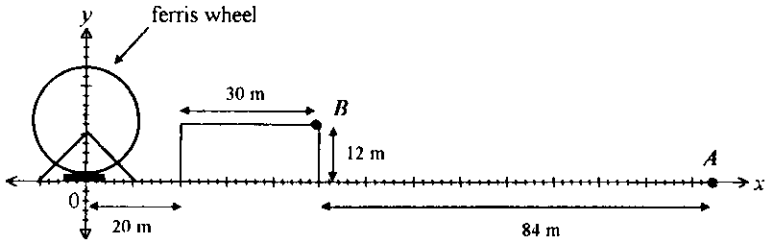
c. The passengers enter the wheel from a platform. What height must this platform be so that passengers enter the wheel without stepping up or down?

1 mark

d. Show that it takes 1.41 minutes for passengers on the wheel to reach a height of 18 metres for the first time?

1 mark

The ferris wheel and its surroundings can be represented on a set of cartesian axes as shown below. In this diagram, y is defined as the height above ground and x is the horizontal distance from the centre of the platform. The distances are measured in metres.



e. Show that the equation of the line joining the point A on the ground and the point B on the edge of the top of a group of buildings can be given by

$$y = \frac{1}{7}(134 - x)$$

2 marks

f. The position of a passenger can be given by the relation

$$(y - 12)^2 + x^2 = 10^2$$

Show that the points of intersection between this relation and the line in part **f** are (-6, 20) and (8, 18).

3 marks

g. For what percentage of time on the ferris wheel will a passenger be able to see the point A? Assume that a passenger's line of sight begins from a point on the ferris wheel making no allowance for how tall the passenger is. Write your answer to the nearest whole percentage.

3 marks

Total 13 marks

Question 3

The height, $h(x)$, of a water slide above the ground is given by

$$h_1(x) = 3 - \frac{1}{4}x^2, \qquad 0 \le x \le 1$$

$$h_2(x) = Ae^{-k(x-1)} - 0.5, \qquad 1 < x \le 7,$$

where x is the horizontal distance in metres from the entrance point at the top of the slide and A and k are real constants

a. Given that the functions h_1 and h_2 intersect at $x = 1$, show that $A = 3.25$.

2 marks

b. **i.** Find $h_1'(x)$, the derivative of $h_1(x)$.

1 mark

ii. Find $h_2'(x)$, the derivative of $h_2(x)$.

1 mark

iii. The water slide is continuous and smooth at the intersection between the functions, h_1 and h_2. Use this information to find the constant k correct to two decimal places.

2 marks

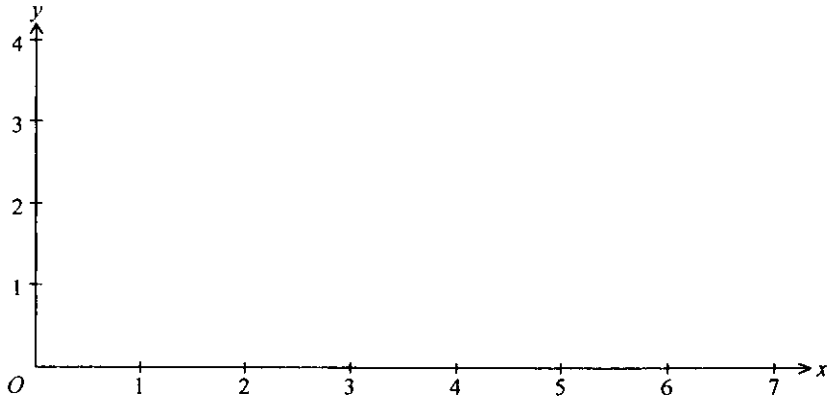
c. At what height, h, would a person be if they slide along the slide for a horizontal distance of 2 metres? Write your answer correct to two decimal places.

2 marks

d. What horizontal distance would a person travel after having descended a height of 2 metres? Write your answer accurate to the nearest second decimal place

2 marks

e. Draw a graph of the functions, h_1 and h_2 on the following set of axes.



2 marks

f. To gain maximum distance before entering the water, a person should jump off in a straight line where $x = 7$, at the end of the slide, in a direction which is perpendicular to the direction of the slide at $x = 4$. Find the equation of this straight line. Write constants of the equation correct to two decimal places.

4 marks

Total 16 marks

[working space]

Question 4

Sally Stone throws a javelin at distances which are normally distributed with a mean distance of 65 metres and a standard deviation of y metres. She qualified for the Olympics with a throw of 64.2 metres. Sally's throwing distance is given by X metres and $Pr(X > 64.2) = 0.6$.

a. What is the probability that Sally will throw

i. more than 65 metres with any one throw?

1 mark

ii. less than 65.8 metres with any one throw?

1 mark

b. Write an expression in terms of y for the standard normal score, z, for Sally's probability of throwing more than 64.2 metres.

2 marks

c. Use the expression from **b.** to find y, Sally's standard deviation. Give your answer correct to two decimal places.

2 marks

The qualifying mark for the Olympic final is expected to be around 63 metres. The record is 72.3 metres.

d. What is the probability, accurate to two decimal places, that Sally will make the final?

3 marks

e. Find, correct to one decimal place, the 95% probability interval for the distance of Sally's throws. Do you think she is likely to create a new record? Explain.

3 marks

Sally's coach, Albert Jones, suspected that Sally was injured 2 weeks before the games and secretly recorded 200 of her throws. He recorded the results to the nearest metre as shown in the following table.

Distance	58	59	60	61	62	63	64	65	66	67	68
Number of Throws	5	11	22	27	39	30	27	25	7	6	1

f. Find $\hat{p}$, the sample proportion, for the number of throws 65 metres or greater. Write your answer correct to three decimal places

1 mark

g. Find, correct to three decimal places, the standard error of $\hat{p}$.

1 mark

h. Find, correct to three decimal places; the 95% confidence interval for the population proportion, p, for the final two weeks of Sally's performance.

1 mark

i. With reference to p or $\hat{p}$ explain whether Albert's suspicions are true.

2 marks

Total 17 marks

END OF TRIAL CAT 3