Trial CAT 3 Solutions

Question 1

a. Reading from the graph, cedar = (2, 0), gum = (5, 4) **[2A]**

b. Gradient = $\frac{4-0}{5-2} = \frac{4}{3}$ **[1M]**

$y - 0 = \frac{4}{3}(x-2)$

$y = \frac{4}{3}x - \frac{8}{3}$

$3y = 4x - 8$ **[1A]**

c. $\sqrt{4^2+3^2} = \sqrt{25} = 5$ **[1A]**

d. Neither. They are both the same distance from the cedar **[1A]**

e. $(0, 0) \Rightarrow 0 = c$ **[1M]**

$(5, 4) \Rightarrow 4 = 25a + 5b$ ①

$(7, 0) \Rightarrow 0 = 49a + 7b$ ②

① × 7 $28 = 175a + 35b$

② × 5 $0 = 245a + 35b$ **[1M]**

Subtract ② from ①

$28 = -70a$

$\frac{-28}{70} = a$

$\frac{-2}{5} = a$ **[1A]**

Substitute into ①

$4 = 25\left(\frac{-2}{5}\right) + 5b$

$4 = -10 + 5b$

$14 = 5b$

$\frac{14}{5} = b$

$y = \frac{-2}{5}x^2 + \frac{14}{5}x$ **[1A]**

f. Many methods possible: One method involves integrating the curve and subtracting the triangular area.

$\int_0^5 \frac{-2}{5}x^2 + \frac{14}{5}x\,dx - \frac{1}{2}(3 \times 4)$ **[1M]**

$= \left[\frac{-2}{15}x^3 + \frac{14}{10}x^2\right]_0^5 - 6$

$= \left[\frac{-50}{3} + \frac{70}{2}\right] - [0+0] - 6$ **[1M]**

= [-16.66 + 35] – 6

= [18.33] - 6 **[1M]**

Area of Mark's farm ≈ 12.33 km^2 **[1A]**

Total 14 marks

Question 2

a. The period is given by one revolution

$T = \frac{2\pi}{n}$ **[1M]**

$= \frac{2\pi}{\frac{\pi}{2}}$

= 4 minutes

So *two* revolutions will take 8 minutes **[1A]**

b. 12 + 10 = 22 metres **[1A]**

c. 12 – 10 = 2 metres **[1A]**

d. $12 - 10\cos\frac{\pi}{2}t = 18$ **[1M]**

$-10\cos\frac{\pi}{2}t = 6$

$\cos\frac{\pi}{2}t = \frac{-3}{5}$

$\frac{\pi}{2}t \approx 2.21$

$t \approx \frac{2.21 \times 2}{\pi}$

$t \approx 1.41$ as required

e. A (134, 0) B(50, 12) from diagram

$m = \frac{12-0}{50-134} = \frac{12}{-84} = \frac{-1}{7}$ **[1M]**

$y - 0 = \frac{-1}{7}(x - 134)$ **[1M]**

$y = \frac{1}{7}(134 - x)$ as required

f. $(y-12)^2 + x^2 = 10^2$ ①

$y = \frac{1}{7}(134 - x)$

$7y = 134 - x$

$x = 134 - 7y$ ②

substitute ② into ① **[1M]**

$(y-12)^2 + (134-7y)^2 = 100$

$y^2 - 24y + 144 + 1756 - 1876y + 49y^2 = 100$

$50y^2 - 1900y - 1800 = 0$

$y^2 - 38y - 360 = 0$

$(y-20)(y-18) = 0$

$y = 20, 18$ **[1M]**

when $y = 20$, $x = 134 - 7 \times 20 = -6$

when $y = 18$, $x = 134 - 7 \times 18 = 8$ **[1M]**

The points of intersection are (-6, 20) and (8, 18) as required.

g. $12 - 10\cos\frac{\pi}{2}t = 20$

$-10\cos\frac{\pi}{2}t = 8$

$\cos\frac{\pi}{2}t = \frac{-4}{5}$

$\frac{\pi}{2}t = \pi - 0.64,\ \pi + 0.64$

$\frac{\pi}{2}t = \pi + 0.64$

$t \approx 2.41$ **[1M]**

From part we have t = 1.41 minutes when h = 18

The difference between the times at h = 20 and h = 18 is 2.41 – 1.41 = 1.00 minute **[1M]**

One revolution takes 4 minutes, so the percentage of time will be

$\frac{1}{4} \times \frac{100}{1} = 25\%$ **[1A]**

Total 13 marks

Question 3

a. When $x = 1$

$h_1(1) = 3 - \frac{1}{4}$

$= \frac{11}{4}$ **[1M]**

$h_2(1) = Ae^{-k(1-1)} - 0.5 = \frac{11}{4}$

$Ae^0 = \frac{11}{4} + \frac{1}{2}$

$A = \frac{13}{4}$ **[1M]**

$A \approx 3.25$ as required

b. i. $h_1'(x) = -\frac{1}{2}x$ **[1A]**

ii. $h_1'(x) = -Ake^{-k(x-1)}$ **[1A]**

iii. When $x = 1$, $h_1'(1) = -\frac{1}{2}$

If the slide is smooth at x = 1, then

$h_2'(1) = -\frac{1}{2}$

$-Ake^{-k(x-1)} = -\frac{1}{2}$ **[1M]**

$-\frac{13}{4}ke^0 = -\frac{1}{2}$

$k = \frac{2}{13} \approx 0.15$ **[1A]**

c. $h_2(2) = \frac{13}{4}e^{-\frac{2}{13}(2-1)} - 0.5$ **[1M]**

$= \frac{13}{4}e^{-\frac{2}{13}} - 0.5$

≈ 2.29 **[1A]**

d. when $x = 0$, $h_1(x) = 3$ metres. After a descent of 2 metres, $h_1(x) = 1$, so $h_2(x) = 1$ metre also.

$h_2(x) = \frac{13}{4}e^{-\frac{2}{13}(x-1)} - 0.5 = 1$ **[1M]**

$\frac{13}{4}e^{-\frac{2}{13}(x-1)} = 1.5$

$e^{-\frac{2}{13}(x-1)} = \frac{6}{13}$

$-\frac{2}{13}(x-1) = \log_e\left(\frac{6}{13}\right)$

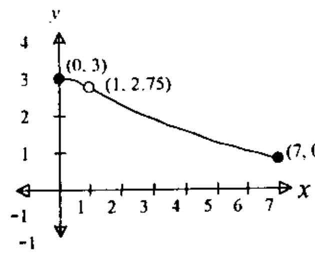
$x - 1 = -\frac{13}{2}\log_e\left(\frac{6}{13}\right)$

$x = 1 - \frac{13}{2}\log_e\left(\frac{6}{13}\right)$

$= 1 + \frac{13}{2}\log_e\left(\frac{13}{6}\right)$

$x \approx 6.03$ **[1A]**

e.



For correct shape [1M], for correct labels [1M]

f. When $x = 4$, $h_2'(4) = -\frac{13}{4}\left(\frac{2}{13}\right)e^{-\frac{2}{13}(4-1)}$

$= -\frac{1}{2}e^{-\frac{6}{13}}$

≈ -0.315 **[1M]**

gradient of the tangent is

$\frac{-1}{-0.315} = 3.173$ **[1M]**

$h_2(7) = \frac{13}{4}e^{-\frac{2}{13}(7-1)} - 0.5 \approx 0.791$ **[1M]**

$(y - 0.791) = 3.173(x - 7)$

$y = 3.173x - 21.42$ **[1A]**

Total 16 marks

Question 4

a. i. Since $\mu = 65$ $\sigma = y$

$$\Pr(X > 65) = \Pr(Z > \frac{65-65}{y})$$
$$= \Pr(Z > \frac{0}{y})$$
$$= \Pr(Z > 0) = 0.5$$ **[1A]**

ii. By Symmetry,

$\Pr(X < 65.8) = \Pr(X > 64.2) = 0.6$ **[1A]**

b. $z = \frac{64.2-65}{y}$ **[1M]**

$z = \frac{-0.8}{y}$ **[1A]**

c. If $\Pr(X > 64.2) = 0.6$, then

$\Pr(z > \frac{-0.8}{y}) = 0.6$

By symmetry, $\Pr(z < \frac{0.8}{y}) = 0.6$

From the normal distribution tables,

$\frac{0.8}{y} = 0.253$ **[1M]**

$y \approx 3.16$ **[1A]**

d. $\Pr(X > 63) = \Pr(z > \frac{63-65}{3.162})$ **[1M]**

$\Pr(X > 63) = \Pr(z > \frac{-2}{3.162})$

$\Pr(X > 63) = \Pr(z < \frac{2}{3.162})$ **[1M]**

$\Pr(X > 63) = \Pr(z < 0.6325)$

≈ 0.74 **[1A]**

e. The 95% probability interval can be expressed as

$\mu - 2\sigma \le x \le \mu + 2\sigma$

$65 - 2(3.162) \le x \le 65 + 2(3.162)$ **[1M]**

$58.7 \le x \le 71.3$ **[1A]**

Since the record is well outside the 95% probability interval, it is unlikely she will break the record. **[1A]**

f. $\hat{p} = \frac{39}{200} = 0.195$ **[1A]**

g. $se(\hat{p}) = \sqrt{\frac{\frac{39}{200}\left(\frac{161}{200}\right)}{200}} = 0.028$ **[1A]**

h. $\hat{p} - 2se(\hat{p}) \le p \le \hat{p} - 2se(\hat{p})$

$0.195 - 2(0.0280) \le p \le 0.195 + 2(0.0280)$

$0.139 \le p \le 0.251$ **[1A]**

i. p is the proportion of throws 65 m or greater in population.

$p = 0.5$

$\hat{p}$ is the proportion of throws 65 m or greater in a sample of 200.

$\hat{p} = 0.195$ **[1A]**

Since $\hat{p}$ is more than two standard errors below p, Albert's suspicions are true. **[1A]**

Total 17 marks

Total 60 Trial CAT 3 marks