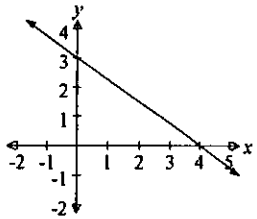
Part I (Multiple-choice Questions)

Question 1
The equation of the line shown here is given by



A. $3y = -4x + 3$

B. $4y = -3x + 16$

C. $3y = -4x + 12$

D. $4y = -3x + 12$

E. $3y = -4x + 7$

Question 2
The function $f(x) = -2 + 3e^{2x-1}$, where $x \in R$ has an inverse with the equation given by

A. $f^{-1}(x) = 1 + \log_e\left(\frac{x+2}{3}\right), x \in [3, \infty)$

B. $f^{-1}(x) = \frac{1}{2} + \frac{1}{2}\log_e\left(\frac{x+2}{3}\right), x \in (3, \infty)$

C. $f^{-1}(x) = 1 + \log_e\left(\frac{x+2}{3}\right), x \in [-2, \infty)$

D. $f^{-1}(x) = \frac{1}{2} + \frac{1}{2}\log_e\left(\frac{x+2}{3}\right), x \in (-2, \infty)$

E. $f^{-1}(x) = 1 + \log_e\left(\frac{x+2}{3}\right), x \in (-2, 3)$

Question 3
The graph of $y = f(x)$ is shown in figure 1.

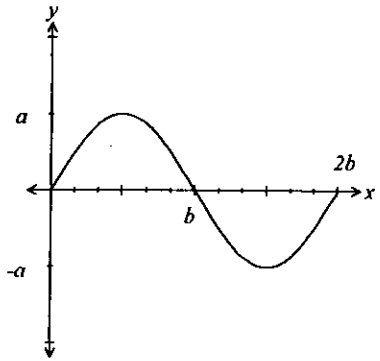
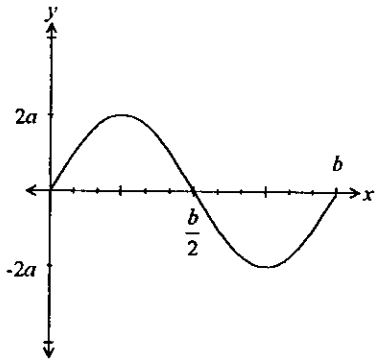


figure 1

Which one of the following best represents the transformed graph of f below?



A. $\frac{2f(2x)}{a}$

B. $2f\left(\frac{x}{2}\right)$

C. $2f(2x)$

D. $\frac{2}{a}f\left(\frac{x}{2}\right)$

E. $\frac{f(2x)}{a}$

Question 4

If $f(x) = a\cos\left(\frac{x}{2}\right)$, where a is a positive constant, and $f'\left(\frac{\pi}{3}\right) = -\sqrt{3}$, then a is given by

A. 4

B. -4

C. $4\sqrt{3}$

D. $\frac{1}{4}$

E. $-4\sqrt{3}$

Question 5

If $f(x) = (x + a)(x - b)$ where a and b are positive constants, then the minimum value of f is given by

A. $-ab$

B. 0

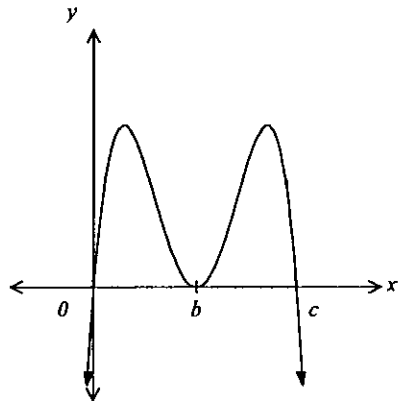
C. $f\left(\frac{-a+b}{2}\right)$

D. $f\left(\frac{a+b}{2}\right)$

E. $f\left(\frac{a-b}{2}\right)$

Question 6

The graph of the function $y = f(x)$ is shown below.



Given that $b > 0$ and $c > 0$, the factorised equation of this graph can be expressed as

A. $y = ax^2(x - b)(x - c)$, where $a > 0$

B. $y = ax(x - b)^2(x - c)$, where $a > 0$

C. $y = ax(x - b)^2(x + c)$, where $a < 0$

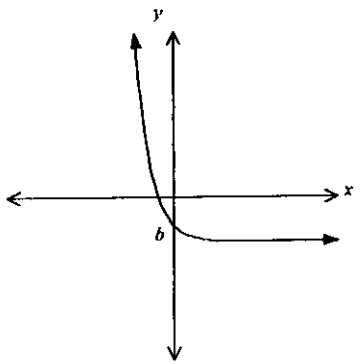
D. $y = ax(x - b)^2(x - c)$, where $a < 0$

E. $y = ax(x - b)(x - c)^2$, where $a < 0$

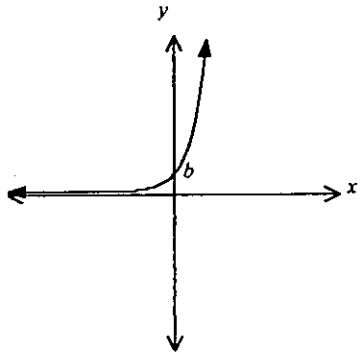
Question 7

Which one of the following graphs would fit the equation $f(x) = e^{x+b} + c$ if $b > 0$ and $c < 0$?

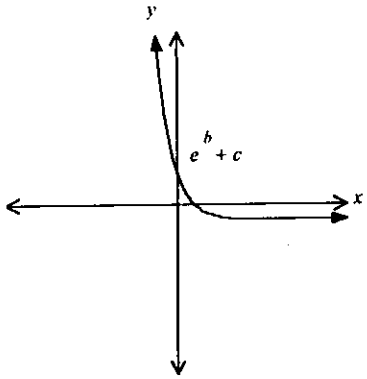
A.



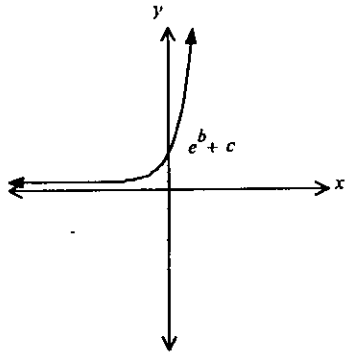
B.



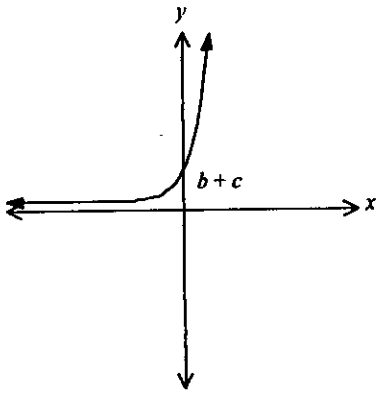
C.



D.



E



Question 8

The parabola with the equation $y = x^2$ is transformed into $y = -(x - 3)^2 + 4$ by

A. A reflection in the x-axis, a translation of 3 units along the x-axis in a positive direction and a positive translation of 4 units parallel to the y-axis

B. A reflection in the x-axis, a translation of 3 units along the x-axis in a negative direction and a positive translation of 4 units parallel to the y-axis

C. A reflection in the x-axis, a translation of 3 units along the x-axis in a positive direction and a negative translation of 4 units parallel to the y-axis

D. A reflection in the x-axis, a translation of 3 units along the x-axis in a negative direction and a negative translation of 4 units parallel to the y-axis

E. A reflection in the y-axis, a translation of 3 units along the x-axis in a positive direction and a negative translation of 4 units parallel to the y-axis

Question 9

The range of the graph of the function $f(x) = 2(x - 2)^2 + 2$, $x \in [0, 7)$ is given by

A. $f(x) \in [2, 52)$

B. $f(x) \in [2, 52]$

C. $f(x) \in [10, 27]$

D. $f(x) \in [10, 52)$

E. $f(x) \in [10, 27)$

Question 10

The following data is most likely linked by which equation, if $a > 0$, $b > 0$ and $c > 0$?

x	-3	-2	-1	0	1	2	3
y	1.56	2.77	6.25	25	Undefined	25	6.25

A. $y = a\log_e bx$

B. $y = ae^{(x+b)} + c$

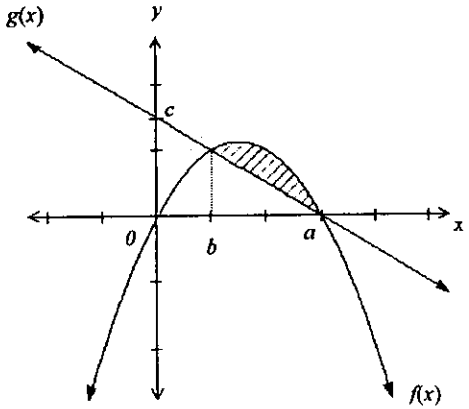
C. $y = \dfrac{a}{(x+b)}$

D. $y = ax^2 + bx + c$

E. $y = \dfrac{a}{(x+b)^2}$

Question 11

The area of the shaded region between the two graphs $f(x)$ and $g(x)$, where the curved graph is $f(x)$ and the linear graph is $g(x)$, can be calculated by



A. $\int_0^a f(x)dx - \int_b^a g(x)dx$

B. $\int_a^b f(x)dx + \int_a^b g(x)dx$

C. $\int_c^a (f(x) - g(x))dx$

D. $\int_0^a f(x)dx - \int_0^a g(x)dx$

E. $\int_b^a (f(x) - g(x))dx$

Question 12

The value of the definite integral $\int_{-2}^{1} (3x+2)^3 dx$ is given by

A. $6\frac{7}{9}$

B. 15

C. 21

D. $30\frac{3}{4}$

E. $73\frac{5}{12}$

Question 13

The equation $f(x) = \frac{1}{2}(x-3)(x+2)$ has a gradient of -1 at which point?

A. (0,-3)

B. $(-\frac{1}{2}, \frac{21}{8})$

C. (-2,0)

D. $(\frac{3}{2}, -\frac{21}{8})$

E. $(-\frac{1}{2}, -\frac{21}{8})$

Question 14

The equation of the normal to the curve $y = 2\sin(3x)$ at the point where the curve crosses the y-axis is given by

A. $y = -\frac{1}{6}x$

B. $y = 6x$

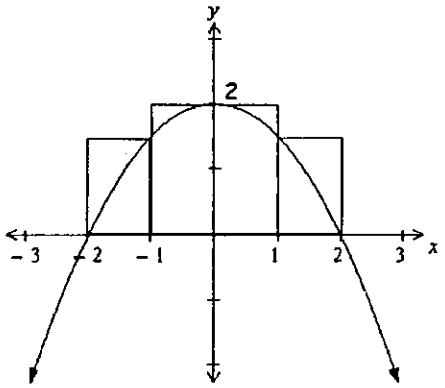
C. $y = -\frac{2}{3}x$

D. $y = \frac{3}{2}x$

E. $y = 0$

Question 15

The approximate area under the curve $y = -\frac{1}{2}x^2 + 2$ between $x = -2$ and $x = 2$ using the upper rectangles approximation shown is given by



A. 4 square units

B. $5\frac{1}{3}$ square units

C. 7 square units

D. 8 square units

E. $10\frac{2}{3}$ square units

Question 16

The function, $f : R \to R$ where $f(x) = 3\cos\left(\frac{x}{4}\right) - 1$ has a minimum value and period respectively of

A. -1, 8π

B. -4, 8π

C. -4, $\frac{\pi}{2}$

D. -3, $\frac{\pi}{2}$

E. -3, $\frac{\pi}{4}$

Question 17

A solution to the equation $\cos(2x) + \sqrt{3}\sin(2x) = 0$ is

A. $\frac{-\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{12}$

D. $\frac{7\pi}{12}$

E. $\frac{5\pi}{12}$

Question 18

The **sum** of the solutions of $\sin(3x) = 1$ where $0 \le x \le \pi$ is given by

A. $\frac{\pi}{6}$

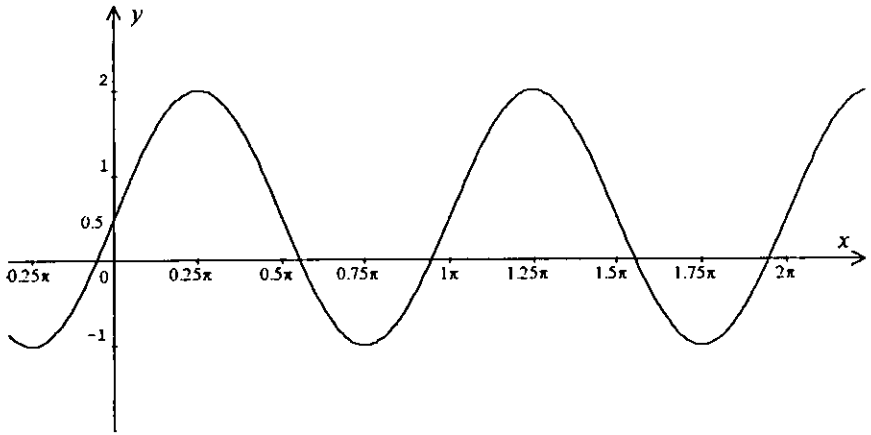
B. $\frac{2\pi}{3}$

C. π

D. $\frac{5\pi}{6}$

E. 2π

Question 19



For this graph the equation is $f(x) = a\cos b(x + c) + \frac{1}{2}$. The unknowns a, b and c are given by

	a	b	c
A.	$\frac{3}{2}$	-2	$\frac{\pi}{4}$
B.	$\frac{3}{2}$	2	$-\frac{\pi}{4}$
C.	2	2	$-\frac{\pi}{4}$
D.	2	$\frac{1}{2}$	$\frac{\pi}{4}$
E.	$\frac{3}{2}$	$\frac{1}{2}$	0

Question 20

$3x\left(1+\log_3 x^3 - 2\log_3 x\right)$ is equivalent to

A. $3x+\log_3 x$

B. $3x\log_3 3x$

C. $3x+\log_3 x^5$

D. $3x+\log_3\left(\frac{3}{2}\right)$

E. 1

Question 21

The fourth term in the expansion of $(2x-3)^9$ is equal to

A. -145152

B. $-145152x^6$

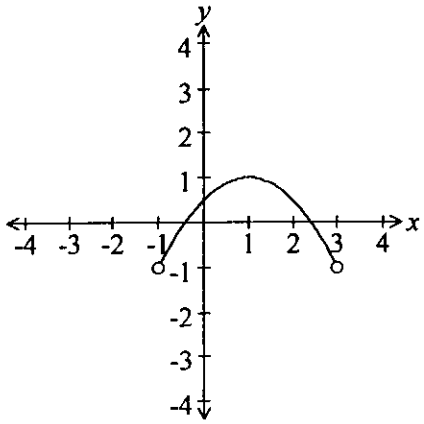
C. $145152x^6$

D. -2268

E. $-2268x^6$

Question 22

The graph of the function f is shown below



In order for the inverse f^{-1} to exist, a possible restricted domain of f is

A. (-1,3)

B. [0,3)

C. [0,1]

D. [-1,1]

E. [1,3]

Question 23

If $y = x\log_e(2-x)$, $x<2$, then $\frac{dy}{dx}$ is equal to

A. $\log_e(2-x) - \frac{x}{2-x}$

B. $\log_e(2-x) + \frac{1}{2-x}$

C. $\log_e(2-x) - \frac{1}{2-x}$

D. $(1-x)\log_e(2-x)$

E. $\frac{-1}{2-x}\log_e(2-x)$

Question 24

An antiderivative of $\sin(3x)+(3x-2)^4$ is

A. $\frac{1}{5}(3x-2)^5-\cos(3x)$

B. $\frac{1}{3}\left(\frac{(3x-2)^5}{5}-\cos(3x)\right)$

C. $\cos(3x-2)+\frac{1}{5}(3x)^5$

D. $\frac{1}{3}\left(\frac{1}{5}(3x-2)^5+\cos(3x)\right)$

E. $\frac{1}{3}(\cos(3x)+\log_e(3x-2))$

Question 25

If $y=\frac{\sin(x^2)}{2x}$, $x\neq 0$ then $\frac{dy}{dx}$ is equal to

A. $\frac{(2x^2-1)\sin(x^2)}{2x^2}$

B. $\frac{(2x^2-1)\sin(x^2)}{x^2}$

C. $\frac{2x^2\cos(x^2)-\sin(x^2)}{2x^2}$

D. $\frac{2x^2\cos(x^2)+\sin(x^2)}{2x^2}$

E. $\frac{2x^2\cos(x^2)-\sin(x^2)}{4x^2}$

Question 26

Given that $f'(x)=\frac{1}{2x-1}-2x^3$, $x\neq\frac{1}{2}$ and $f(1)=0$ then $f(x)$ is equal to

A. $\frac{1}{2}(1-x^4)+\log_e(2x-1)$

B. $x^2-\frac{x^4}{2}$

C. $\frac{1}{2}(\log_e(2x-1)+1-x^4)$

D. $\frac{1}{2}(\log_e(2x-1)-1-x^4)$

E. $x^2-\frac{x^4}{2}-\frac{1}{2}$

Question 27

In an isolated town, the probability of the number of cars that drive through during a particular period of time is given by the following table

Number of Cars	0	1	2	3	4 or more
Probability	0.1	0.2	0.3	0.2	0.2

The probability that more than 2 cars pass through the town during this time is

A. 0.7

B. 0.4

C. 0.3

D. 0.9

E. 0.8

Question 28
Jill is a casual employee at the local supermarket which is always open. She has a 30% chance of being called for work on any day. The probability that she will work two days next week is

A. 0.130

B. 0.310

C. 0.110

D. 0.015

E. 0.318

The following refers to question 29 and 30

In a particular country the proportion of people with green eyes is known to be 0.2. A random sample of 100 people is drawn from this population.

Question 29
Using the normal distribution as an approximation to the binomial distribution, the probability that more than 10 people out of the sample of 100 have green eyes can be written as

A. $1 - \Pr(Z < 10)$

B. $\Pr(Z > 10)$

C. $\Pr(Z < \frac{10.5}{4})$

D. $\Pr(Z < \frac{9.5}{4})$

E. $\Pr(Z < \frac{10}{4})$

Question 30
Let random variable X be the number of people with green eyes in the sample of 100. The values of a and b such that $\Pr(a < X < b)=0.67$ could be

A. $a = 16$ $b = 24$

B. $a = 18$ $b = 22$

C. $a = 12$ $b = 28$

D. $a = 8$ $b = 32$

E. $a = 15$ $b = 25$

Question 31
In a random sample of 300 people it was found that 124 could name at least 2 members of the band the Spice Girls. An approximate 95% confidence interval for p, the proportion of the total population that could name at least 2 members of the Spice girls, is given by

A. $0.385 < p < 0.442$

B. $0.356 < p < 0.413$

C. $0.30 < p < 0.50$

D. $0.356 < p < 0.470$

E. $0.413 < p < 0.470$

Question 32
Sam Sneed, a champion putt-putt golfer, had a probability of 0.7 of making a hole in one. Sam had a quick 9 holes before he went to his bagpipes lesson. The probability that Sam scored no more than two holes in one is given by

A. $1 - {}^9C_2(0.7)^2(0.3)^7$

B. ${}^9C_1(0.7)^1(0.3)^8 + {}^9C_2(0.7)^2(0.3)^7$

C. ${}^9C_2(0.7)^2(0.3)^7$

D. $(0.3)^9 + {}^9C_1(0.7)^1(0.3)^8 + {}^9C_2(0.7)^2(0.3)^7$

E. ${}^9C_0(0.7)^9(0.3)^0 + {}^9C_1(0.7)^8(0.3)^1 + {}^9C_2(0.7)^7(0.3)^2$

Question 33
The following table displays the number of stations (x) travelled by 50 train passengers who boarded the train at Caulfield train station and travelled towards Flinders street station.

x	1	2	3	4	5	6	7
No. of Passengers	5	6	10	8	11	4	6

Based on this evidence, the expected number of stations travelled by a passenger is

A. 3

B. 3.5

C. 4

D. 4.5

E. 5

Part II (Short Answer Questions)

Question 1
If the coordinates of the point of intersection between the two curves

$$y = \sqrt{3}\cos ax, \quad 0 \le x \le \pi \quad \text{and} \quad y = \sin ax, \quad 0 \le x \le \pi, \quad 0 \le a \le 1$$

are (π, b), find a and b.

2 marks

Question 2
Find $f'(x)$ given $f(x) = (3x + 1)(2x - 3)^3$, giving the answer in a factorised form.

2 marks

Question 3
Find the distance between the two midpoints of lines PQ and RQ, where the cartesian coordinates of P, Q and R are: (-6, 7), (2, 3) and (-2, -5) respectively. Write your answer correct to two decimal places.

3 marks

Question 4
The height above sea level $h(x)$ metres of a road x kilometres out from a particular town is given by

$$h(x) = 30\cos\left(\frac{\pi x}{100}\right) + 40, \quad 0 \le x \le 100$$

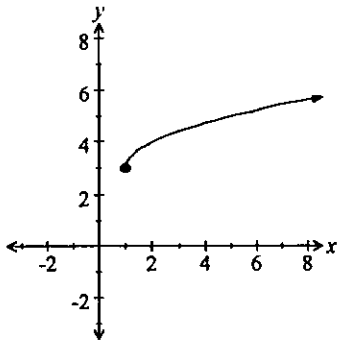
a. What is the height above sea level of the road 25 km out from the town? Write your answer accurate to the nearest two decimal places.

1 mark

b. How far out from the town will the road be if it is 40 metres above sea level?

2 marks

Question 5
The graph of the function $f : [1, \infty) \to R$, where $f(x) = 3 + \sqrt{x - 1}$ is shown below



a. On the same graph sketch the inverse function

1 mark

b. Find the rule for the inverse function, f^{-1}.

1 mark

Question 6

If X is a binomial random variable with $n = 15$ and a mean of 12, find

a. The variance, correct to one decimal place.

1 mark

b. $\Pr(X < 14)$, correct to two decimal places.

2 marks

Question 7

X is a normally distributed random variable with a mean of 30 and a standard deviation of $2a$.

If $\Pr(X > b) = 0.30$, find b in terms of a.

2 marks

TOTAL 17 marks

END OF TRIAL CAT 2