VCE 1998

Board of Studies
Report for Teachers

Mathematics

Cycle 3

Page

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VCE REPORT FOR TEACHERS 1998

Further Mathematics CAT 2: Written examination (Facts, skills and applications)

GENERAL COMMENTS

The number of students presenting for Further Mathematics CAT 2 in 1998 was 14 341, a slight increase over the number who sat in 1997 (13 777). Most students completed three modules in the multiple-choice section and clearly identified these modules.

The multiple-choice paper format, as in previous years, generally moves from easier questions to more difficult questions.

SPECIFIC INFORMATION

Multiple-choice solutions

Core:

The third column gives the percentage of correct responses.

1	C	82
2	E	66
3	D	49
4	D	71
5	D	35
6	D	42
7	A	64
8	D	22
9	C	47

Modules:

The third column in each set gives the percentage of correct responses.

Arithmetic and Applications

1	C	90
2	A	87
3	D	49
4	C	38
5	D	33
6	B	66
7	D	52

Probability and Statistics

1	C	46
2	C	82
3	A	46
4	D	35
5	A	48
6	E	42
7	C	26

Geometry and Trigonometry

1	D	86
2	C	69
3	D	45
4	E	32
5	C	12
6	C	33
7	B	72

Graphs

1	D	66
2	B	28
3	E	89
4	D	58
5	D	51
6	D	26
7	B	39

Business-related Mathematics

1	B	88
2	B	46
3	C	80
4	B	23
5	D	38
6	A	36
7	A	60

Networks

1	C	46
2	B	56
3	C	48
4	E	36
5	C	53
6	C	70
7	A	81

Short-answer section

Areas of Weakness

Students should be able to use calculators efficiently to produce required univariate and bivariate statistics. They should also be able to specify results to a required accuracy. There were many instances where answers were not specified to the required accuracy.

Marking Policy

Decimal place errors drew only one penalty per paper and this was common.

Question 1

1a (Mean score .35, Available marks: 1)

Answer: 0.8564 (4 decimal places)

The main problem was accuracy.

1b (.82; 3)

Answer: number sold = –16.97 + 3.69 × temperature

The two problems most obvious in this were accuracy and reversing the coefficients. The latter suggests there was little effort made to relate the line equation to the data in this question, as reversed coefficients give a very poor fit (as could be noted by a visual check of a 'by eye' line of fit). This is also true of most other erroneous answers. Some students also dropped the negative sign on the first coefficient.

1c (.35; 1)

Answer: 61

Should be a whole number but consequential solutions on part b were accepted.

1d (.37; 1)

Answer: 0.733

Again some consequential result on part a was paid if the answer was between 0 and 1.

1e (.40; 1)

Answer: 73.3% or 73%

This was dependent on part d. No accuracy was specified here.

Question 2

2a (.12; 1)

Answer: 1.18

Where there are 12 seasons the sum of the seasonal indices is 12. So the December index is

12 – 10.82 = 1.18. This was the most common response.

2b (.35; 1)

Answer: 74%

The most common error was to continue on and use the December figure whereas the question asks for June. Ironically, this was to avert sequential errors for those unable to do part a.

2c (.54; 1)

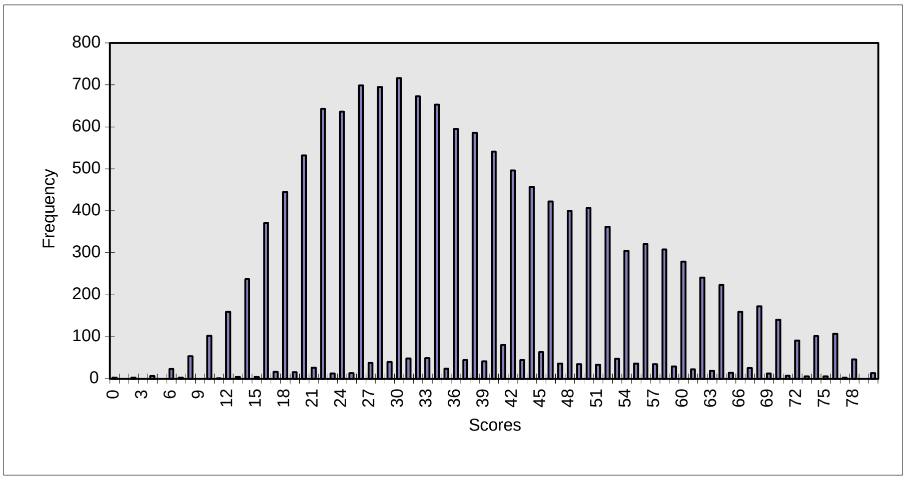
Answer: Increasing trend over the year.

Students gave a variety of answers here. One confusion existed over the relationship between this data ('deseasonalised') and the data in part a which was raw. Some answers involving both years had data falling in January. The other problem was students merely describing what happened. It needs to be noted that an increasing trend does not demand an increase each time—it is a pattern over time. Finally a very large number of students described the data as having a seasonal trend despite being told it was 'deseasonalised'.

CAT MA072 MATHEMATICS: Further Mathematics WRITTEN EXAMINATION

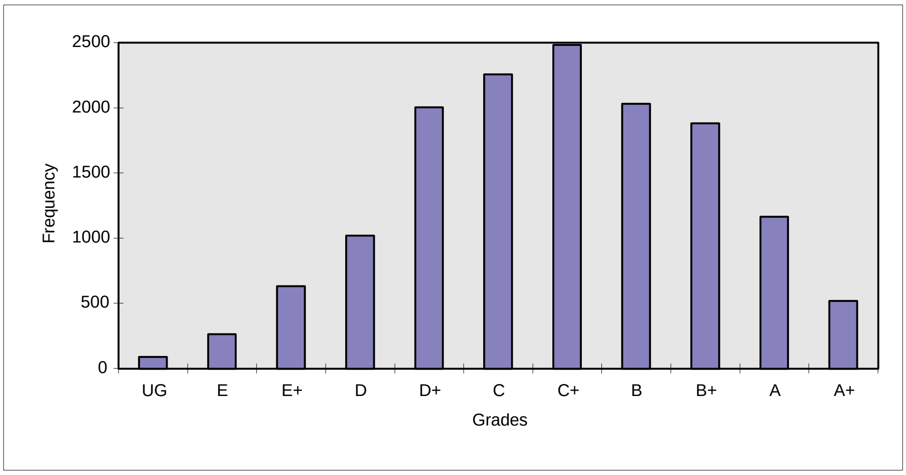
HISTOGRAM OF TOTAL SCORES 1998

Count **14341** Mean **37.96** Standard Deviation **15.71** NA Result **1550**



HISTOGRAM OF TOTAL GRADES 1998

Count **14341** Mean **5.84** Standard Deviation **2.16** NA Result **1550**



ENROLMENTS		%
Female	8160	51.4
Male	7722	48.6
Total	15882	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the ‘average’ score; that is all scores totalled then divided by the ‘Count’.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Further Mathematics CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

The number of students presenting for Further Mathematics CAT 3 in 1998 was 14 325 an increase of 583 or 4.2 per cent over the number who sat in 1997 (13 742). The great majority of students attempted exactly three modules as required. In cases where this was not complied with and students attempted more than three modules, the results in all modules were very poor. The marking scheme allowed for students to gain many marks through displaying correct working—even from an initial error. Showing full working will help students checking work in the exam itself and assists markers in attempting to distribute consequential and method marks.

The paper format, as in previous years, generally moved from easier questions to more difficult questions which hopefully spread student results effectively and rewarded the able student.

Areas of Weakness

Although raw results improved over the previous year there are still many common areas of weakness in all modules. There is evidence of inefficient use of calculators – either inappropriate use or poor use in generating the required accuracy. Decimal place problems still occurred regularly, with a standard rule of no more than one mark per paper being deducted for this to prevent repeated accuracy marks being lost.

Too often simple algebraic manipulations, including substitutions, still generated errors. Ratios were often poorly handled.

There is evidence that many students cannot apply mathematical methods to unfamiliar problems, indicating that concepts may not be sufficiently well established. Many students are evidently applying methods used on easier problems seen in similar contexts without giving much thought to the processes involved.

As in previous years, individual modules generate their own familiar pattern of common errors. In Arithmetic and Applications, the difference equations, ratio, and infinite series all cause problems; in Probability the notion of probabilities greater than 1 does not bother too many students and simulations are often executed carelessly; in Geometry and Trigonometry a surprising number of students have difficulty with work they have been doing for years. The popular belief that all triangles are right angled persists as does difficulty in reading diagrams. Module 4 generated many impossible or inappropriate answers which went unremarked by students including those in the early part, where simple linear graphs were involved – work they have covered several times in earlier years.

In Module 5 some students displayed very poor work on basic money problems, similar to those that they will have to handle all their lives. Annuities work was generally handled poorly despite similar questions occurring in past papers.

In Module 6 the understanding of basic definitions was often poor and their application incorrect or inaccurate.

Choice of Options

The pattern of option selection matched previous years. The pattern is seen in the following table.

Module	**1996**	**1997**	**1998**
Arithmetic	7971	7715	8267
Probability	1160	2026	2043
Geom & Trig	10830	11353	11367
Graphs	6054	5890	6492
Bus Maths	9367	8967	9453
Networks	4562	4670	4899

The selection patterns have shown only limited variation over these years.

Marking Policy

As noted, decimal place errors drew only 1 penalty mark per paper. If students crossed out work and replaced it, the replacement work was assessed. It was not possible to award marks if the original was correct unless it was not replaced. This was to reduce the problem of dealing with a selection of answers.

SPECIFIC INFORMATION

Module 1: Arithmetic and applications

Q1a (Mean score: 3.28; Available marks: 5)

1ai

Answer: 7m

Usually well done – designed to be read off the diagram.

1aii

Answer: 13m

Generally well answered

1aiii

Answer: $t_n = 4 + 1.5(n – 1)$ or $2.5 + 1.5n$

Often $t_n = 4 + 1.5n$ or $t_n = 4 + 1.5n – 1$ was given as the answer where the construction or brackets made problems.

1aiv

Answer: 13

Usually satisfactory but often 12 was given as the answer by formal or informal solution of

$22 = 4 + 1.5n$.

1b (.53; 2)

Answer: 4

This question was poorly answered. Most students used numbers inappropriately and thought nothing of using 100 litres of concentrate to dilute to get 20 litres. Previous evidence shows these problems are intrinsically difficult and yet they are realistic problems for everyday life. This type of problem needs further practise.

Q2 (3.39; 7)

2a

Answer: a = 1.08; b = 100; c = 400

Often poorly answered. Clearly students find translating into mathematics and difference equations difficult.

2b

Answer: 332

Students with the wrong difference equation often produced this correct result anyway.

2c

Answer: Replacement rate is smaller than selling level.

Any similar response was satisfactory. Sequential errors allowed for other answers, for example; if the student's difference equation gave unlimited growth they could say they ended up with too many cows.

2d

Answer: 32

Sequential answers accepted but this was more likely to be correct than part a.

Q3 (1.12; 6)

3a

Answer: 2.7m

Usually correct.

3bi

Answer: 9.5m

This question was poorly done. One problem was the initial height. Students often could not work through an infinite series. Sequence ratio sizes exceeding 1 were frequently used.

3bii

Answer: 13

Sequential results from previous question acceptable, but this was not done well.

Module 2: Probability and Statistics

Q1 (1.96; 6)

1ai

Answer: 0.01024 or 0.4^5

Not generally well done.

1aii

Answer: .3456 or ${}^5C_2.4^2.6^3$

Many errors were made in response to this question.

1aiii

Answer: $1 – 0.6^5 = 0.92224$

Not well done.

1b

Answer: 2

This part was the most successfully answered in the question.

Q1c (.95; 2)

1ci

Answer: 0–3 free, for example

1cii

Answer: 4–9 busy, for example

These required any suitable complementary combination of digits. An improved performance compared to previous questions of a similar type.

Q2 (1.98; 5)

2a

Answer: 0.2

A common mistake was to include the three items and get 0.3.

2b

Answer: 4.9

Generally satisfactory.

2c

Answer: 0.3

Q3 (3.39; 5)

3a

Answer:

3	7				
				19	
					360

The numbers 19 and 360 were easily supplied – the others less so.

3b

Answer: 327 seconds or 5 minutes 27 sec

Surprisingly many only averaged four simulations – even if they answered part a. satisfactorily.

Q3c (.2; 2)

Answer: 294 sec

Not well done.

Module 3: Geometry and Trigonometry

Q1 (3.79; 7)

1a

Answer: 360/8

Most students gave some equivalent of this. e.g. $45 \times 8 = 360$.

1bi

Answer: 25

Common errors included 25×2.

1bii

Answer: 22.5 degrees

Common errors included 45 degrees.

1biii

Answer: 20.71

Common errors included incorrect use of trigonometric ratios, or rounding problems as well as sequential mistakes from other parts.

1c

The area is 2071 cm^2 (nearest square cm).

A common approach was to evaluate $8 \times$ area $\Delta\, POQ$. Some sequential errors occurred from part b.

Q2 (1.34; 2)

Answer: 34.4

Units were often 'lost' here. 3440 cm was satisfactory as it gave the correct answer and accuracy. 3440 was too indefinite and 3440 m suggested a lighthouse a little too high.

Q3 (2.46; 7)

3ai

Answer: 78 degrees

Many students could not relate text and picture, so the angle was misplaced.

3aii

Answer: 864 m

Better done than expected given other errors. Sequential problems did occur from part i. and, of course, the desire to create right triangles.

3b

Answer: 125 degrees true

Incorrect rule use was the main problem. Some students completed part of this question successfully but forgot to find the bearing angle.

Q4 (1.17; 4)

4a

Answer: 82 deg

$\tan\theta = \dfrac{250}{25}$ was the popular starting point. The denominator error still gives some sequential work.

4b

Answer: 50 cm

No sense of ratios. Some did this by trigonometry to generate decimal place errors.

Module 4: Graphs and Relations

Q1 (1.63; 3)

1ai

Answer: 52

Answers derived by various means, between 50–54, were satisfactory.

1aii

Answer: y = 240 + 52t

Intercept was definite but slope of part i., if satisfactory, was accepted.

1b

Answer: 864

A mark was allowed for a graph based estimate so answers between 840– 890 were accepted.

Q2 (2.95; 6)

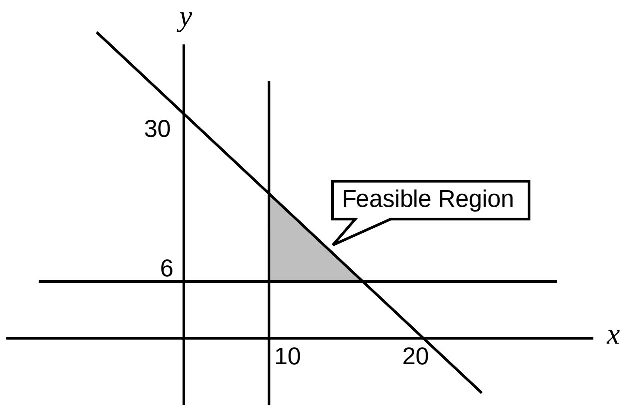
2a

Answer: $y = 6$; $39x + 26\,y = 780$ or $3x + 2y = 60$

Inequality directions a problem. Equations used. There was a recurrence of the problems identified in last year's report.

2b

Answer:



2c–d (2.33; 5)

2ci

Answer: R = 3000x + 1800y

2cii

Answer: Maximum is 58 800 at (16, 6)

Often only one vertex was checked. Most did this by vertex checking, not a sliding line method. This question was a problem for students with errors in part b.

2di

Answer: $1980

Generally well done.

2dii

Answer: $1220

Generally well done.

Q3 (1.01; 6)

3ai

Answer: C = 17320 + 1980x

Could be in expanded unsimplified form.

3aii

Answer: T = 3000x + 10800

Sequential on 2ci.

3b

Answer: x = 7 (As this is covered by constraint, no loss is possible in feasible set).

More stumbled towards this than obtained correct details in part a. Not very well done overall. Students sometimes confused revenue and profit.

Module 5: Business-related mathematics

Q1 (2.76; 6)

1a

Answer: 10%

Surprisingly, many failed to find this. A preferred error being 150/1350 × 100%.

1b

Answer: $450

Most found this result although some answered $150.

1c

Answer: 33.3%

A number of students still have difficulties with percentages.

1d

Answer: 61.5%

As this was formula based it was possible to get sequential marks if part c. was wrong.

Q2 (2.6; 5)

2a

Answer: 8 years

This question was well done, although often only an answer was given.

2b

Answer: 6 years

Also quite well done although, once again, sometimes only the answer was given.

Q3 (1.66; 6)

3a

Answer: A = 0   n = 12   R = 1.02

This question was poorly executed despite having been given before. The A always confuses students and the R required them to think about both quarterly adjustment and how to express it as a multiple not a percent.

3b

Answer: $141.84

This question required inversion of the formula with A = 0. Students often do this even if they do not set A = 0 in part a. The link may be obscure to students but the need to set A = 0 to save effort is more obvious here.

3c

Answer: $202.07

Could have rounding problems involved. $202.08 was satisfactory.

Q4 (.9; 3)

4a

Answer: $400

Well done overall, although some students still read this graph incorrectly.

4b

Answer: 9% or $135

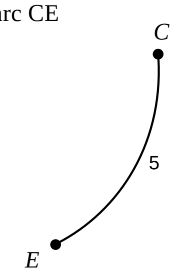
Both answers are acceptable as rates. This question was poorly done.

Module 6: Networks and decision mathematics

Q1 (3.28; 6)

1ai

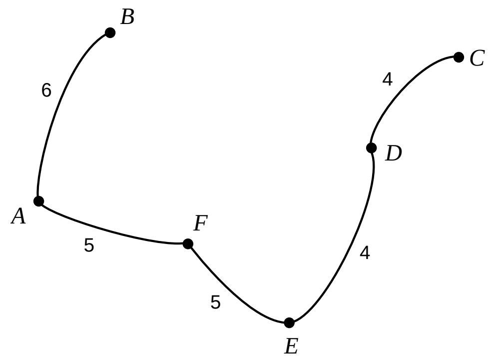
Answer: Add the arc CE



Many students did not attempt this question – and/or added other arcs to the diagram. Others missed labelling the distance.

1aii

Answer: The answer was the minimum length spanning tree



The length: 24 km.

Students who tried to draw this over their existing diagram usually made a mess. The intention was for them just to sketch and label the tree.

1aiii

Answer: Matrix entries are as follows:

1	0	1	1
1	1	0	1
0	1	1	0

Q1b (2.66; 4)

1bi

Answer: 45 km

Many errors related to part a.

1bii

Answer: Repeated visits to sites nodes

1biii

Answer: ABCDEFA or similar

Most students answered this well.

1biv

Answer: 33 km or whatever matched their circuit.

Usually done well.

Q1c (.17; 1)

Answer: F and C

Few students gave this answer.

Q2 (1.31; 4)

2a

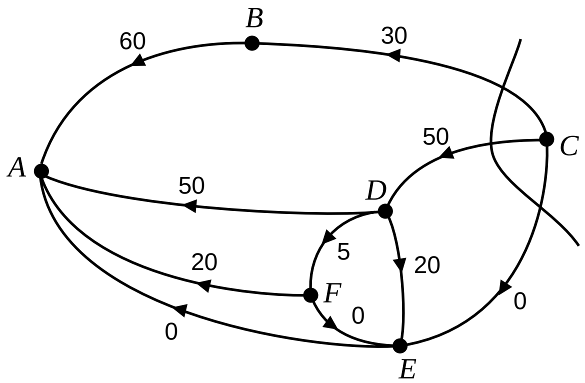
Answer: 100

Students could use a minimum cut or show their answer by arc capacities. This was not done well at all.

2b

Answer: 80

The question required students to label the given graph, as a directed graph for travel in the reverse direction, using the data in the table supplied for 2b. A minimal cut would then determine the maximum flow through the network or it could be done by inspection of the graph.



Students have difficulty identifying the flow associated with cut sets and even with transferring information from the table to the network. In part 2a. this flow procedure was addressed directly with a graph where arc capacities were given on the graph. In part 2b. the arc capacities had to be obtained from the table. The question aimed to show that in directed graphs, forward and reverse capacities may differ but the method of solution is the same.

Q3 (2.13; 5)

3a

Answer: A-Z, B-W, C-X, D-Y and A-Z, B-X, C-W, D-Y

The Hungarian algorithm gives a guaranteed optimum. Strictly speaking the alternative is exhaustive enumeration. Both methods were done often enough and the results overall were fairly good.

3b

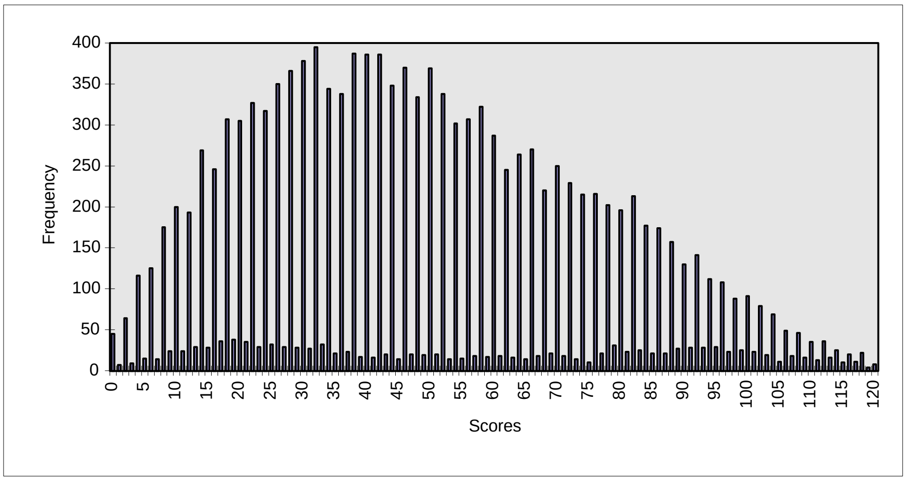
Answer: $130

Sequential on part a.

CAT MA073 MATHEMATICS: Further Mathematics **WRITTEN EXAMINATION**

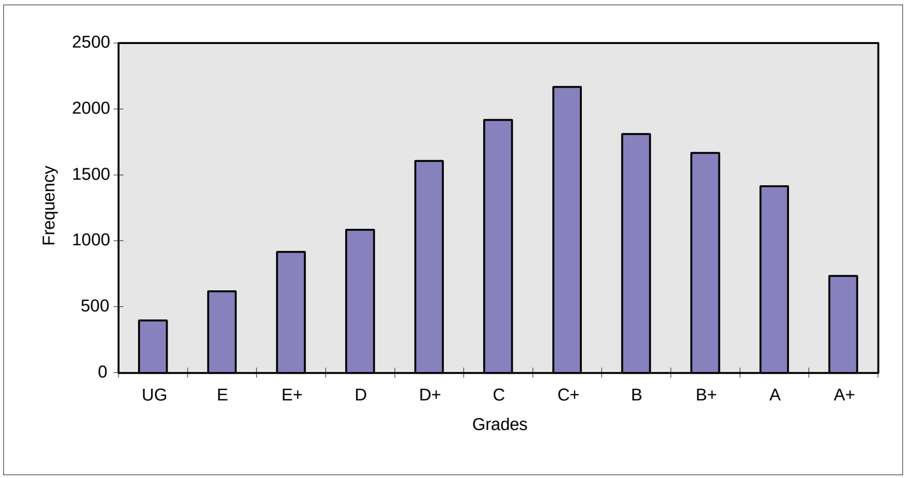
HISTOGRAM OF TOTAL SCORES **1998**

Count **14325** Mean **49.39** Standard Deviation **26.90** NA Result **1566**



HISTOGRAM OF TOTAL GRADES **1998**

Count **14325** Mean **5.64** Standard Deviation **2.55** NA Result **1566**



ENROLMENTS		**%**
Female	**8160**	**51.4**
Male	**7722**	**48.6**
Total	**15882**	

GLOSSARY OF TERMS

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Mathematical Methods CAT 2: Written examination (Facts, skills and applications task)

GENERAL COMMENTS

The number of students who sat for the 1998 examination was 17 346. Just over 5 per cent scored 90 per cent or more of the available marks, with 53 students scoring full marks on the paper.

As in the past, students performed reasonably well on the multiple-choice questions. The short-answer questions were designed to be accessible to all students and there appeared to be little discernible difference in students' abilities to respond to these questions compared to 1997 and 1996. The ability to sketch a graph of a derived function, having been given the graph of the original function, again posed problems for many students; as did the algebraic manipulation of simple logarithmic and polynomial expressions.

Continued difficulties in these aspects of the course indicate a need for teachers to pay more attention to these methods in class.

As in previous years some students found it difficult to follow instructions such as 'correct to two decimal places', 'probability', 'exact area', 'in the form of'. In some cases this may indicate students' lack of knowledge of basic mathematical terms. Presentation of responses ranged from outstanding to poor. Some students displayed 'sloppy' mathematics. Some appeared careless in defining important points, for example, points on a graph or terminals for integrals. Overall students need to pay more attention to the logical, clear and precise presentation of mathematics expected at this level.

SPECIFIC INFORMATION

Multiple-choice questions (Total 33 marks)

Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	B	55	18	B	50
2	D	60	19	C	62
3	C	84	20	B	46
4	E	49	21	B	31
5	E	47	22	B	61
6	D	57	23	E	29
7	E	50	24	D	34
8	D	39	25	C	18
9	B	44	26	A	67
10	D	36	27	E	53
11	E	82	28	A	66
12	A	73	29	C	54
13	C	55	30	B	56
14	C	77	31	D	46
15	A	71	32	D	48
16	C	56	33	B	61
17	A	23			

Short-answer questions (Total 17 marks)

Question 1 (Mean mark: 1.39; Available marks: 3)

Part a = 1; Part b = 2

a Many students failed to use the information provided, despite the graph showing an intersection with the x-axis at $x = 2$. Those who did obtain the $(x - 2)$ factor found it difficult, or impossible to correctly divide the polynomial to obtain the quadratic required. Some students attempted to work backwards having obtained three solutions using a graphics calculator, and invariably came unstuck with the algebra and a failure to recognise that the calculator provided decimal approximations not exact answers and therefore values for b. and c. were left as decimals instead of whole numbers.

A correct response is:

$(x-2)(x^2-3x-5)$

b Many students obtained the $x = 2$ intersection point and stopped. Those who tried to find the other two points were often unsuccessful due to their inability to correctly factorise a quadratic. Use of the quadratic formula was generally poor and some students even tried to 'complete the square', usually unsuccessfully. Others ignored the 'hence' and stated the three solutions, using decimal approximations probably obtained from a graphics calculator. No marks were awarded for an answer obtained in this manner.

A correct response is:

$2, \left(3 \pm \sqrt{29}/2\right)$ or $-1.19, 2, 4.19$

Question 2 (0.65; 2)

Although many students handled this question very well, there were also many who experienced difficulties. Quite a few believed that $\log_a$ was an algebraic expression that stood alone while others believed that it has the properties of a linear function. Significant weaknesses with arithmetic and algebra were often evident, as was knowledge of log laws. The disappearing 2's, the disappearing log's, disappearing a's were all evident. Some students changed the base to 10 or e. Those students who did know what to do generally solved for x in terms of a efficiently. Some students who knew their log laws solved for a in terms of x.

A correct response is:

$2\log_a x - \log_a 9 = 2$

$\log_a(x^2/9) = 2$

$x^2/9 = a^2$

$x^2 = 9a^2$

$x = 3a \ (a > 0, x > 0)$

Question 3 (1.3; 3)

Part a = 1; Part b = 1; Part c = 1

a This was the most poorly answered part of the question. Many students mentioned a vertical line test only, others suggested the inverse existed simply because f was a function. Some students believed f^{-1} existed because of something to do with reflecting the graph in the line $y = x$.

A correct response is:

f is a 1 – 1 function.

b This was generally well answered. Common mistakes occurred in expressing the domain using $\cap$ instead of $\cup$ and the use of square brackets where round brackets were required. A small number of students stated that the range of f = the domain of f^{-1} and then stopped.

A correct response is:

$R\backslash\{2\}$ or $(-\infty, 2) \cup (2, \infty)$

c Some students simply rearranged the expression with A, B and b without substituting the values. Otherwise this question was reasonably well done, apart from a few algebraic errors.

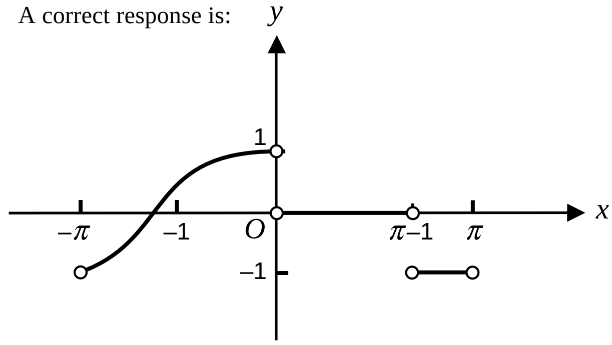
A correct response is:

$f^{-1}(x) = 1/(x-2) + 1$

Question 4 (0.26; 2)

Some students continue to have difficulty answering this style of question. Many do not seem to appreciate the association between gradient and the derived function. Auite a few students sketched either the inverse function or the reciprocal function. Many students ignored the interval (0, π – 1), not recognising that f' was zero there.

A correct response is:



Question 5 (1.74; 4)

Part a = 2; Part b = 1; Part c = 1

a The majority of students obtained an equation in the form $\sum p(x) = 1$ (some gave $\sum xp(x) = 1$). However the difficulty then arose in how to solve this equation to find the required value of k. Quite a number of students obtained $k = 1$ by various methods. Some students obtained values of k resulting in probabilities greater than one or less than zero, and proceeded to use this value.

A correct response is :

$$\frac{2k^2 - 1 + 4k + 3k + k}{9} = 1$$

$2k^2 + 8k - 10 = 0$

$(k+5)(k-1) = 0$

$k = 1$ since $0 \le p(x) \le 1$

b This was among the best answered questions on the paper. The vast majority of students knew to apply $E(X) = \sum xp(x)$. The only real problem was remembering the 'correct to two decimal places' required in the answer.

A correct response is:

$E(X) = 1 \times 4/9 + 2 \times 3/9 + 3 \times 1/9 = 13/9 = 1.44$

c This question was poorly answered. Many students attempted to use the binomial theorem or normal distribution. Another common error was to have

$\Pr(X \le 2) = \Pr(X = 0) + \Pr(X = 1)$.

A correct response is:

$\Pr(X \le 2) = 1 - \Pr(X = 3)$

$= 8/9 = 0.89$

or

$\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$

$= 1/9 + 4/9 + 3/9 = 8/9 = 0.89$

Question 6 (0.87; 3)

Most students recognised what was required was the anti-differentiation of a difference of two functions. However, some students divided the area up incorrectly or into far too many parts to be able to successfully complete the integration. Errors were also made with the terminals, in particular 1 occurred quite frequently as the lower terminal in the first integral instead of 0. As expected some students differentiated cos*x* and/or cos(2*x*). Frequently errors occurred in the substitution of terminals, resulting in an incorrect final answer. ‘Exact area’ was ignored by some students.

A correct response is:

$$2\int_0^{\frac{2\pi}{3}} \left(\cos x - \cos(2x)\right)\, dx + \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(\cos(2x) - \cos x\right)\, dx$$

$$= 2\left[\sin x - \frac{1}{2}\sin(2x)\right]_0^{\frac{2\pi}{3}} + \left[\frac{1}{2}\sin(2x) - \sin x\right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

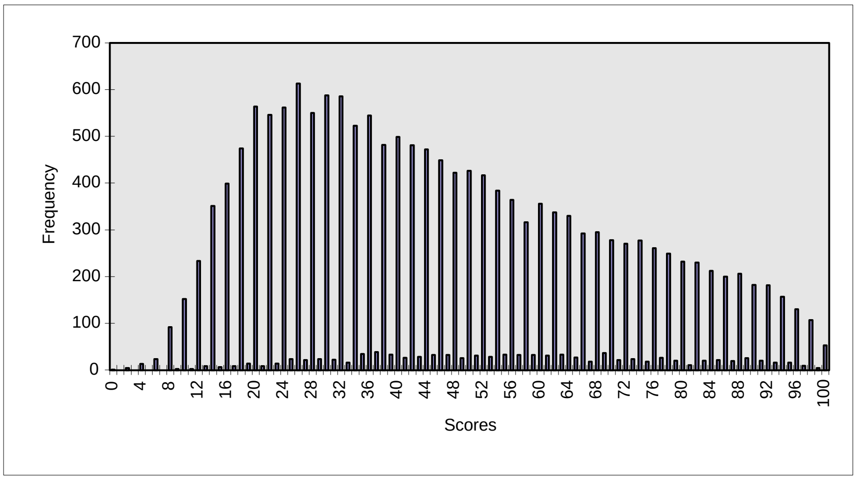
$$= 2\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right] + \left[\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right]$$

$$= 3\sqrt{3}$$

CAT MA082 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION

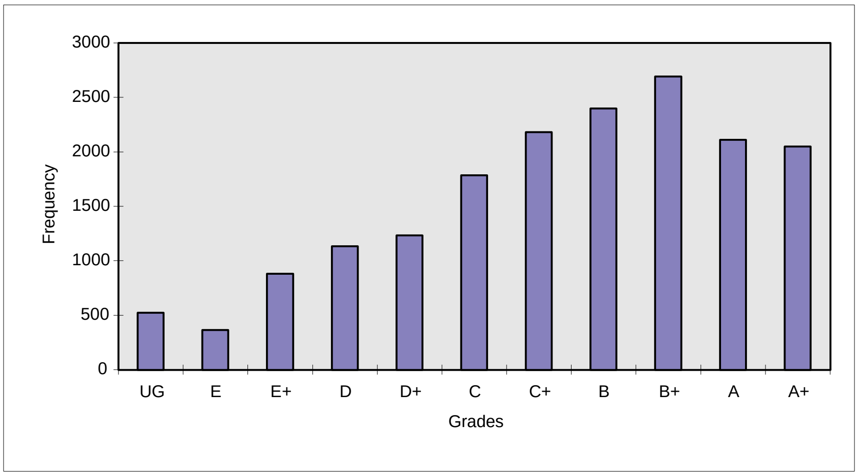
HISTOGRAM OF TOTAL SCORES 1998

Count **17350** Mean **46.82** Standard Deviation **23.11** NA Result **681**



HISTOGRAM OF TOTAL GRADES 1998

Count **17350** Mean **6.36** Standard Deviation **2.65** NA Result **681**



ENROLMENTS		**%**
Female	**8361**	**46.4**
Male	**9648**	**53.6**
Total	**18009**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the ‘average’ score; that is all scores totalled then divided by the ‘Count’.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Mathematical Methods CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

The entire range of marks from zero to 55 were present. There were highly competent students who showed their abilities by submitting perfect or near-perfect papers, although these were fewer than last year. There continued to be a high proportion of students who could gain only a very small number of marks, despite there being marks available which required only Units 1 and 2 level work.

Students' abilities to correctly manipulate algebraically is a major weakness. Many students lost marks through carelessness in their algebra and through gaps in their understanding of basic algebra and probability.

All too often, students lost marks because they failed to directly address the question asked or because they did not answer the question in the specified way. This was a particular problem where exact answers were required but approximate answers given and where a specified number of decimal places was required. In some cases, students lost marks because they did not read the question carefully enough. For example:

- Question 1g., some students did not correctly interpret 'at least two Jumbo oranges';
- Question 2c., some students did not find both intercepts;
- Question 3d., it was common to find the South Paddock not shaded.

Use of graphics calculators was noted, sometimes resulting in few marks being awarded as the direction on the front of the paper (Calculus must be used to evaluate derivatives and integrals) was not observed by some students. Students who gave the correct answer for Question 3b. but showed no derivative, or an incorrect one, gained no marks. Similarly in Questions 2e., 3f. and 3g., the correct answer was not rewarded unless the correct anti-derivative was shown.

This year there were a number of questions which required students to show a particular result (usually so that the result was available to use later in the question). Many students were very poor at explaining or setting out arguments in these questions and frequently resorted to 'manufacturing' the required answer out of their ineffective and incorrect use of algebra. Teachers should provide students with substantial practice in questions of this type.

SPECIFIC INFORMATION

Question 1

	Mean marks	Possible marks
a	.72	1
b	1.46	2
c	1.24	2
d	.75	2
e	.46	3
f	.91	2
g	1.25	3

Many students were able to handle most of this standard probability question. It was clear that many students were using calculator programs rather than normal distribution tables and some students did not do this correctly, particularly when the inverse was required. There were many students who did not provide answers to the required accuracy, presumably through not reading the question carefully enough. Part d. was poorly done; the concepts presented being too difficult for many students to cope with. In part e., few students recognised that there was a reduced sample space involved; many did not score any marks as a result. Conditional probability seems to have been rarely met in the context of normal distributions. Part f. was generally well done and in part g., the majority of students knew to use binomial probability, but not all were successful in their application. Parts e. and g. again showed the weaknesses commented on in the 1997 and 1996 reports.

Question 2

	Mean marks	Possible marks
a	.36	1
b	.74	2
c	1.44	2
di	1.37	2
dii	.49	1
e	1.45	4

The usual weaknesses in nomenclature and algebra were apparent in this question. While most students knew that –2 was somehow involved in part a., far too many could not express the domain in correct set notation and hence 64 per cent of students were not awarded the mark. Part b. was poorly done, the most common mark being 1 out of 2. Many students were not able to express the transformations correctly, both translations and the dilation requiring a direction to be specified. Teachers should provide practice for students at writing this kind of question, checking that the transformations are correctly expressed in an unambiguous manner. The dilation and the order of the transformations were not understood well by many students. A significant number of students indicated that the asymptotes moved rather than the graph. In part c., some students only found one intercept (misreading 'axes' as singular) but the question was generally well done, with marks being mainly lost for poor algebra. A common error was to give the answer as (2, 3). In part di., some students confused f^{-1} with f', which often prevented a good score. It was clear that there were many students unable to anti-differentiate in part e., while those who could had trouble with the algebra, resulting in low scores for this question.

Question 3

	Mean marks	Possible marks
a	1.5	1.19
b	1.19	2
c	.71	1
d	.48	1
e	1.65	3
f	1.29	4
g	.5	2

This question was generally understood by students, but again many lost marks through poor algebra and calculus. Parts a., b. and c. were generally well done, as they required little more than Units 1 and 2 level mathematics. A number of students were unable to complete part b. where it was only necessary to substitute $\pi/6$ into the derivative and show this was zero (a significant number of students wanted to find the required derivative using the product rule). The latter half of the question was very poorly handled by a large group of students, careless algebra and calculus costing many marks. Many students could not correctly identify the way in which they attempted to find the area, many more losing the mark for a careless graph of $y = \sin x$, too often without a horizontal tangent at $x = \pi/2$ or without the South Paddock shaded. In part e., marks were often lost for inexact answers and for not finding the y-coordinate. Part f. presented the usual problems with negative answers for areas, poor knowledge of exact values of trigonometric functions of 0, $\pi/4$, etc. and careless anti-differentiation. Many students managed to give the correct answer after some inappropriate algebraic manipulation but were not awarded marks for this.

3a $A = \left(0, \frac{1}{2}\right), \quad B = \left(\frac{\pi}{2}, \frac{1}{2}\right)$

b $\frac{dy}{dx} = \cos(x) - \sin(2x)$

At a stationary point, $\frac{dy}{dx} = 0$.

If $x = \frac{\pi}{6}, \frac{dy}{dx} = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$, so there is a stationary point when $x = \frac{\pi}{6}$.

c $c = \sin\left(\frac{\pi}{6}\right) + 0.5\cos\left(\frac{\pi}{3}\right) = 0.75$

d Sketch – must pass horizontally through $\left(\frac{\pi}{2}, 1\right)$ and through O. South paddock must be shaded.

e $\sin(x) = \sin(x) + 0.5\cos(2x)$

$\Rightarrow 0.5\cos(2x) = 0$

$\Rightarrow 2x = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{4}, y = \frac{1}{\sqrt{2}}$, So coordinates are $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

f

$$A = \int_0^{\frac{\pi}{4}} \sin(x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\sin(x) + 0.5\cos(2x)\right] dx$$

$$= \left[-\cos(x)\right]_0^{\frac{\pi}{4}} + \left[-\cos(x) + 0.25\sin(2x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(-\frac{\sqrt{2}}{2} + 1\right) + (0 + 0) - \left(-\frac{\sqrt{2}}{2} + \frac{1}{4}\right)$$

$$= \frac{3}{4}$$

g Total area

$$= \int_0^{\frac{\pi}{2}} \left(\sin(x) + 0.5\cos(2x)\right) dx$$

$$= \left[-\cos(x) + 0.25\sin(2x)\right]_0^{\frac{\pi}{2}} = 1$$

$\Rightarrow$ Area North Paddock = $\frac{1}{4}$ km^2

$\Rightarrow$ Area of South Paddock = $\frac{3}{4}$ km^2 = 3 × area of North Paddock.

Question 4

	Mean marks	Possible marks
a	.68	2
b	.75	4
c	.4	2
di	.38	1
dii	.24	2
e	.12	2

Although this question is similar to questions found in most textbooks, many students struggled to understand it. Only the better students made a lot of headway with the question, but a large number of students at least attempted to find solutions. Few students clearly showed that they knew where the given terms came from in part a. Part b. required the chain rule for differentiation and this was poorly handled by many students. In part c., attempts to find the correct value of p for $w = \sqrt{2}$ were usually successful, but few students reasoned well for $w > \sqrt{2}$. Students who attempted part d. were often let down by careless arithmetic. Part e. proved to be too difficult for most students.

4a $AX^2 = 10^2 + p^2$

$\Rightarrow$ Distance on water = $AX = \sqrt{\left(100 + p^2\right)}$

Distance on beach = $20 - p$

C = cost on water + cost on beach

$= w\sqrt{\left(100 + p^2\right)} + 20 - p$

b

$$\frac{dC}{dp} = \frac{2pw}{2\sqrt{\left(100 + p^2\right)}} - 1$$

= 0 for a minimum

$\Rightarrow pw = \sqrt{\left(100 + p^2\right)}$

$\Rightarrow p^2w^2 = 100 + p^2 \Rightarrow p^2(w^2 - 1) = 100$

$\Rightarrow p^2 = \dfrac{100}{w^2 - 1}$

c The pipeline will pass in front of the resort if $p \le 10$

$w = \sqrt{2} \Rightarrow p = 10$, so the pipeline will pass in front of the resort.

If $w > \sqrt{2}, w^2 - 1 > 1$ and $p^2 < 100$, so $p < 10$ and the pipeline will pass in front of the resort.

di $w = \sqrt{5} \Rightarrow p = 5$

dii Cost when $p = 10$ (A at C) is $10 + 10\sqrt{10}$ = \$416 000

Cost when $p = 5$ is \$400 000

Difference in cost < \$20 000, so cheaper to build to miss the resort.

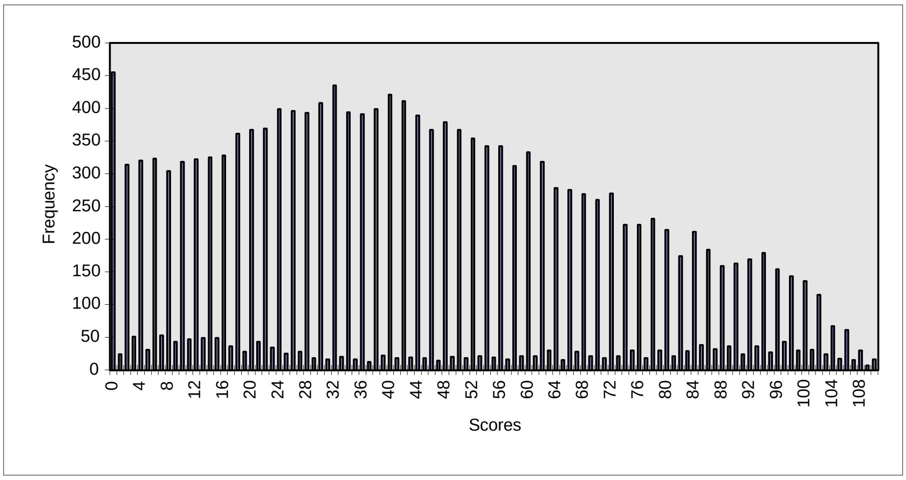
e $p = 20 \Rightarrow w = \dfrac{\sqrt{5}}{2}$

If $1 < w \le \dfrac{\sqrt{5}}{2}$, the cost will be minimum

CAT MA083 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION

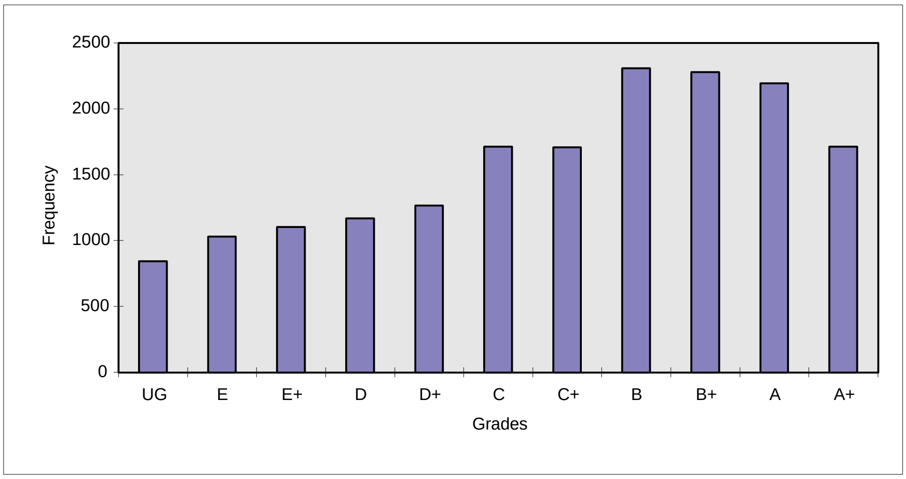
HISTOGRAM OF TOTAL SCORES 1998

Count **17329** Mean **44.98** Standard Deviation **28.32** NA Result **702**



HISTOGRAM OF TOTAL GRADES 1998

Count **17329** Mean **5.88** Standard Deviation **2.94** NA Result **702**



ENROLMENTS		%
Female	8361	46.4
Male	9648	53.6
Total	18009	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Specialist Mathematics CAT 2: Written examination (Facts, skills and applications task)

GENERAL COMMENTS

In 1998, 6140 students sat for the Specialist Mathematics examination; 153 less than the number (6293) who sat in 1997. About 5 per cent of students scored more than 90 per cent of the available marks (up from less than 3 per cent last year), with 19 students scoring full marks. It seems that the length and level of difficulty of this year's examination was about right. The correlation between scores on Part I (multiple-choice questions) and Part II (short-answer questions) was 0.78.

In some scripts students provided clearly written, accurate, succinct and logical answers. These students demonstrated an understanding of content that went beyond the ability to obtain the correct answers. However, as in previous years, the work of many students was poorly set out and featured sloppy use of mathematical notation—for example, misuse of = and ⇒, omission of *dx* or *dy* in integrals and writing vectors without tildes. Students and teachers need to recognise the importance and value of fostering accurate expression so that its routine use becomes an aid to thinking and to efficient production of good work.

Finally, to repeat last year's advice, students should be strongly advised to use a pen to answer Part II of the CAT. Work done in pencil is sometimes so faint that it is difficult to assess and can even be misinterpreted by the student. In addition, work in pencil is often erased whereas if it is left it may gain some credit.

SPECIFIC INFORMATION

Part I: Multiple-choice questions

The correct responses with percentages of correct responses were as follows:

Section 1

Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	C	77	12	E	55
2	B	42	13	A	60
3	A	60	14	D	56
4	C	71	15	A	65
5	A	75	16	D	73
6	C	42	17	B	40
7	D	63	18	B	69
8	E	79	19	D	51
9	B	62	20	E	47
10	E	66	21	C	56
11	E	70	22	D	53

Section 2

Module 1: Mechanics			***Module 2: Geometry***		
Question number	Correct answer	Percentage correct	Question number	Correct answer	Percentage correct
1	E	47	1	D	70
2	E	58	2	C	47
3	B	56	3	A	33
4	B	57	4	E	29
5	C	20	5	D	28
6	A	83	6	A	38
7	E	56	7	C	41
8	A	26	8	D	35
9	D	48	9	B	51
10	D	38	10	E	33
11	C	48	11	B	36

Each question in Section A was answered correctly by at least 40 per cent of students. Two questions from Module 1 (Mechanics) in Section B did not meet this criterion. Only 26 per cent of students answered Question 8 correctly (A), with a majority of students (58 per cent) selecting alternative C. It is probable that many students did not read this question carefully enough and thought all they had to do was identify the equation of the curve shown. Question 5 had the lowest success rate (19.7 per cent), with alternative A being the most popular choice (44.7 per cent) and alternatives D (13.5 per cent) and E (17.5 per cent) both having substantial support. The students who scored highly overall tended to favour E over C. The overall poor performance of students on this question indicates that more attention needs to be paid to the concept of vector equality. For Module 2 (Geometry), about half of the questions did not meet the '40 per cent' criterion.

Analysis of scores by gender on each question showed no significant difference between the performance of girls and boys in nearly all cases. The exceptions appear to be that the boys performed better on Question 21 (Section A) and Question 2 (Mechanics), whereas the girls performed better on Question 9 (Geometry).

Part II: Short-answer questions

Question 1 (Mean marks: 1.49; Available marks: 3)

Answer: a. $-\dfrac{\pi}{6}$ b. $-\dfrac{5\pi}{12}$

The most common error in this question was the failure to give the principal value of the argument in one or both parts. In part a., $\dfrac{5\pi}{6}$ was sometimes given as the answer; in part b., $\dfrac{19\pi}{12}$ was often converted to $\dfrac{7\pi}{12}$. Other common mistakes were multiplying arguments to find Arg(w^2z) and giving the complete polar form as the answer.

Question 2 (0.72; 2)

Answer: $-\dfrac{1}{2}e^{\frac{2}{x}}$

Having the wrong sign or a factor of 1 or 2 instead of $\dfrac{1}{2}$ were common errors due to careless working. The most common incorrect approach was to attempt the substitution $u = x^2$.

Question 3 (1.49; 3)

Answer: $\dfrac{t^3}{3}+\cos t-2t+2$

This question was more successfully answered than similar ones in previous years, with few students using constant acceleration formulas. Most students included integration constants but the vast majority of them used $v = 2$ instead of $v = -2$ to find the first antiderivative expression and got the answer $x=\dfrac{t^3}{3}+\cos t+2t+2$

Question 4 (1.28; 3)

Answer: $2\pi(e^2 - 1)$

Many students had difficulty in setting up the correct integral with a surprisingly large number of them including an extra term in an otherwise correct expression. For example, $V=\pi\int_0^1\left(1-2e^y\right)^2 dy$. A common careless error was to simplify $2\pi(e^2 - e^0)$ to $2\pi e^2 - 1$.

Question 5 (1.19; 2)

Answer: Pulling force (10 N) is less than the maximum possible magnitude of the frictional force (12 N).

Most students calculated μN (correctly) but their explanations of why the box does not move were inadequate because they thought that μN gave the value of the frictional force instead of its *limiting* (maximum) value. The most common answer was that the box does not move because the friction force is greater than the pulling force (reluctantly accepted). Some students claimed it does not move because its acceleration is negative (not accepted). A better understanding of friction is expected in the future.

Question 6 (1.43; 4)

Answer: a. $\ddot{x}=-2(x+3)$ b. 6 m

Most students knew they had to find $\dfrac{d}{dx}\left(\dfrac{1}{2}\left(-2x^2-12x+54\right)\right)$ although some omitted the $\dfrac{1}{2}$. Many students did not go beyond $\ddot{x} = -2x - 6$ or factorised the left side incorrectly as $-2(x - 3)$. The simplest way to find the amplitude is to solve $\dot{x}=0$ but a variety of other methods were used (with mixed success).

Question 7 (0.97; 3)

Answer: a. $\overrightarrow{AB}=\underset{\sim}{b}-\underset{\sim}{a}$, $\overrightarrow{OC}=\underset{\sim}{a}+\underset{\sim}{b}$

b. Use $(\underset{\sim}{b}-\underset{\sim}{a}).(\underset{\sim}{a}+\underset{\sim}{b})=0$ to show that $b = a$. Hence $OACB$ is a rhombus because it is a parallelogram with four equal sides.

Part b. was done very poorly, with most students apparently not knowing what was required. A common 'answer' was to write $(\underset{\sim}{b}-\underset{\sim}{a}).(\underset{\sim}{a}+\underset{\sim}{b})=b^2-a^2=1-1=0.$

Question 8 (0.38; 3)

Answer: a. Line with equation Im z = Re z – 4

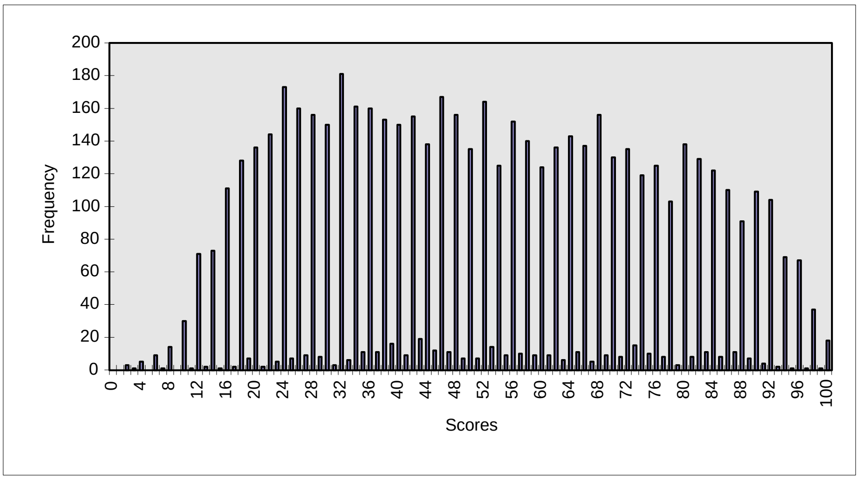
b. S is the perpendicular bisector of OP

Most students were unable to sketch S. Many of those who could sketch S recognised that it is perpendicular to OP, but hardly anyone noted that S bisected OP.

CAT MA092 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION

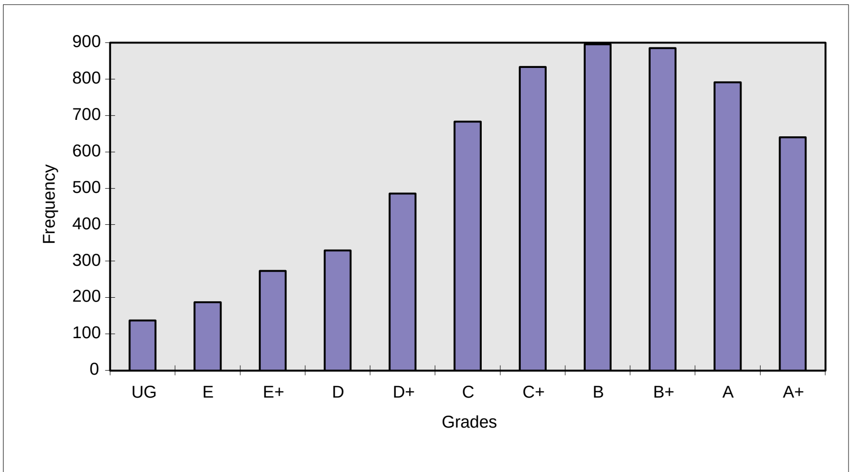
HISTOGRAM OF TOTAL SCORES 1998

Count **6140** Mean **52.15** Standard Deviation **23.56** NA Result **188**



HISTOGRAM OF TOTAL GRADES 1998

Count **6140** Mean **6.34** Standard Deviation **2.57** NA Result **188**



ENROLMENTS		**%**
Female	**2323**	**36.8**
Male	**3997**	**63.2**
Total	**6320**	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the ‘average’ score; that is all scores totalled then divided by the ‘Count’.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Specialist Mathematics CAT 3: Written examination (Analysis task)

GENERAL COMMENTS

In 1998, 6130 students sat for the Specialist Mathematics examination; 158 less than the number (6288) who sat in 1997. The vast majority of students (97.8 per cent) attempted Question 4 (Mechanics) as their optional question, with 1.9 per cent of students attempting Question 5 (Geometry) and 0.3 per cent not attempting either question. About 2.2 per cent of students scored 90 per cent or more of the marks, with 14 students scoring full marks. It appeared as though the challenges posed by Questions 1bvi., 1bvii. and 3c., rather than a lack of time, prevented more students from obtaining high marks. On the other hand, Questions 1biv., 4bi. and 4bii. provided 3 readily accessible marks for most students.

It was noted that 11 per cent of students scored less than 10 per cent of the available marks. Students need to consider carefully the selection of appropriate courses of study in mathematics. Analysis of scores by gender on each question showed no obvious significant difference between the performance of girls and boys, with the possible exception that the boys appear to have performed better on the geometry question (Question 5). The correlation between scores on CAT 3 and Part I (multiple-choice questions) of CAT 2 was 0.81.

While there was some excellent, clearly presented work, many students could improve their working in this regard. Particularly noticeable this year was the frequent omission of *dx* (and even the $\int$ sign itself) from integrals. Students who persist in writing such things as

$$\frac{d}{du}\left(u\,\mathrm{Tan}^{-1}u\right)=\mathrm{Tan}^{-1}u + \frac{u}{1+u^2} \Rightarrow u\,\mathrm{Tan}^{-1}u = \int \mathrm{Tan}^{-1}u + \frac{u}{1+u^2}$$

should not necessarily expect sympathetic treatment from the assessors.

This year there were a number of questions that required students to establish results given on the paper. The answers to such ‘show that’ questions are often poorly presented and it needs to be stressed to students they must show, clearly and logically, every step that leads to the result. The setting out of such questions needs to be explicitly taught and practised during the year.

Even allowing for the difficulty of parts vi. and vii., the overall standard of answers to Question 1b. (complex numbers) was not as high as it may have been. More attention needs to be given to fundamental complex number concepts such as representation on an Argand diagram and the principal value of arguments, and to routine tasks such as finding *n* th roots.

The use of graphics calculators was clearly evident in the work of some students, especially when answering Questions 1aii. (for graphing), 3b. (for numerical integration) and 3c. (for numerical differentiation). In many of these cases no marks were awarded because the appropriate working was not shown (see individual question comments). However, many other students would have found graphics calculators helpful in these questions to confirm results they had obtained in the appropriate way.

SPECIFIC INFORMATION

Question 1ai (Mean marks: 1.56; Available marks: 3)

$2(\sin t + \cos t)$; $t = \frac{3\pi}{4}$

A common error was to give the answer $t = \mathrm{Tan}^{-1}(-1) = -\frac{\pi}{4}$, ignoring the domain of t.

Question 1aii (1.87; 4)

$x^2 + \frac{y^2}{4} = 1$; ellipse, centre O, through (0, 2) and (1, 0)

Many students gave the answer $y = 2\cos(\mathrm{Sin}^{-1}x)$, which only gives the ‘top half’ of the path. Many students obtained the correct cartesian equation but apparently did not recognise it as the equation of an ellipse and relied on their graphics calculator. This often resulted in students sketching only the top half of the ellipse and/or omitting the near vertical sections.

Question 1bi (1.46; 3)

$8\,\mathrm{cis}\dfrac{3\pi}{4}$

$8\,\mathrm{cis}(-\dfrac{\pi}{4})$ was a very common incorrect answer. When doing polar conversions, students should be encouraged to check the quadrant of the argument with a quick Argand diagram plot.

Question 1bii (0.49; 1)

The easiest approach is to show that $u^3 = 8\,\mathrm{cis}\dfrac{3\pi}{4}$. Many of the students who had $8\,\mathrm{cis}(-\dfrac{\pi}{4})$ as their part i. answer wrote $u^3 = 8\,\mathrm{cis}\dfrac{3\pi}{4} = 8\,\mathrm{cis}(-\dfrac{\pi}{4})$, apparently believing that these two arguments are equal instead of being alerted to the incorrectness of their earlier answer.

Question 1biii (0.40; 2)

$2\,\mathrm{cis}\dfrac{11\pi}{12},\ 2\,\mathrm{cis}\dfrac{19\pi}{12}$

Most students thought that the conjugate of *u*, $2\,\mathrm{cis}(-\dfrac{\pi}{4})$, had to be a root. The fact that the conjugate root theorem applies only to polynomials with *real* coefficients needs to be stressed by teachers.

Question 1biv (0.79; 1)

$\sqrt{2} + \sqrt{2}\,i$

Very well done.

Question 1bv (0.64; 2)

Very poorly done with many students not even being able to plot *u* correctly and most students not recognising that the roots needed to be evenly spaced around a circle.

Question 1bvi (0.29; 3)

$w = \sqrt{2}\,i$

Most students did not know what to do in this question and many omitted it altogether. Some students found $w = \sqrt{2}\,i$ by equating **a** pair of coefficients, or stumbled upon it some other way, but few verified that $(z - \sqrt{2}\,i)^3 = -4\sqrt{2} + 4\sqrt{2}\,i$.

Question 1bvii (0.07; 2)

$\sqrt{2} + 2\sqrt{2}\,i$

Many of the students who completed part vi. successfully could not complete this part because they did not notice the link to part ii.

Question 2a (1.16; 2)

A common error was to obtain $k = 2500$, but this did not deter many students from 'proving' the given result.

Question 2b (1.50; 5)

$t = 2.5\log_e \dfrac{99N}{100 - N}$

Most students did not recognise the need to use partial fractions and either took $\dfrac{1}{N}$ out of the integrand as a 'constant' factor or applied the 'universal log rule' that the antiderivative of any quotient is equal to the log of the denominator. The most common error amongst those who proceeded correctly was to write: $\displaystyle\int \frac{1}{1000 - N}\,dN = \log_e(1000 - N)$.

Question 2c (0.44; 2)

$t = 15$

Question 3ai (1.42; 2)

$\mathrm{Tan}^{-1}u + \dfrac{u}{1+u^2}$

This should have been 2 readily accessible marks yet just over one quarter of students either omitted it or otherwise scored 0.

Question 3aii (0.96; 2)

The better students clearly indicated $\displaystyle\int \frac{u}{1+u^2}\,du = \frac{1}{2}\int \frac{2u}{1+u^2}\,du = \frac{1}{2}\log_e(1+u^2)$. A minority of students chose to differentiate the given result to show that this gave $\mathrm{Tan}^{-1}\,u$.

Question 3b (1.78; 5)

773 m^3

The majority of students realised that the solution involved finding the area of *OABC* by integration and wrote down the correct integral, namely $\displaystyle\int_0^4 (15 - 2x\,\mathrm{Tan}^{-1}(x^2))\,dx$, although some candidates felt it necessary to add to this the area of the rectangular base of *OABC*. Most students, however, had trouble with the integration. The major mistake was to not realise that it is necessary to make the substitution $u = x^2$ before the rule given in part aii. can be used to antidifferentiate $2x\,\mathrm{Tan}^{-1}(x^2)$ – it was common to see

$$\int_0^4 2x\,\mathrm{Tan}^{-1}(x^2)\,dx = [2x(x^2\,\mathrm{Tan}^{-1}(x^2) - \frac{1}{2}\log_e(1+x^4))]_0^4.$$

A common mistake amongst those students who did substitute correctly was either to not change the terminals to 0 and 16 or to also evaluate $15x$ at these values instead of at 0 and 4.

A small number of students apparently used their graphics calculator to evaluate the integral since the numerical value of the area (or the final answer for the volume) appeared without any further working. No marks were awarded for this evaluation as the question directs students to *use calculus* to find the volume and, in addition, the *Directions to students* at the start of the paper include: 'Calculus must be used to evaluate derivatives and definite integrals. A decimal value, no matter how accurate, will not be rewarded unless the appropriate working is shown'.

Question 3c (0.45; 4)

–9.0 m/s

Many students did not attempt this question. A common error in finding $\dfrac{dy}{dx}$ was to use $\dfrac{d}{dx}\mathrm{Tan}^{-1}(x^2) = \dfrac{1}{1+x^4}$. Some students produced the correct value for $\dfrac{dy}{dx}$ at $x = 2$ without any working, having presumably used their graphics calculator, but did not get the corresponding marks in line with the direction to students quoted above. Some students thought that the value of $\dfrac{dy}{dx}$ at $x = 2$ was the required answer; many of the students who knew that they had to find $\dfrac{dy}{dt}$ thought that they could do so by applying Pythagoras' theorem in the form $(\dfrac{dy}{dx})^2 = (\dfrac{dx}{dt})^2 + (\dfrac{dy}{dt})^2$. Most students who got the correct answer did so using a gradient argument ($\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$) rather than by direct use of the chain rule from a 'related rates' perspective.

Question 4ai (0.64; 1)

8 s

Lack of knowledge of terminology seemed to cause most of the difficulties in this question, with the value of *n* or the frequency being given as the answer instead of the period.

Question 4aii (1.10; 2)

$\frac{\pi}{8}$ m/s

Most students knew that the answer was given by *nA* and had *A* correct (0.5), but many students either confused *n* with the period or calculated it incorrectly.

Question 4aiii (1.34; 3)

0.82 s

Most students wrote down a rule of the form $x = 0.5 \sin (nt)$ and those who got as far as solving $0.5 \sin (nt) = 0.4$ generally completed the question successfully.

Question 4bi (0.87; 1)

Very well done.

Question 4bii (0.86; 1)

12 m/s

Very well done.

Question 4ci (0.69; 1)

Well done.

Question 4cii (1.02; 4)

4.1 s

Many students tried to use constant acceleration formulas; others could not invert $\frac{dv}{dt} = \frac{1}{10}(40 - v)$ correctly. Of the minority who inverted correctly and integrated, many used $v = 0$ as the initial velocity instead of $v = 12$.

Question 4di (1.24; 3)

$$(5t - 0.1t^2)\underset{\sim}{i} + (6t - 0.1t^2)\underset{\sim}{j} + (20 - 0.2t^2)\underset{\sim}{k}$$

Most students integrated the acceleration expression to find the correct expression for the velocity, but many students did not take into account the initial altitude of 20m when integrating the velocity expression and hence obtained $(5t - 0.1t^2)\underset{\sim}{i} + (6t - 0.1t^2)\underset{\sim}{j} - 0.2t^2\underset{\sim}{k}$ as their answer. A significant number of students got the same answer by using constant acceleration formulas.

Question 4dii (0.40; 2)

64 m

Most students were severely hampered in their attempts at this question by not having the correct $\underset{\sim}{k}$ component in their answer to part i.

Question 5a (0.69; 1)

$\frac{1}{\sqrt{3}}$

Well done.

Question 5b (0.68; 1)

Well done.

Question 5c (0.43; 1)

$-\sqrt{\frac{2}{3}} \cot\theta$

Reasonably well done.

Question 5d (0.53; 3)

Poorly done.

Question 5e (0.56; 4)

$\angle FQP = \frac{\pi}{3}$

Very poorly done.

Question 5f (0.66; 4)

centre $\left(\frac{1}{2},\ 0\right)$

The better students got as far as finding the equation of the ellipse but few were able to express it in a form which enabled each semi-axis length to be read off correctly.

Question 5g (0.70; 4)

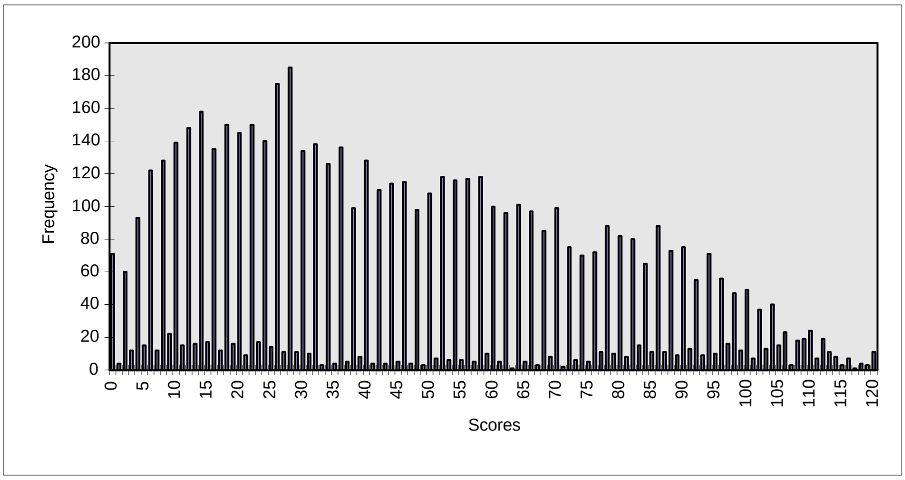
RQ has equation $y = -\frac{1}{\sqrt{3}}x + \sqrt{3}$

The better students obtained the correct equation from the complex relation, but few went on to check that it was the equation of the tangent at *P*.

CAT MA093 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION

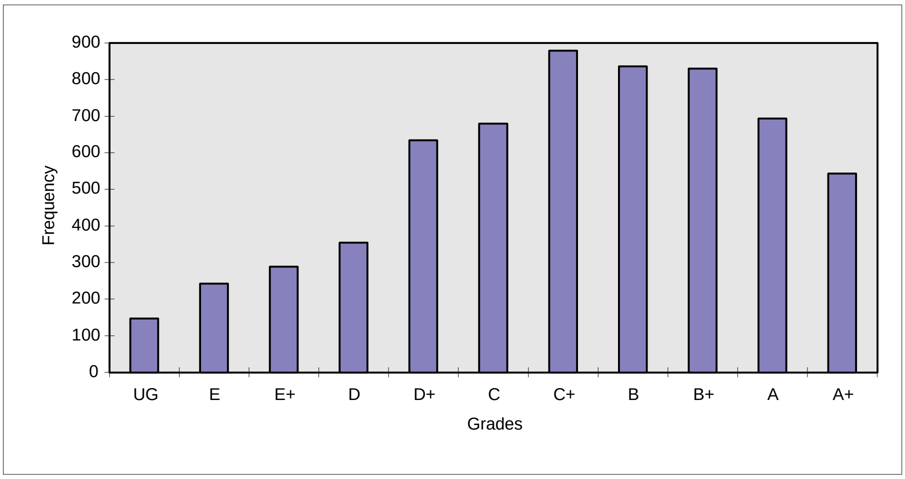
HISTOGRAM OF TOTAL SCORES 1998

Count **6128** Mean **46.70** Standard Deviation **29.95** NA Result **200**



HISTOGRAM OF TOTAL GRADES 1998

Count **6128** Mean **6.08** Standard Deviation **2.60** NA Result **200**



ENROLMENTS		%
Female	2323	36.8
Male	3997	63.2
Total	6320	

GLOSSARY OF TERMS

Count Number of students undertaking the CAT. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).

Notes

Notes

‘Report for Teachers’ series of booklets contain reports from Chief Assessors and State Reviewers for the common assessment tasks undertaken in 1998. Each report contains an overview of student performance on individual CATs. Chief Assessors and State Reviewers have commented on such matters as the assessment criteria and student performance on the CATs.

Users of these reports should be aware that these reports are for 1998 CATs. Changes may have been made to study designs, CATs and assessment criteria since the completion of the reports.

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