Trial CAT 3 Solutions

Question 1

a. (10, 10) [1A]

b. (40, 25) [1A]

c. when $x = 0$, $y = 0$

$\therefore c = 0$ [1A]

when $x = 10$, $y = 10$

$\therefore 10 = 100a + 10b$ ①

when $x = 40$, $y = 25$

$\therefore 25 = 1600a + 40b$ ② [1M]

$25 = 1600a + 40b$

$-\ 40 = 400a + 40b$ ① × 4

$-15 = 1200a$

$\therefore a = \frac{-15}{1200}$

$\therefore a = \frac{-1}{80}$ [1A]

Substitute a into ①:

$10 = 100a + 10b$

$10 = 100(\frac{-1}{80}) + 10b$

$10 = \frac{-10}{8} + 10b$

$\frac{90}{8} = 10b$

$\frac{9}{8} = b$

$\therefore y = \frac{-1}{80}x^2 + \frac{9}{8}x$ [1A]

d. when $x = 0$, $y = 11$

when $x = 10$, $y = 10$

Gradient $= \frac{10-11}{10-0} = \frac{-1}{10}$ [1A]

$\therefore y - 11 = \frac{-1}{10}(x - 0)$

$\therefore y = \frac{-1}{10}x + 11$ [1A]

e. Substitute $x = 88$ into both the line and curve

Line: $y = \frac{-1}{10}(88) + 11$

$y = \frac{-88}{10} + 11$

$y = \frac{11}{5}$ [1M]

Curve: $y = \frac{-1}{80}x^2 + \frac{9}{8}x$

$y = \frac{-1}{80}(88)^2 + \frac{9}{8}(88)$

$y = \frac{-7744}{80} + 99$

$y = \frac{11}{5}$ [1M]

Or equate the line and curve....

f. $\int_{10}^{88} -\frac{1}{80}x^2 + \frac{9}{8}x - [-\frac{1}{10}x + 11]\ dx$ [1M]

$= \int_{10}^{88} -\frac{1}{80}x^2 + \frac{9}{8}x + \frac{1}{10}x - 11\ dx$

$= \int_{10}^{88} -\frac{1}{80}x^2 + \frac{98}{80}x - 11\ dx$

$= \left[-\frac{1}{240}x^3 + \frac{98}{160}x^2 - 11x\right]_{10}^{88}$ [1M]

$= [-2839.466 + 4743.2 - 968] - [-4.166 + 61.25 - 110]$

$= [935.73] + 52.92$ [1M]

$= 988.65$

$x = 988.7$ square metres [1A]

g. Volume = Height × Area

$= 988.7 \times 1.2$

$= 1186.44\ m^3$

$= 1186\ m^3$ [1A]

Total 15 marks

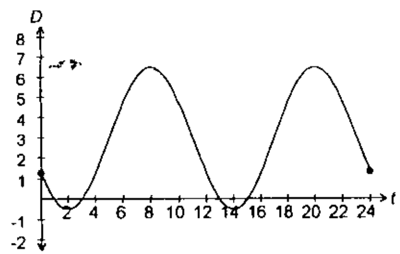
Question 2

a. i. Period $= \frac{2\pi}{\frac{\pi}{6}} = 12$ hours [1A]

ii. Amplitude $= 3\frac{1}{2}$

Height $= 3 + 3\frac{1}{2} = 6\frac{1}{2}$ metres [1A]

b.



1 mark for the scale

1 mark for the shape

1 mark for correct position of graph. [3A]

c. $\cos\frac{\pi}{6}(t - 8) = 1$

$\therefore \cos(0) = 1$ or $\cos(2\pi) = 1$

$\therefore t - 8 = 0$ or $t - 8 = 12$

$\therefore t = 8$ or $t = 20$ [1M]

$\therefore$ times are 8 am and 8 pm [1A]

Alternatively, use the graph ...

d. Half the Period $= \frac{12}{2} = 6$ hours [1A]

e. High tide is at 8 am and there is a six hour difference between high and low tides

$\therefore 8 - 6 = 2$ 2 am is the first time. [1A]

f. Depth = 0.

$3 + 3\frac{1}{2}\cos\frac{\pi}{6}(t - 8) = 0$ [1A]

$\cos\frac{\pi}{6}(t - 8) = \frac{-3}{3.5}$

$\frac{\pi}{6}(t - 8) = \text{Cos}^{-1}(\frac{-3}{3.5})$

$\frac{\pi}{6}(t - 8) = 2.6005$ [1M]

$t - 8 = 4.966$

$t = 12.966$

$t = 12$ hours and 58 mins [1A]

By symmetry the other value is 15 hours and 2 mins

Time is between 12:58 pm and 3:02 pm [1A]

g. Between 9 am and 12 noon [1A]

Reasons: (Need three of the following)

- Low Tide at 2 p.m.
- No water in the bay during lunchtime
- The afternoon does not have enough time or water to operate the shop
- Need time to pack up the boats and get them to the beach before the tide goes out.
- Cannot sail without water, etc. [3A]

Total 17 marks

Question 3

a. When $t = 0$, $P = 100e^0 = 100$ [1A]

b. $90 = 100e^{-0.000121t}$ [1M]

$0.9 = e^{-0.000121t}$

$\ln(0.9) = -0.000121t$

$t = 870.748 \approx 871$ years [1A]

c. $\frac{dP}{dt} = -0.0121e^{-0.000121t}$ [1M]

when $t = 5730$,

$\frac{dP}{dt} = -0.0121e^{-0.000121 \times 5730} = -0.0061$ [1A]

d. $t = 1000$, $D = 111.57 \Rightarrow 111.57 = D_0e^{-1000k}$ ①

$t = 2000$, $D = 24.90 \Rightarrow 24.90 = D_0e^{-2000k}$ ② [1M]

① ÷ ② $\Rightarrow 4.48 = \frac{e^{-1000k}}{e^{-2000k}}$ [1M]

$= e^{k(-1000+2000)}$

$= e^{1000k}$

$1000k = \ln(4.48)$

$k \approx 0.0015$ [1A]

Substitute k into ①:

$111.57 = D_0e^{-1000 \times 0.0015}$ [1M]

$D_0 \approx 500$ [1A]

e. $D = 500e^{-0.0015t}$

$\ln D = \ln 500e^{-0.0015t}$ [1M]

$\ln D = \ln 500 - 0.0015t$

$0.0015t = -\ln D + \ln 500$ [1M]

$t = \frac{-1}{0.0015}\ln D + \frac{\ln 500}{0.0015}$

gradient $= \frac{-1}{0.0015}$

y-intercept $= \frac{\ln 500}{0.0015}$ [1A]

Total 13 marks

Question 4

a. i. $\mu = 61$ $\sigma = 8$

Let E be the weight of an egg

$\Pr(E > 67) = \Pr(Z > \frac{67-61}{8})$ [1M]

$= \Pr(Z > 0.75)$

$= 1 - \Pr(Z \le 0.75)$

$= 1 - 0.7734$

$= 0.2266$ [1A]

ii. $\Pr(E < 59) = \Pr(Z > \frac{59-61}{8})$ [1M]

$= \Pr(Z > -0.25)$

$= 1 - \Pr(Z \le 0.25)$

$= 1 - 0.5987$

$= 0.4013$ [1A]

iii. $\Pr(E > 67 / E > 61) = \frac{\Pr(x > 67)}{\Pr(x > 61)}$ [1M]

$= \frac{0.2260}{\Pr(z > 0)}$

$= \frac{0.2260}{0.5}$

$= 0.4532$ [1A]

b. n = 6

$\Pr(E > 67) = \Pr(Z > \frac{67-61}{8})$ [1M]

$= \Pr(Z > 0.75)$

$= 1 - \Pr(Z \le 0.75)$

$= 1 - 0.7734$

$= 0.2266$ [1A]

Let x be the number of eggs which weigh more than 67 grams.

$\Pr(X \ge 2) = 1 - [\Pr(X = 0) + \Pr(X = 1)]$ [1M]

$= 1 - [{}^6C_0(0.3085)^0(0.6915)^6 + {}^6C_1(0.3085)^1(0.6915)^5]$

$= 1 - [0.1093 + 0.2927]$

$= 0.5980$ [1C]

c. $\sigma = \sqrt{4} = 2$

$\Pr(X > 67) = 0.98$

$\Pr(Z > \frac{67-\mu}{2}) = 0.98$

$\Pr(Z > c) = 0.98$

$c = -2.054$ [1M]

$\frac{67-\mu}{2} = -2.054$

$-\mu = -4.108 - 67$

$\mu = 71.108$

$\mu = 71.1$ grams [1A]

d. $\hat{p} = \frac{7}{12} = 0.5833$

$se(\hat{p}) = \sqrt{\frac{\frac{7}{12}(\frac{5}{12})}{12}} = 0.1423$ **[1M]**

95% Confidence Interval = (0.2987, 0.8679)
= (29.87%, 86.79%) **[1A]**

e. The proportion of Victor's Eggs which weigh more than 67 grams is between 29.87% and 86.79%. International statistics suggest only 25% of eggs weigh more than 67 grams. So yes. Victor's chickens *do* lay larger eggs. Bad luck, Albert. **[1M]**

Total 15 marks

Total 60 Trial CAT 3 marks