MAY Mathematical Methods Trial CAT 2 1998

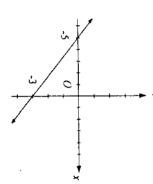
Part I (Multiple-choice Questions)

Question I
The equation of the line shown here is

3.
$$-5x-3y=15$$

3.
$$5y + 3x = -15$$





Question 2

The domain of the relation whose graph is shown

$$[-3,-1)\cup(-1,3]$$

- (-3, 3)
- [-3, 1] + [0, 3]
- [-3, 1]
- [-3, 3]

Question 3

A possible equation for the graph shown would be

A.
$$f(x) = (x-a)(x-b)(x-c)$$

B.
$$f(x) = -(x-a)(x-b)(x+c)^2$$

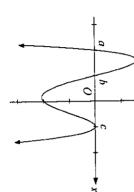
C.
$$f(x) = (x - a)(x - b)(x + c)^2$$

D.
$$f(x) = (x-a)(x-b)(x-c)^2$$

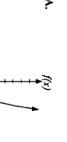
$$0. \quad f(x) = (x-a)(x-a)(x-c)$$

E.
$$f(x) = -(x-a)(x-b)(x-c)^2$$



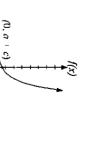


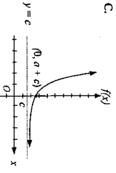
Question 4 Which one of the following graphs would fit the equation $f(x) = ae^{(c+b)} + c$ if a > 0, b = 0 and c > 0?



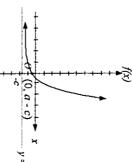


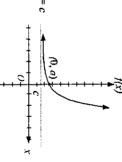
(1,c)











Question 5
The parabola with the equation $y = x^2$ is translated so that its image has a vertex (a, b) where a < 0 and b > 0. The equation of the image is

A.
$$y = (x - a)^2 + b$$

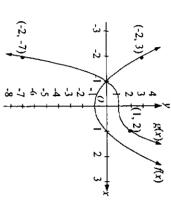
B.
$$y' = (x+a)^2 - b$$

C.
$$y = (x+a)^2 + b$$

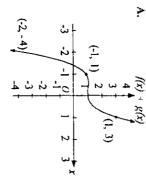
D.
$$y = (x - a)^2 - b$$

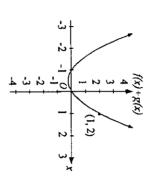
$$\mathbf{E}. \quad y = -ax^2 + b$$

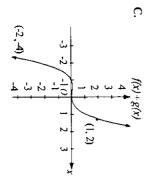
separate graphs f(x) and g(x)The axes shown on the right display two

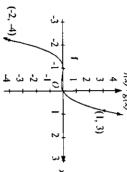


Which of the following graphs would best display the combination of the graphs, f(x) + g(x)?

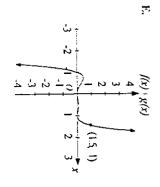


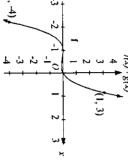






Ď.







The solutions between 0 and π for which $\cos(3x) + \sqrt{3}\sin(3x) = 0$ are Question 9

A.
$$\frac{5\pi}{18}, \frac{11\pi}{12}$$

B.
$$\frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$$

$$\frac{5\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}$$

$$\mathbf{D.} \quad \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$$

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Question 7

In comparison with the graph of $y = \sin(x)$, the graph of $y = 2\sin(2x + \frac{\pi}{2})$ has

- double the amplitude, half the period and is shifted $\frac{\pi}{2}$ units to the left
- half the amplitude, half the period and is shifted $\frac{\pi}{2}$ units to the right
- double the amplitude, half the period and is shifted $\frac{\pi}{4}$ units to the right
- double the amplitude, half the period and is shifted $\frac{\pi}{4}$ units to the left
- double the amplitude, half the period and is shifted $\frac{\pi}{2}$ units up

Question 8
A possible equation of the graph shown is

A.
$$y = 2a\sin(x)$$

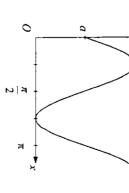
B.
$$y = a\sin(2x+1)$$

$$\mathbf{p} \cdot \mathbf{y} = a \sin(2x + t)$$

C.
$$y = a + \sin(x - \pi)$$

D.
$$y = -a + a\sin(2x)$$

$$E. \quad y = a + a\sin(2x)$$



$y = \frac{-x}{e} + \frac{1}{c} + e$

Question 11 The equation of the normal of $f(x) = e^x$ at the point (1, e) is

- $y = -e^x + 2e$
- $y = e^x$
- $y = e^{\epsilon}x e^{\epsilon} + e$
- $y = \frac{-x}{e} + 2$

Question 12 If $f(x) = 4e^{3x}\sin(2x)$ then f'(x) equals

A.
$$4e^{x}(3\sin(2x) + 2\cos(2x))$$

B.
$$\frac{2}{3}e^{3x}(2\sin(2x)+3\cos(2x))$$

3
C.
$$2e^{3x}(6\sin(2x) + \cos(2x))$$

D.
$$\frac{3}{2}e^{3x}(2\sin(2x) + 3\cos(2x))$$

E.
$$4e^{3x}(3\sin(2x) + 2\cos(2x))$$

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Question 13

The area of the shaded region of the graph is given by

A.
$$\int_{1}^{x} -f(x)dx$$

 $\int f(x)dx$

$$\int_{a}^{b} x - f(x) dx$$

$$\mathbf{D.} \quad \int_{0}^{a} f(x) - x dx$$

$$\mathbf{E.} \quad \int_{a}^{0} -f(x)dx$$

Question 14

The derivative of $\frac{3x^2-x}{x}$ is equal to

A.
$$3 + \frac{2}{x}$$

2

- 3x 1

Question 15

An antidervative of $\frac{1}{(3x+2)^5}$ is

$$\frac{-3}{(3x+2)^4}$$

8.
$$\frac{-1}{12(3x+2)^4}$$

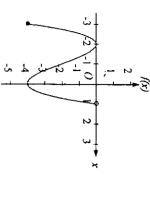
$$5 \log_{2}(3x+2)$$

$$D. \quad \frac{6}{(3x+2)^6}$$

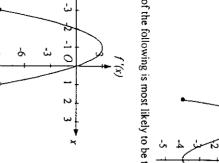
$$\frac{1}{6(3x+2)^4}$$

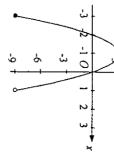
Question 16

The graph of the function $f: [-3,1) \to \mathbb{R}$ is shown below

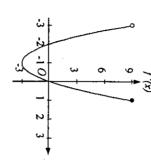


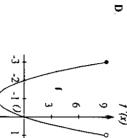
Which of the following is most likely to be the graph of the derived function f'?

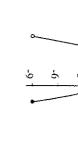


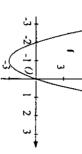


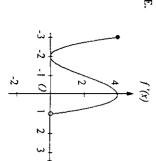
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Question 17

If $y = \frac{\log_e(2x)}{2x}$ then $\frac{dy}{dx}$ is

$$\frac{1-2\log_e(2x)}{4x^2}$$

$$\frac{1 - \log_e(2x)}{2x^2}$$

$$\frac{2[1-\log_{e}(2x)]}{x^{2}}$$

Question 18 The coefficient of x^{10} in the expansion of $(3-2x)^{12}$ is

A.
$$-{}^{12}C_{2}(2)^{10}(9)$$

B.
$${}^{12}C_2(-2)^{10}(9)$$

C.
$${}^{12}C_2(-2)^{12}(9)^{10}$$

D.
$${}^{12}C_{10}(-2)^{10}(9)^{2}$$

E.
$$-{}^{12}C_{10}(2)^{10}(3)^2$$

Question 19 Which of the following is a solution of the equation, $3e^{2x} = 6$?

21og, 2

$$\frac{1}{2}\log_{2} 2$$

E.
$$\frac{1}{2}\log_6 3$$

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Let $h: [a, 2] \to \mathbb{R}$ where $h(x) = 2x - 2x^2$. If a is the smallest real value such that h has an inverse function h^{-1} , then a equals

Question 21

If $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = e^{(2x-1)}$ then $f^{-1}(x)$ equals

- €-21 + 1
- -e^{2x-1}
- $\frac{1}{5} \log_{1}(x+1)$
- $\frac{1}{2}\log_{e}(x+\frac{1}{2})$
- $\frac{1}{5}\log_{e}(x) + \frac{1}{2}$

Question 22 If $2 + \log_{10}(3x) = \log_{10}(y)$, then

- y = 100 + 3x
- $y = \frac{3x}{100}$
- y = 600x
- y = 300x
- $y = \frac{100}{3x}$

Question 23
If $P(x) = x^3 + ax^2 - 6x + 8$ has a remainder of 24 when divided by (x + 2), then the value of a is

- .≂ 12
- 0
- P.

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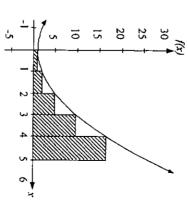
Question 24

The derivative of xe^{2x} is $(2x+1)e^{2x}$. Hence $\int xe^{2x} dx$ is equal to

- $\int (2x+1)e^{2x}dx$
- $\int (2x-1)e^{2x}dx$
- $2xe^{2x} + \int e^{2x} dx$
- $(2x+1)e^{x}+c$
- $\frac{e^{2x}}{2}(x-\frac{1}{2})+c$

rectangular strips as shown in the diagram. The area of this approximation is Question 25
The area under the graph $f(x) = x^2 + 1$ between x = 0 and x = 5 can be approximated by

- 30 square units
- 35 square units
- Ç 36 square units
- 40 square units
- $46\frac{2}{3}$ square units



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The following information refers to questions 26 and 27

choc-chips in each of the 50 choc-chip cookies she bought recently. Her results are as follows Fearful that she is not getting value for money, Marita decides to count the number of

	Cookie	Training of once on the	Number of choc-chips	
	_		0	,
	2	1	_	•
ļ	5	1	7	,
	0		u	د
	5		+	`
	[3		ن	'n
	ø	,	c	٨

Question 26

What proportion of cookies had less than 5 choc chips?

	>	•
50	1	œ

B.
$$\frac{13}{50}$$

D.
$$\frac{42}{50}$$

Question 27

If Marita chose a cookie at random, what is the expected number of choc-chips in a cookie?

Question 28

success p, the mean and standard deviation are 10 and $\sqrt{6}$ respectively. Which of the following gives the correct values for n, p and the variance of this distribution? For a particular binomial distribution with n independent trials, each with a probability of

A.
$$n = 16$$
, $p = 0.6$, variance = 10

B.
$$n = 16$$
, $p = 0.6$, variance = 6

C.
$$n = 25$$
, $p = 0.6$, variance = 36

D.
$$h = 25$$
, $p = 0.4$, variance = $\sqrt{6}$

E.
$$\mu = 25$$
, $p = 0.4$, variance = 6

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Question 29

Which of the following is not an example of a discrete random variable?

- The numbers of students sitting Mathematical Methods CATs over various years
- The numbers of beans in a particular brand of bean bag
- Ç The daily numbers of men seen wearing blue jeans in the Bourke Street Mall
- Ģ. The numbers of girls born at the Royal Children's Hospital
- The lengths of tapes in 30-minute audiocassettes

Question 30

than I goal? 0.3. In one game, he shoots for goal 10 times. What is the probability that he will score more Whenever Aris shoots for goal in a game of soccer, the probability that he will score a goal is

- · (0.3)¹⁰
- $1 [{}^{10}C_1(0.3)(0.7)^9]$
- Ç $1 - [(0.7)^{10} + 10(0.3)(0.7)^{9}]$
- $(0.7)^{10} + {}^{10}C_1(0.3)(0.7)^9$
- $^{10}C_1(0.3)(0.7)^9$

Question 31

Bradkins, a statistical surveying company, has been hired to a 95% confidence interval for the proportion of a population who like vegemite. What is minimum number of people that they need to interview to ensure that the margin of error of their result is no more that 5%?

- 144
- 256
- Ç 384

757

200

- 400

475

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The following information refers to questions 32 and 33

The maximum temperature, T° , of a November day in Melbourne has been found to be normally distributed with a mean of 24°C and a standard deviation of 2°C.

- What would the standard normal expression be for the probability of the maximum temperature being 27°C or more? Question 32
- Pr(Z>1)Pr (Z < 1)
- Ç $1 - \Pr(Z < \frac{3}{2})$
- 1 Pr $(Z > \frac{3}{2})$
- $\Pr\left(-\frac{1}{2} < Z < \frac{1}{2}\right)$

Question 33 The values of c_1 and c_2 such that $Pr(c_1 < T < c_2) = 0.95$ would be closest to

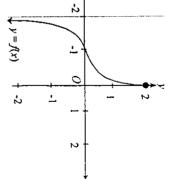
- $c_1 = 20$ and $c_2 = 26$
- $c_1 = 21$ and $c_2 = 28$
- \circ $c_1 = 18$ and $c_2 = 30$
- $c_1 = 20$ and $c_2 = 28$
- $c_1 = 21$ and $c_2 = 26$

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Part II (Short Answer Questions)

Question 1

The graph of a function, f_i is shown below.



On the same set of axes, sketch the graph of f^{-1} .

2 marks

What is the range of f^{-1} ?

mark

Question 2

Hassan's intellectual biorhythm is modelled by the function

$$P = 2 + \sin \frac{2\pi}{28}I$$

where t is the number of days after today and P is Hassan's biorhythm level (no units are necessary).

a. What will Hassan's biorhythm level be in 7 days' time?

l mark

b. How many days will it take for Hassan to reach a biorhythm level of 1? 2 marks Question 3 For the equation $f(x) = \sin(2x)$ $0 \le x \le \pi$, find the exact coordinates of the first point on the curve where $f'(x) = 1$ 3 marks Question 4 At a certain company the total cost, in dollars, of manufacturing x calculators is given by $f(x) = 0.3x^2 + 20x + 200$, where $0 \le x \le 100$. a. Calculate the cost of producing 50 calculators.	l mark
How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will it take for Hassan to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days will be seen to reach a biorhythm level of 1? How many days	
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, biorhythm level of 1 indicates that Hassan is at his lowest intellectual potential.	
	A biorhythm level of 1 indicates that Hassan is at his lowest intellectual potential.

3 marks

Question If a is the ratio $\frac{a}{b}$.			MAY Mad
n S e coefficient of x^4 and b is the coefficient of x^2 in the expansion of $(2x-1)^4$, fin	2 mari	What is the maximum total cost of manufacturing calculators?	MAV Mathematical Methods Trial CAT 2 1998

Question 6
Astrid is a darts player attempting to hit the dark bulls-eye section of the circular borad shown in the diagram. She is allowed three throws, each of which hits the board at a random point.



มllseye is hole dart board is	
\ 0	H

The area of the buthe area of the wh

136.5 cm² 910.0 cm²

What is the probability that she scores at least one bullseye? Give your answer correct to two decimal places.

l mark		

Ċ. What is the probability that she scores exactly one bullseye? Give your answer correct to two decimal places.

l mark		

Total 17 marks

; ,