The Mathematical Association of Victoria

1998

MATHEMATICAL METHODS

Trial Examination 1

(Practice papers based on MAV 1998 trial CATs)

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **30 marks** available for this part.

Part II consists of seven questions. Answers all questions in **Part II** in the spaces provided. There are **20 marks** available for this part.

There are **50 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

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Multiple-Choice Answer Sheet

Student's Name: _____

Cross through the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**
28.	**A**	**B**	**C**	**D**	**E**
29.	**A**	**B**	**C**	**D**	**E**
30.	**A**	**B**	**C**	**D**	**E**

Mathematical Methods

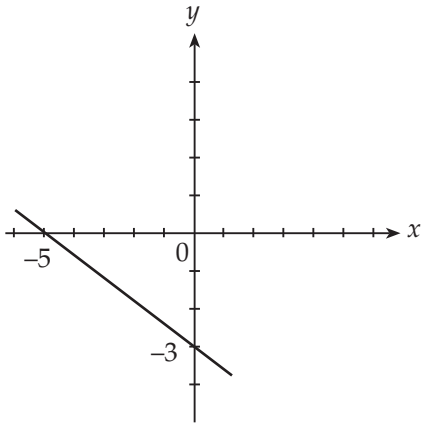
Practice papers based on MAV 1998 trial CATs

Examination 1 (Facts, skills and applications)

Part I (Multiple-choice questions)

Question 1

The equation of the line shown here is



A. $y = 3x - 5$

B. $-5x - 3y = 15$

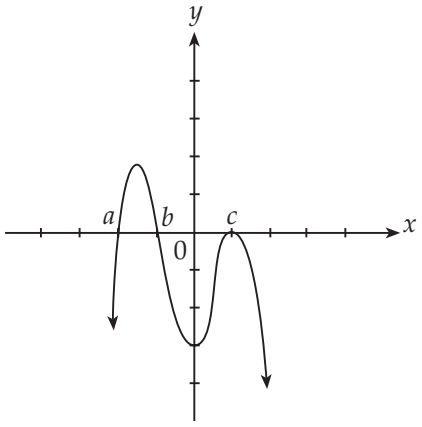
C. $y = \frac{3}{5}x - 3$

D. $5y + 3x = -15$

E. $y = -5x - 3$

Question 2

A possible equation for the graph shown would be



A. $f(x) = (x - a)(x - b)(x - c)$

B. $f(x) = -(x - a)(x - b)(x + c)^2$

C. $f(x) = (x - a)(x - b)(x + c)^2$

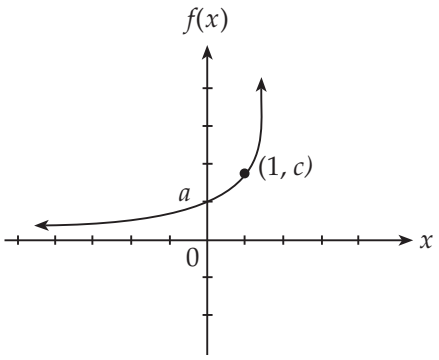
D. $f(x) = (x - a)(x - b)(x - c)^2$

E. $f(x) = -(x - a)(x - b)(x - c)^2$

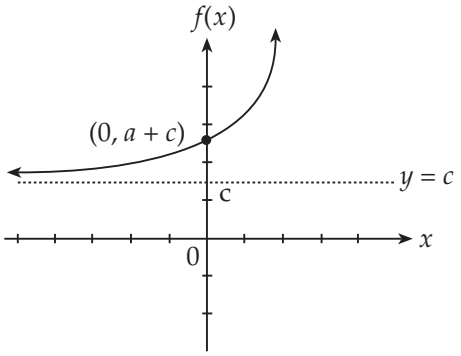
Question 3

Which one of the following graphs would fit the equation $f(x) = ae^{(x+b)} + c$ if $a > 0$, $b = 0$ and $c > 0$?

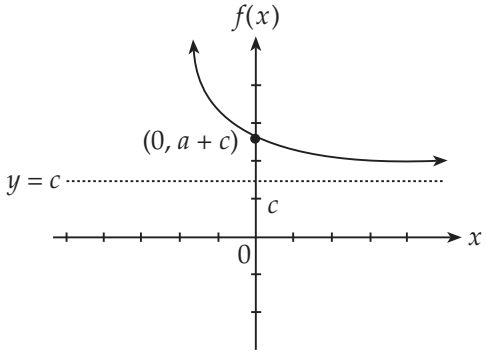
A.



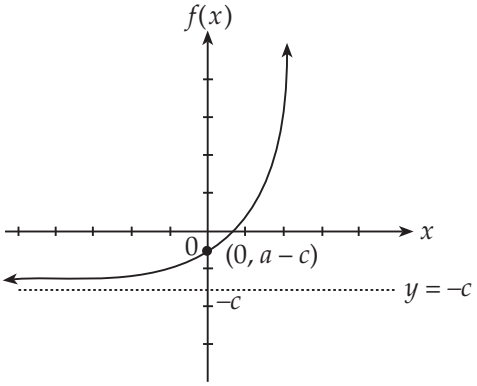
B.



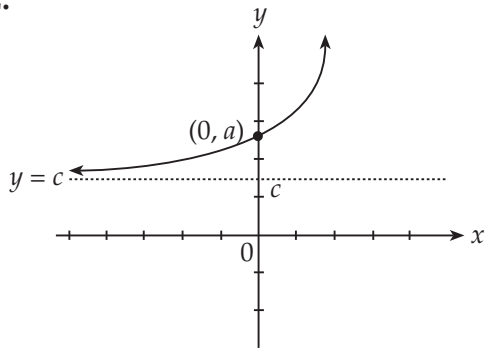
C.



D.



E.



Question 4

The parabola with the equation $y = x^2$ is translated so that its image has a vertex (a, b) where $a < 0$ and $b > 0$. The equation of the image is

A. $y = (x - a)^2 + b$

B. $y = (x + a)^2 - b$

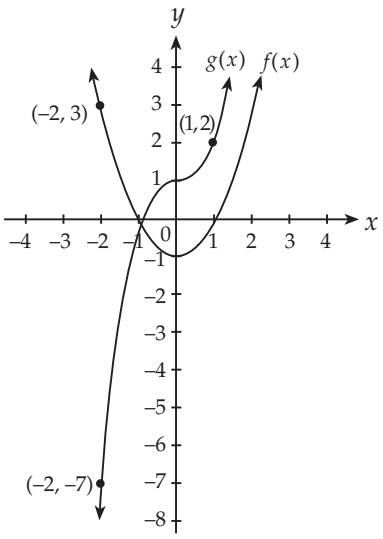
C. $y = (x + a)^2 + b$

D. $y = (x - a)^2 - b$

E. $y = -ax^2 + b$

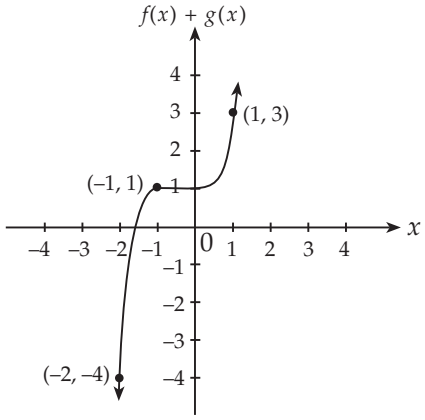
Question 5

The axes shown on the right display two separate graphs $f(x)$ and $g(x)$

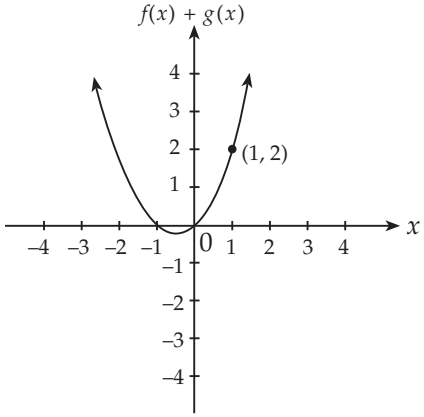


Which of the following graphs would best display the combination of the graphs, $f(x) + g(x)$?

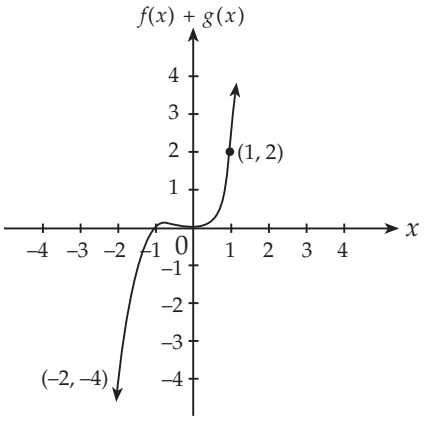
A.



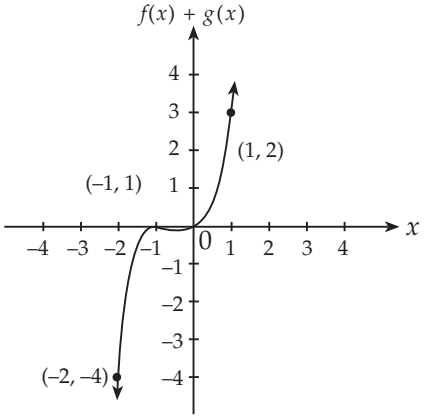
B.



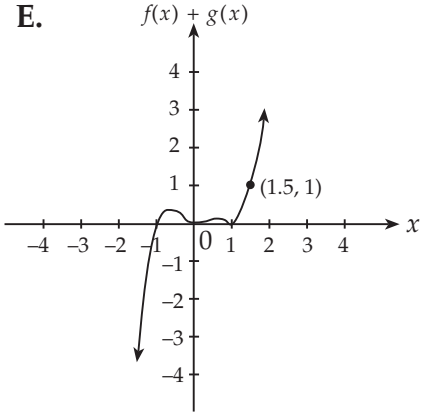
C.



D.



E.



Question 6

In comparison with the graph of $y = \sin(x)$, the graph $y = 2\sin\left(2x + \frac{\pi}{2}\right)$ has

A. double the amplitude, half the period and is shifted $\frac{\pi}{2}$ units to the left

B. half the amplitude, half the period and is shifted $\frac{\pi}{2}$ units to the right

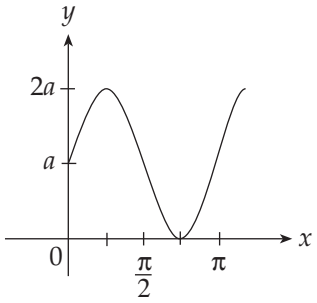
C. double the amplitude, half the period and is shifted $\frac{\pi}{4}$ units to the right

D. double the amplitude, half the period and is shifted $\frac{\pi}{4}$ units to the left

E. double the amplitude, half the period and is shifted $\frac{\pi}{2}$ units up

Question 7

A possible equation of the graph shown is



A. $y = 2a\sin(x)$

B. $y = a\sin(2x + 1)$

C. $y = a + \sin(x - \pi)$

D. $y = -a + a\sin(2x)$

E. $y = a + a\sin(2x)$

Question 8

The solutions between 0 and π for which $\cos(3x) + \sqrt{3}\sin(3x) = 0$ are

A. $\frac{5\pi}{18}, \frac{11\pi}{12}$

B. $\frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$

C. $\frac{5\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}$

D. $\frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$

E. 0, 3, 5, 7

Question 9

The equation $m\sin(4x) = 2m\cos(4x)$, where $m > 0$, has how many soluions in the interval $[0, \pi]$?

A. 0

B. 1

C. 4

D. 2

E. 3

Question 10

The equation of the normal to the curve $y = e^x$ at the point $(1, e)$ is

A. $y = \dfrac{-x}{e} + \dfrac{1}{e} + e$

B. $y = -e^x + 2e$

C. $y = ex$

D. $y = e^e x - e^e + e$

E. $y = \dfrac{-x}{e} + 2$

Question 11

If $f(x) = 4e^{3x}\sin(2x)$ then $f'(x)$ equals

A. $4e^x\left(3\sin(2x) + 2\cos(2x)\right)$

B. $\dfrac{2}{3}e^{3x}\left(2\sin(2x) + 3\cos(2x)\right)$

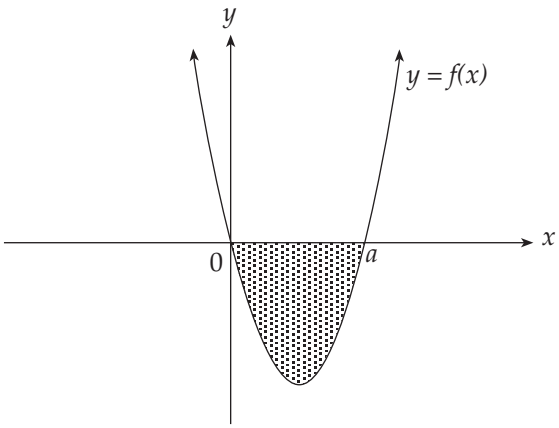
C. $2e^{3x}\left(6\sin(2x) + \cos(2x)\right)$

D. $\dfrac{3}{2}e^{3x}\left(2\sin(2x) + 3\cos(2x)\right)$

E. $4e^{3x}\left(3\sin(2x) + 2\cos(2x)\right)$

Question 12

The area of the shaded region of the graph is given by



A. $\int_0^a \big(-f(x)\big)dx$

B. $\int_0^a f(x)dx$

C. $\int_0^a \big(x - f(x)\big)dx$

D. $\int_0^a \big(f(x) - x\big)dx$

E. $\int_a^0 \big(-f(x)\big)dx$

Question 13

The derivative of $\dfrac{3x^2 - x}{x}$ is equal to

A. $3 + \dfrac{2}{x}$

B. 3

C. $3x - 1$

D. $\dfrac{3x^2 - 2x}{x^2}$

E. $6x$

Question 14

An antiderivative of $\dfrac{1}{(3x+2)^5}$ is

A. $\dfrac{-3}{(3x+2)^4}$

B. $\dfrac{-1}{12(3x+2)^4}$

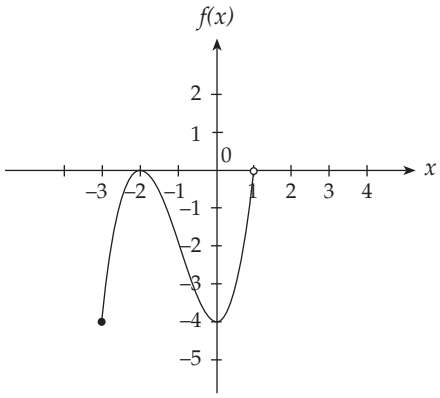
C. $5\log_e(3x+2)$

D. $\dfrac{6}{(3x+2)^6}$

E. $\dfrac{1}{6(3x+2)^4}$

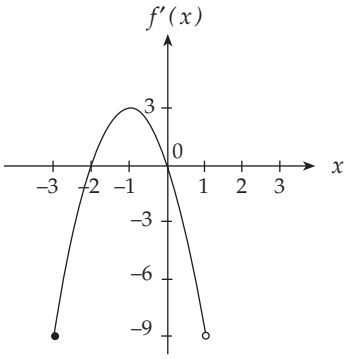
Question 15

The graph of the function $f : [-3, 1) \rightarrow R$ is shown alongside.

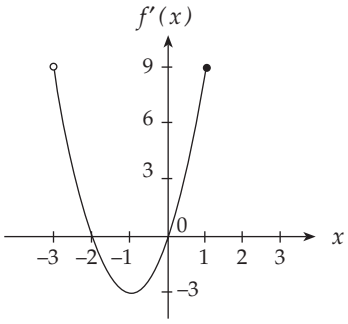


Which of the following is most likely to be the graph of the derived function f'?

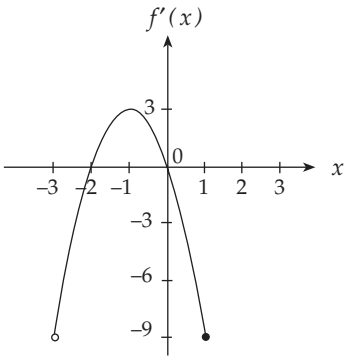
A.



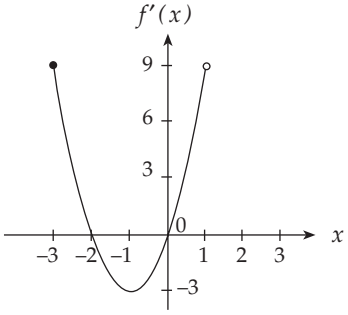
B



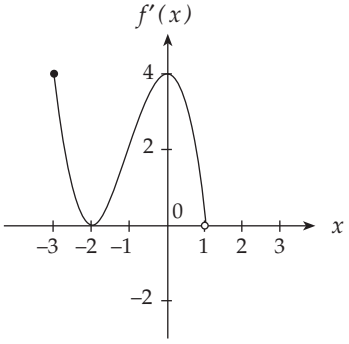
C.



D.



E.



Question 16

If $y = \dfrac{\log_e(2x)}{2x}$ then $\dfrac{dy}{dx}$ is

A. $\dfrac{x}{2}$

B. $\dfrac{-x^3}{2}$

C. $\dfrac{1-2\log_e(2x)}{4x^2}$

D. $\dfrac{1-\log_e(2x)}{2x^2}$

E. $\dfrac{2[1-\log_e(2x)]}{x^2}$

Question 17

The coefficient of x^{10} in the expansion of $(3 - 2x)^{12}$ is

A. $-{}^{12}C_2\,(2)^{10}\,(9)$

B. ${}^{12}C_2\,(-2)^{10}\,(9)$

C. ${}^{12}C_2\,(-2)^{12}\,(9)^{10}$

D. ${}^{12}C_{10}\,(-2)^{10}\,(9)^2$

E. $-{}^{12}C_{10}\,(2)^{10}\,(3)^2$

Question 18

Which of the following is a solution of the equation, $3e^{2x} = 6$?

A. $2\log_e 2$

B. $2\log_e 3$

C. 0

D. $\dfrac{1}{2}\log_e 2$

E. $\dfrac{1}{2}\log_e 3$

Question 19

Let $h:[a, 2] \to R$ where $h(x) = 2x - 2x^2$. If a is the smallest real value such that h has an inverse function h^{-1}, then a equals

A. –1

B. 0

C. 1

D. –2

E. $\frac{1}{2}$

Question 20

If $f: R \to R$, where $f(x) = e^{(2x-1)}$ then $f^{-1}(x)$ equals

A. e^{-2x+1}

B. $-e^{2x-1}$

C. $\frac{1}{2}\log_e(x+1)$

D. $\frac{1}{2}\log_e(x+\frac{1}{2})$

E. $\frac{1}{2}\log_e(x)+\frac{1}{2}$

Question 21

If $2 + \log_{10}(3x) = \log_{10}(y)$, then

A. $y = 100 + 3x$

B. $y = \frac{3x}{100}$

C. $y = 600x$

D. $y = 300x$

E. $y = \frac{100}{3x}$

Question 22

The derivative of xe^{2x} is $(2x+1)e^{2x}$. Hence $\int xe^{2x}dx$ is equal to

A. $\int (2x+1)e^{2x}dx$

B. $\int (2x-1)e^{2x}dx$

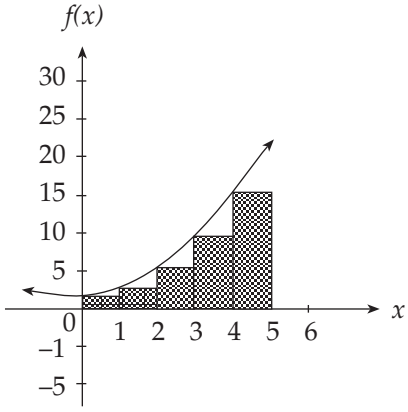
C. $2xe^{2x}+\int e^{2x}dx$

D. $(2x+1)e^{x}+c$

E. $\frac{e^{2x}}{2}\left(x-\frac{1}{2}\right)+c$

Question 23

The area under the graph $f(x) = x^2 + 1$ between $x = 0$ and $x = 5$ can be approximated by rectangular strips as shown in the diagram. The area of this approximation is



A. 30 square units

B. 35 square units

C. 36 square units

D. 40 square units

E. $46\frac{2}{3}$ square units

The following information relates to questions 24 and 25.

Fearful that she is not getting value for money, Marita decides to count the number of choc-chips in each of the 50 choc-chip cookies she bought recently. Her results are as follows

Number of choc-chips	0	1	2	3	4	5	6
Number of cookies with this number of choc-chips	1	2	5	6	15	13	8

Question 24

What proportion of cookies had less that 5 choc chips?

A. $\frac{8}{50}$

B. $\frac{13}{50}$

C. $\frac{29}{50}$

D. $\frac{42}{50}$

E. $\frac{28}{50}$

Question 25

If Marita chooses a cookie at random, what is the expected number of choc-chips in a cookie?

A. 4.08

B. 203

C. 4.06

D. 3

E. 7.14

Question 26

For a particular binomial distribution with n independent trials, each with a probability of success p, the mean and standard deviation are 10 and $\sqrt{6}$ respectively. Which of the following gives the correct values for n, p and the variance of this distribution?

A. $n = 16, p = 0.6,$ variance = 10

B. $n = 16, p = 0.6,$ variance = 6

C. $n = 25, p = 0.6,$ variance = 36

D. $n = 25, p = 0.4,$ variance = $\sqrt{6}$

E. $n = 25, p = 0.4,$ variance = 6

Question 27

Which of the following is **not** an example of a discrete random variable?

A. The numbers of students sitting Mathematical Methods Examinations over various years

B. The numbers of beans in a particular brand of bean bag

C. The daily numbers of men seen wearing blue jeans in the Bourke Street Mall

D. The numbers of girls born at the Royal Children's Hospital

E. The lengths of tapes in 30-minute audiocassettes

Question 28

Whenever Aris shoots for goal in a game of soccer, the probability that he will score a goal is 0.3. In one game he shoots for goal 10 times. Assuming indpendence, what is the probability that he will score more than one goal?

A. $(0.3)^{10}$

B. $1-\left[{}^{10}\mathrm{C}_1(0.3)(0.7)^9\right]$

C. $1-\left[(0.7)^{10}+10(0.3)(0.7)^9\right]$

D. $(0.7)^{10}+{}^{10}C_1(0.3)(0.7)^9$

E. ${}^{10}C_1(0.3)(0.7)^9$

The following information relates to questions 29 and 30

The maximum temperature T°C on a November day in Melbourne has been found to be normally distributed with a mean of 24°C and a standard deviation of 2°C.

Question 29

What would the standard normal expression be for the probability of the maximum temperature being 27°C or more?

A. $\Pr(Z > 1)$

B. $\Pr(Z < 1)$

C. $1 - \Pr\left(Z < \frac{3}{2}\right)$

D. $1 - \Pr\left(Z > \frac{3}{2}\right)$

E. $\Pr\left(-\frac{3}{2} < Z < \frac{3}{2}\right)$

Question 30

On approximately 95% of November days in Melbourne, the maximum temperature could be expected to be between c_1°C and c_2°C, where

A. $c_1 = 20$ and $c_2 = 26$

B. $c_1 = 21$ and $c_2 = 28$

C. $c_1 = 18$ and $c_2 = 30$

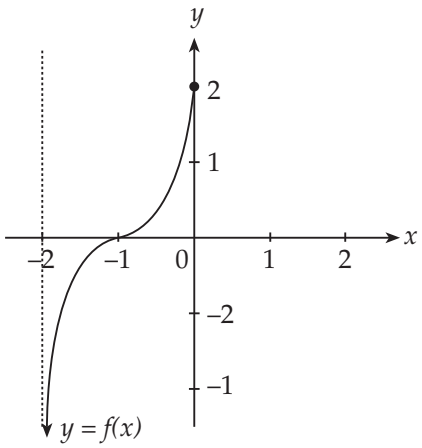
D. $c_1 = 20$ and $c_2 = 28$

E. $c_1 = 21$ and $c_2 = 26$

Part II (Short-answer questions)

Question 1

The graph of a function, f, is shown below.



a. On the same set of axes, sketch the graph of f^{-1}. [2 marks]

b. What is the range of f^{-1}? [1 mark]

Question 2

Hassan's intellectual biorhythm is modelled by the function

$$P = 2 + \sin\frac{2\pi}{28}t$$

where t is the number of days after today and P is Hassan's biorhythm level (no units are necessary).

a. What will Hassan's biorhythm level be in 7 days time? [1 mark]

A biorhythm level of 1 indicates that Hassan is at his lowest intellectual potential.

b. How many days will it take for Hassan to reach a biorhythm level of 1? [2 marks]

Question 3

For the curve with equation $f(x) = \sin(2x)$, $0 \le x \le \pi$, find the exact coordinates of the first point on the curve where $f'(x) = 1$. [3 marks]

Question 4

a. At a certain company the total cost, in dollars, of manufacturing x calculators is given by

$$C(x) = 0.3x^2 + 20x + 200, \text{ where } 0 \leq x \leq 100.$$

What is the maximum total cost of manufacturing calculators? [2 marks]

b. Each calculator that is made is sold for \$68.

i. Find an expression for $P(x)$, the total profit in dollars from manufacturing and selling x calculators.

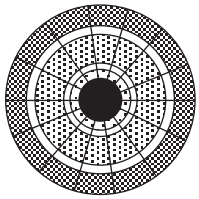
ii. How many calculators should be made and sold for the profit to be a maximum? [1 + 2 = 3 marks]

Question 5

If a is the coefficient of x^4 and b is the coefficient of x^2 in the expansion of $(2x - 1)^4$, find the ratio $\frac{a}{b}$. [3 marks]

Question 6

Astrid is a darts player attempting to hit the dark bulls-eye section of the circular board shown in the diagram. She is allowed three throws, each of which hits the board at a random point.



The area of the bullseye is 136.5 cm^2

The area of the whole dart board is 910.0 cm^2

a. What is the probability that she scores at least one bullseye? Give your answer correct to two decimal places. [2 marks]

b. What is the probability that she scores exactly one bullseye? Give your answer correct to two decimal places. [1 mark]

Part II Total: 20 marks

Mathematical Methods

Practice papers based on MAV 1998 trial CATs

Examination 1 (Facts, skills and applications)

Answers and solutions

Answers are given for multiple-choice questions. For short-answer questions and analysis questions, solutions (or solution outlines) are given to show the steps of *possible* methods and to provide answers. They are NOT intended to be full model solutions.
[A] represents a mark for a correct answer
[M] represents a mark for a correct method.

Part I (Answers to multiple-choice questions)

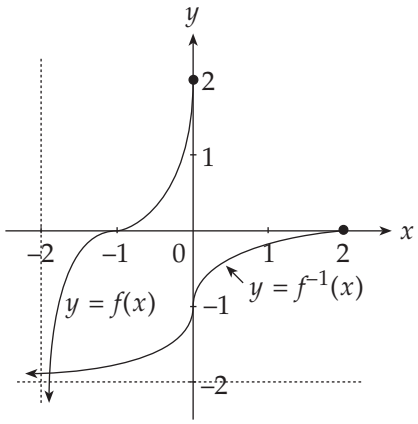
1. **D**	2. **E**	3. **B**	4. **A**	5. **C**
6. **D**	7. **E**	8. **B**	9. **C**	10. **A**
11. **E**	12. **A**	13. **B**	14. **B**	15. **D**
16. **D**	17. **B**	18. **D**	19. **E**	20. **E**
21. **D**	22. **E**	23. **B**	24. **C**	25. **C**
26. **E**	27. **E**	28. **C**	29. **C**	30. **D**

Part II (Solutions to short-answer questions)

Question 1

a. correct shape **[A]**

correct intercepts and asymptote **[A]**



b. $\text{ran}(f^{-1}) = \text{dom}(f) = (-2, 0]$ **[A]**

Question 2

a. $P = 2 + \sin\frac{2\pi}{28} \times 7$

$= 2 + \sin\frac{\pi}{2}$

$= 3$ **[A]**

b. $2 + \sin\frac{2\pi}{28}t = 1$ **[A]**

$\sin\frac{2\pi}{28}t = -1$

$\frac{2\pi}{28}t = \frac{3\pi}{2}$

$t = 21$ days **[A]**

Question 3

$f'(x) = 2\cos(2x)$ **[A]**

$0.5 = \cos(2x)$

$2x = \frac{\pi}{3}$ or …

$x = \frac{\pi}{6}$ or … **[M]**

$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

So, the point is $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$ **[A]**

Question 4

a. Since the function is increasing for $x > 0$, the maximum cost will be at the domain maximum, i.e. $x = 100$. **[M]**

$0.3(100)^2 + 20(100) + 200 = \$5200$ **[A]**

b. **i.** $P(x) = 68x - \left(0.3x^2 + 20x + 200\right), \quad 0 \le x \le 100$

$= -0.3x^2 + 48x - 200$ **[A]**

ii. $P(x)$ is a quadratic function whose stationary value is a maximum.

For maximum $P(x)$, $P'(x) = 0$ **[M]**

$\therefore -0.6x + 48 = 0$

$\therefore \quad x = 80$ **[A]**

Question 5

In the expansion of $(x + a)^n$

$$T_{r+1} = {}^nC_r(x)^{n-r}(a)^r$$

$${}^4C_2(2x)^{4-2}(-1)^2 = 24x^2 \quad \textbf{[M]}$$

$${}^4C_0(2x)^{4-0}(-1)^0 = 16x^4 \quad \textbf{[A]}$$

$$b = 24$$

$$a = 16$$

$$\frac{a}{b} = \frac{16}{24} = \frac{2}{3} \quad \textbf{[A]}$$

Question 6

a. Let X = no. of bullseyes in 3 throws. X is binomial.

$$p = \frac{136.5}{910.0} = 0.15 \qquad q = 0.85 \quad \textbf{[M]}$$

$$\begin{aligned}\Pr(X \geq 1) &= 1 - \left[\Pr(X = 0)\right] \\ &= 1 - {}^3C_0(0.15)^0(0.85)^3 \\ &= 1 - (0.85)^3 \\ &\approx 0.39 \quad \textbf{[A]}\end{aligned}$$

b.
$$\begin{aligned}\Pr(X = 1) &= {}^3C_1(0.15)^1(0.85)^2 \\ &= 3 \times 0.15 \times 0.7225 \\ &\approx 0.33 \quad \textbf{[A]}\end{aligned}$$