Question 1

The linear factors of $x^4 - 16x^2$ are:

A. $x, x, x^2 - 16$

B. $x, x, x - 8, x + 8$

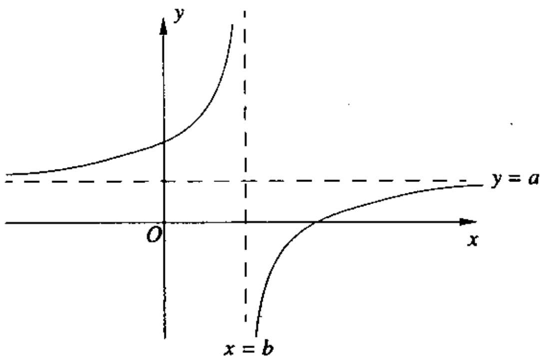
C. $x, x - 3$

D. $x, x, x - 16, x + 16$

E. $x, x, x - 4, x + 4$

Question 2

A possible equation for the graph shown, where $a > 0$ and $b > 0$, is:



A. $y - a = \frac{1}{x + b}$

B. $y - a = \frac{3}{x - b}$

C. $y + a = \frac{2}{x - b}$

D. $y - a = -\frac{2}{x - b}$

E. $y - a = -\frac{3}{x + b}$

Question 3

Given that $f(x + 3) = x^3 - 3$, then $f(x) =$

A. $(x + 3)^3 - 3$

B. $(x + 3)^3 + 3$

C. $(x - 3)^3 - 3$

D. $(x - 3)^3 + 3$

E. $x^3 - 9$

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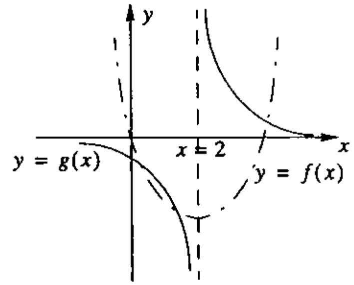
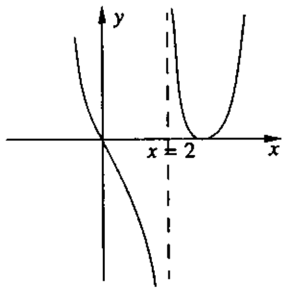
Question 4

The graphs of $y = f(x)$ and $y = g(x)$ are shown.

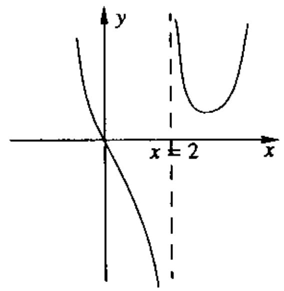


The graph of $y = g(x) - f(x)$ is then given by:

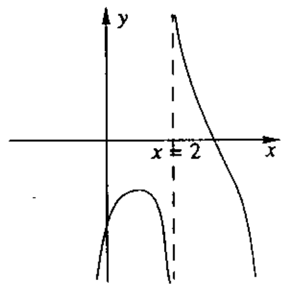
A.



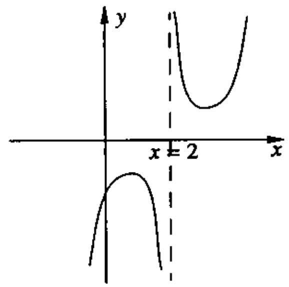
B.



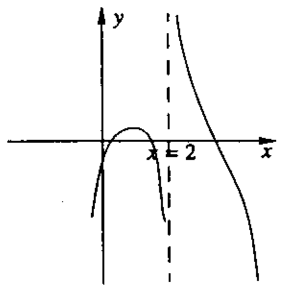
C.



D.



E.



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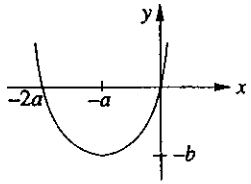
Question 5

The graph of $y = f(x)$ is shown alongside.

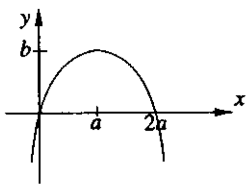


The graph of the function with equation

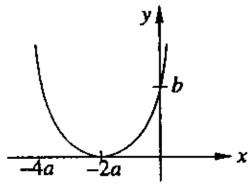
$$y = b - f(x + a)$$

is best represented by:

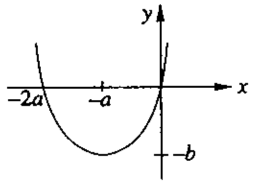
A.



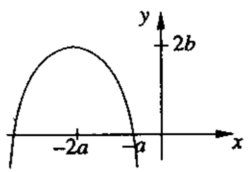
B.



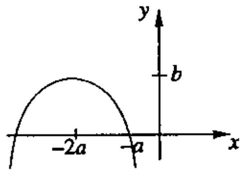
C.



D.



E.



Question 6

The implied domain for the function $f(x) = \dfrac{x}{\sqrt{2-x}}$ is:

A. $[0, \infty)$

B. $(-\infty, 0]$

C. $[2, \infty)$

D. $(-\infty, 2)$

E. $(2, \infty)$

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Question 7

The function $f:\left[0, \frac{\pi}{2}\right] \to R, f(x) = 3\sin\left(x - \frac{\pi}{2}\right)$ has range

A. $\left[0, -\frac{\pi}{2}\right]$

B. $[0, 6]$

C. $[0, 3]$

D. $[-3, 0]$

E. $[-3, 3]$

Question 8

A trigonometry function is given by $f: R \to R, f(x) = -a\cos(2x + \pi) + 1$.

The amplitude, period and range of f are respectively

A. $a, \pi, [-a-1, a+1]$

B. $a, \pi, [0, a+1]$

C. $-a, \frac{\pi}{2}, [-a+1, a+1]$

D. $a, \frac{\pi}{2}, [-a+1, a+1]$

E. $a, \pi, [-a+1, a+1]$

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Question 9

The function $f: R \to R$, $f(x) = e^{2x} + 1$ has, as its inverse, the function $f^{-1}(x) =$

A. $e^{-2x} - 1$

B. $\log_e\left(\frac{x}{2} - 1\right)$

C. $\frac{1}{2}\log_e(x - 1)$

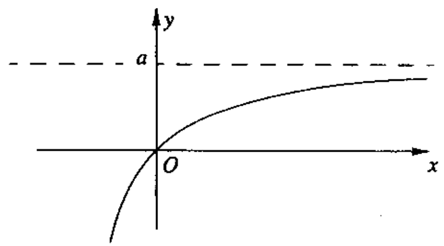
D. $\frac{1}{2}\log_e(x) - 1$

E. $\frac{1}{e^{2x} + 1}$

Question 10

The graph of the function f is shown alongside.

The rule for f is most likely to be:



A. $f(x) = a - e^{-x}$

B. $f(x) = a - e^{x}$

C. $f(x) = a(1 - e^{x})$

D. $f(x) = a + e^{-x}$

E. $f(x) = a(1 - e^{-x})$

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Question 11

The expression $\log_3 \sqrt[3]{\frac{x^3}{y}}$ is equivalent to

A. $\frac{1}{3}[\log_3 x - \log_3 y]$

B. $\log_3 x - \frac{1}{3}\log_3 y$

C. $\frac{1}{3}[\log_3 x + \log_3 y]$

D. $\log_3 x + \frac{1}{3}\log_3 y$

E. $\frac{1}{2}[3\log_3 x - \log_3 y]$

Question 12

Given that a, b and c are positive constants, the equation $a\cos(x + b) - c = 0$ is guaranteed to have at least one solution over the interval $[0, 2\pi]$ as long as

A. $\frac{a}{c} > 1$

B. $\frac{a}{c} < 1$

C. $c > 1$

D. $c < 1$

E. $a > b - c$

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Question 13

The gradient of the secant from $x = 2$ to $x = 4$ for the function $y = f(x)$ is equal to

A. $\frac{f(4) - f(2)}{2}$

B. $\frac{f(4) - f(2)}{6}$

C. $\frac{f(4) + f(2)}{6}$

D. $f(4) + f(2)$

E. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

Question 14

The derivative of $x - \frac{1}{3x}$ is equal to

A. $\frac{2}{3}$

B. $1 - \frac{1}{3x^2}$

C. $1 + \frac{1}{3x^2}$

D. $1 + \frac{3}{x^2}$

E. $1 - \frac{1}{3}\log_e x$

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Question 15

Given that $f(x) = \frac{x+1}{2x-3}$ then $f'(k) =$

A. $\frac{1}{2}$

B. $\frac{5}{(2k-3)^2}$

C. $\frac{1}{(2k-3)^2}$

D. $\frac{5k-1}{(2k-3)^2}$

E. $-\frac{5}{(2k-3)^2}$

Question 16

The gradient of the normal to the curve $y = 3\cos(2x)$ where $y = 1.5$ is

A. $3\sqrt{3}$

B. $-3\sqrt{3}$

C. $\frac{1}{3\sqrt{3}}$

D. $-\frac{1}{3\sqrt{3}}$

E. 3

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Question 17

For the functions f and g where $f'(x) = g'(x)$, then

A. f and g have the same stationary points.
B. f and g have the same maximum.
C. f and g have the same minimum.
D. f and g differ by a constant.
E. f and g have the same x–intercepts.

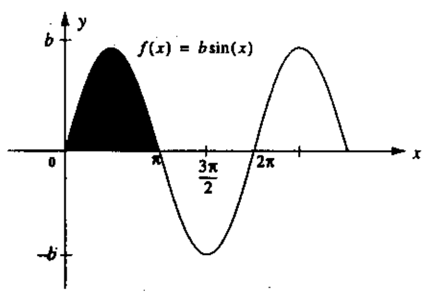
Question 18

Given that $\int_1^3 g(x)dx = 2$ then $\int_1^3 (2g(x)+1)dx$ will equal

A. 6
B. 3
C. 4
D. 6
E. $4+2x$

Question 19

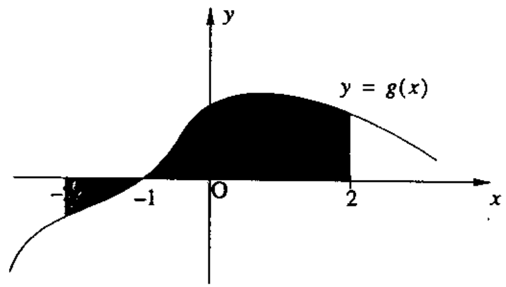
The graph of $f:[0, \pi] \to R$, $f(x) = b\sin(x)$ is shown.
Given that the area of the shaded region is equal to 2, then $b =$



A. 1
B. 2
C. 4
D. $\frac{1}{2}$
E. $\frac{1}{4}$

Question 20

The area of the shaded region in the diagram shown is equal to



A. $\int_2^{-2} g(x)dx$
B. $\int_{-2}^{2} g(x)dx$
C. $\int_{-1}^{2} g(x)dx + \int_{-2}^{-1} g(x)dx$
D. $-\int_{-1}^{2} g(x)dx + \int_{-2}^{-1} g(x)dx$
E. $\int_{-1}^{2} g(x)dx - \int_{-2}^{-1} g(x)dx$

Question 21

An antiderivative of $e^{-2x} - \frac{1}{x}$ is:

A. $-2e^{-2x} + \frac{1}{x^2}$
B. $-\frac{1}{2}e^{-2x} - \log_e x$
C. $-2e^{-2x} - \log_e x$
D. $-\frac{1}{2}e^{-2x} + \log_e x$
E. $-\frac{1}{2}e^{-2x} + \frac{1}{x^2}$

Question 22

An antiderivative of $\dfrac{1}{\sqrt{(5x+4)^5}}$ is equal to

A. $\dfrac{-2}{15\sqrt{(5x+4)^3}}$

B. $\dfrac{-5}{2\sqrt{(5x+4)^3}}$

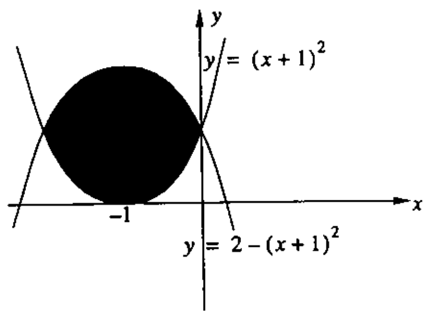
C. $\dfrac{-15}{2\sqrt{(5x+4)^3}}$

D. $\dfrac{15}{2\sqrt{(5x+4)^3}}$

E. $\dfrac{2}{15\sqrt{(5x+4)^3}}$

Question 23

The area of the shaded region shown is given by



A. $\int_{-1}^{1}(2-2(x+1)^2)dx$

B. $\int_{-1}^{1}(2(x+1)^2-2)dx$

C. $\int_{-2}^{0}(2(x+1)^2-2)dx$

D. $\int_{-2}^{0}(2-2(x+1)^2)dx$

E. $\int_{-2}^{0}(2)dx$

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Question 24

Given that $g'(x) = \dfrac{1}{x+1}$ and $g(0) = 2$, then $g(x) =$

A. $\log_e(x+1)+2$

B. $-\dfrac{1}{(x+1)^2}+3$

C. $-\log_e(x+1)+3$

D. $\log_e(x+1)$

E. $\dfrac{1}{(x+1)^2}+1$

Question 25

When $(3a-2b)^k$ is expanded, there will be

A. $k-1$ terms

B. k terms

C. $k+1$ terms

D. 3^k-2^k terms

E. $6(ab)^k$ terms.

Question 26

In a VCE Physics test the class mean was found to be 56 and the variance 16. It is expected that approximately 95% of the students marks in this Physics class will lie in the interval

A. 40 to 72

B. 52 to 69

C. 24 to 88

D. 48 to 64

E. 50 to 62

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Question 27

The random variable X has the following probability distribution :

x	1	2	3	4	5
$Pr(X = x)$	0.1	0.2	a	0.4	0.1

The probability that X is greater than or equal to 3 is:

A. 0.2
B. 0.3
C. 0.5
D. 0.7
E. unable to be determined.

Question 28

A random variable X has a probability distribution with mean 6 and $E(X^2) = 52$. The standard deviation of X would then be equal to

A. 4
B. 16
C. 36
D. $\sqrt{46}$
E. $\sqrt{58}$

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Question 29

If the random variable X has a normal probability distribution with variance 4 and is such that $Pr(X > 8) = 0.3$, then $E(X) =$

A. 5.902
B. 6.484
C. 6.589
D. 6.952
E. 9.049

The following information refers to questions 30 and 31.

The length of chocolate coated licorice 'sticks' produced as being 100 mm long, has been found to be normally distributed with a mean of 105 mm and variance 16.

Question 30

The probability that such a stick is between 100 mm and 107 mm, is closest to

A. 0.4599
B. 0.5858
C. 0.2029
D. 0.4141
E. 0.3968

Question 31

The number of sticks that measure under 100 mm in a sample of 1000 would be closest to

A. 20
B. 106
C. 212
D. 788
E. 894

Question 32

Washing machines produced at KIP–m–KLEAN Pty Ltd, are known to have a *defective rate* of 12%. Ten machines from the plant are selected at random from a large production run. The probability that at most 8 of these machines are defective is given by

A. ${}^{10}C_8(0.12)^8(0.88)^2$

B. $1-{}^{10}C_8(0.12)^8(0.88)^2$

C. $1-{}^{10}C_8(0.12)^8(0.88)^2-{}^{10}C_9(0.12)^9(0.88)^1-{}^{10}C_{10}(0.12)^{10}(0.88)^0$

D. $1-{}^{10}C_9(0.12)^9(0.88)^1-{}^{10}C_{10}(0.12)^{10}(0.88)^0$

E. $1-{}^{10}C_9(0.88)^9(0.12)^1-{}^{10}C_{10}(0.88)^{10}(0.12)^0$

Question 33

Packets of peanuts are labelled as being 250 gm. A random sample of 100 of these packets show that 70 of them had in fact a mass of under 240 gm. An approximate 95% confidence interval for the proportion, p, of packets that have a mass of under 240 gm is

A. [0.63, 0.77]
B. [0.61, 0.79]
C. [0.65, 0.75]
D. [0.21, 0.39]
E. [0.56, 0.84]

Total 33 marks

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PART II SHORT ANSWER QUESTIONS

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	17

Directions to students

Materials
You may use an approved calculator, ruler, protractor, set–square and aids for curve sketching.

The task
Ensure you write your **name** on the space provided on the front cover of this question and answer booklet.
Answer **all** questions.
The marks allocated to each question are indicated at the end of each question.
There is a total of 17 marks available for Part II.
You need not give numerical answers as decimals unless instructed to do so.
Alternative forms may involve, for example, π, e, surds or fractions.
All written responses should be in English.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task
Place the answer sheet for multiple–choice questions (Part I) inside the back cover of this question and answer booklet and hand them in.

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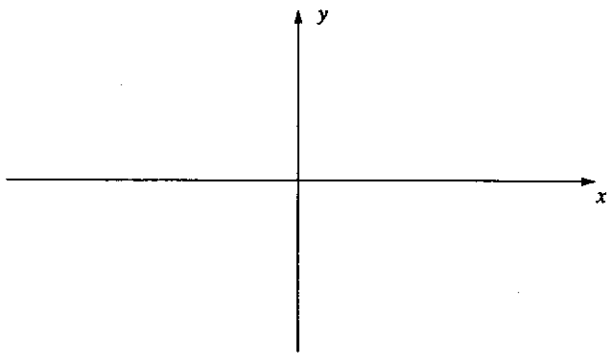
Question 1

a. Find the linear factors of $x^4 - (ax)^2$.

[1 mark]

b. Given that $a > 4$, sketch the graph of $f: R \to R$, where $f(x) = x^4 - a^2x^2$.

***Do not** find the coordinates of any stationary points.*



[2 marks]

Question 2

a. Find $\frac{d}{dx}(e^{x^2+1})$.

[2 marks]

b. Hence, find the exact value of $\int_0^1 xe^{x^2+1}dx$

[2 marks]

Question 3

For the function $f: [0, 2] \to R$, where $f(x) = 2 - 3\sin\left(\frac{\pi}{2}x\right)$, state:

a. i. its amplitude

ii. its range

[2 marks]

b. Find $\left\{x : f(x) = \frac{1}{2}\right\} \cap \{x: 0 < x < 2\}$, giving exact values for x.

[2 marks]

Question 4

Given that the solution to the equation $\log_{10}x - \log_{10}(x-2) = 1$ is $x = \frac{a}{b}$, find the values of a and b, where a and b are both integers.

....

....

....

....

[2 marks]

Question 5

Given that the random variable X has a normal distribution with mean a and variance b^2, show that if $Pr(X < 2) = 0.60$, then $a + 0.2533b = 2$.

....

....

....

[2 marks]

Question 6

A sample of 40 sugar cubes reveals that 12 are too small. If $\hat{p}$ is the proportion of sugar cubes that are too small from this sample, find the standard error of $\hat{p}$.

....

....

[2 marks]

Total 17 marks

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