The Mathematical Association of Victoria

1997

MATHEMATICAL METHODS

Trial Examination 1

(Practice papers based on MAV 1997 trial CATs)

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **30 marks** available for this part.

Part II consists of seven questions. Answers all questions in **Part II** in the spaces provided. There are **20 marks** available for this part.

There are **50 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

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Multiple-Choice Answer Sheet

Student's Name: ____________________________________

Cross through the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**
28.	**A**	**B**	**C**	**D**	**E**
29.	**A**	**B**	**C**	**D**	**E**
30.	**A**	**B**	**C**	**D**	**E**

Mathematical Methods

Practice papers based on MAV 1997 trial CATs

Examination 1 (Facts, skills and applications)

Part I (Multiple-choice questions)

Question 1

Given that $f(x + 3) = x^3 - 3$, then $f(x) =$

A. $(x + 3)^3 - 3$

B. $(x + 3)^3 + 3$

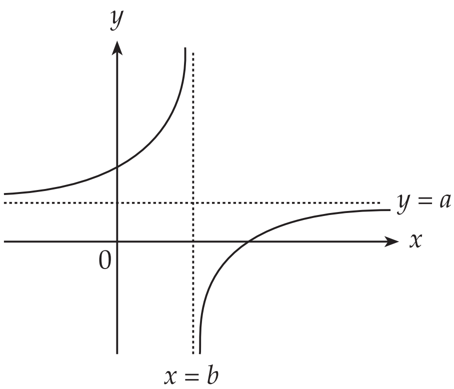
C. $(x - 3)^3 - 3$

D. $(x - 3)^3 + 3$

E. $x^3 - 9$

Question 2

A possible equation for the graph shown, where $a > 0$ and $b > 0$, is



A. $y - a = \dfrac{1}{x+b}$

B. $y - a = \dfrac{3}{x-b}$

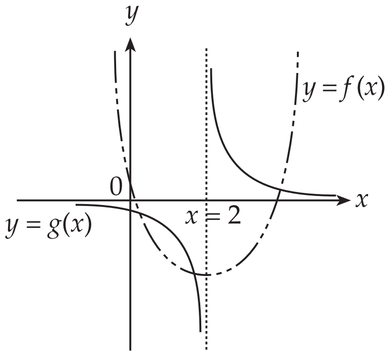
C. $y + a = \dfrac{2}{x-b}$

D. $y - a = -\dfrac{2}{x-b}$

E. $y - a = -\dfrac{3}{x+b}$

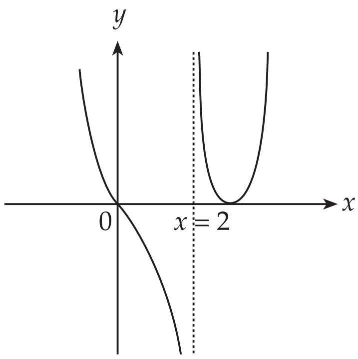
Question 3

The graphs of $y = f(x)$ and $y = g(x)$ are shown.

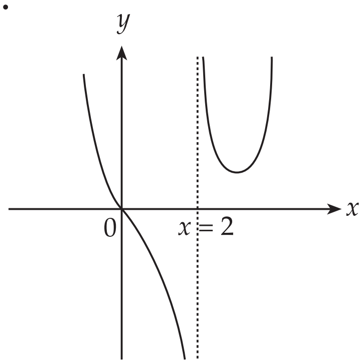


The graph of $y = g(x) - f(x)$ is then given by

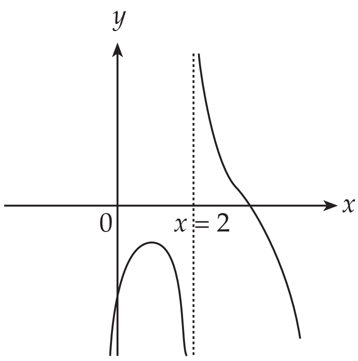
A.



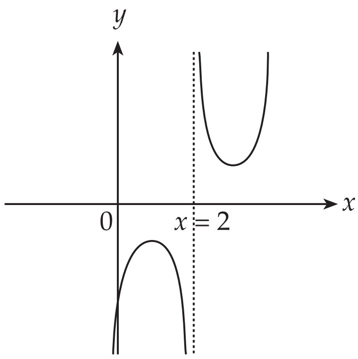
B.



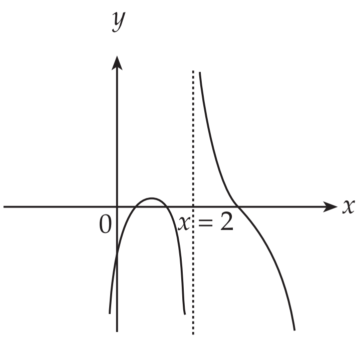
C.



D.



E.



Question 4

The implied domain for the function $f(x) = \dfrac{x}{\sqrt{2-x}}$ is

A. $[0, \infty)$

B. $(-\infty, 0]$

C. $[2, \infty)$

D. $(-\infty, 2)$

E. $(2, \infty)$

Question 5

The function $f : \left[0, \frac{\pi}{2}\right] \to R, f(x) = 3\sin\left(x - \frac{\pi}{2}\right)$ has range

A. $\left[0, -\frac{\pi}{2}\right]$

B. $[0, 6]$

C. $[0, 3]$

D. $[-3, 0]$

E. $[-3, 3]$

Question 6

A trigonometric function is given by $f : R \to R, f(x) = -a\cos(2x + \pi) + 1,$ where $a > 0$. The amplitude, period and range of f are respectively

A. $a, \pi, [-a - 1, a + 1]$

B. $a, \pi, [0, a + 1]$

C. $-a, \frac{\pi}{2}, [-a + 1, a + 1]$

D. $a, \frac{\pi}{2}, [-a + 1, a + 1]$

E. $a, \pi, [-a + 1, a + 1]$

Question 7

The function $f: R \to R, f(x) = e^{2x} + 1$ has, as its inverse, the function $f^{-1}(x) =$

A. $e^{-2x} - 1$

B. $\log_e(\frac{x}{2} - 1)$

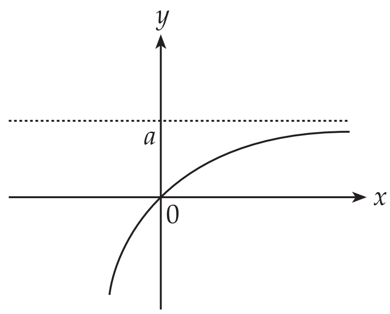
C. $\frac{1}{2}\log_e(x - 1)$

D. $\frac{1}{2}\log_e(x) - 1$

E. $\frac{1}{e^{2x}+1}$

Question 8

The graph of the function f is shown alongside.
The rule for f is most likely to be



A. $f(x) = a - e^{-x}$

B. $f(x) = a - e^{x}$

C. $f(x) = a(1 - e^{x})$

D. $f(x) = \mathrm{a} + e^{-x}$

E. $f(x) = a(1 - e^{-x})$

Question 9

The expression $\log_3\sqrt[3]{\frac{x^3}{y}}$ is equivalent to

A. $\frac{1}{3}[\log_3 x - \log_3 y]$

B. $\log_3 x - \frac{1}{3}\log_3 y$

C. $\frac{1}{3}[\log_3 x + \log_3 y]$

D. $\log_3 x + \frac{1}{3}\log_3 y$

E. $\frac{1}{2}[3\log_3 x - \log_3 y]$

Question 10

Given that a, b and c are positive constants, the equation $a\cos(x+b)-c=0$ must have at least one solution over the interval $[0, 2\pi]$ if

A. $\frac{a}{c}>1$

B. $\frac{a}{c}<1$

C. $c>1$

D. $c<1$

E. $a>b-c$

Question 11

The gradient of the secant from $x = 2$ to $x = 4$ for the graph of the function $y = f(x)$ is equal to

A. $\frac{f(4)-f(2)}{2}$

B. $\frac{f(4)-f(2)}{6}$

C. $\frac{f(4)+f(2)}{6}$

D. $f(4)+f(2)$

E. $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

Question 12

The derivative of $x-\frac{1}{3x}$ is equal to

A. $\frac{2}{3}$

B. $1-\frac{1}{3x^2}$

C. $1+\frac{1}{3x^2}$

D. $1+\frac{3}{x^2}$

E. $1-\frac{1}{3}\log_e x$

Question 13

Given that $f(x)=\dfrac{x+1}{2x-3}$ then $f'(k)=$

A. $\dfrac{1}{2}$

B. $\dfrac{5}{(2k-3)^2}$

C. $\dfrac{1}{(2k-3)^2}$

D. $\dfrac{5k-1}{(2k-3)^2}$

E. $-\dfrac{5}{(2k-3)^2}$

Question 14

The gradient of the normal to the curve $y = 3\cos(2x)$ at the point where $x = \dfrac{\pi}{6}$ is

A. $3\sqrt{3}$

B. $-3\sqrt{3}$

C. $\dfrac{1}{3\sqrt{3}}$

D. $-\dfrac{1}{3\sqrt{3}}$

E. 3

Question 15

For the functions f and g where $f'(x) = g'(x)$,

A. the graphs of f and g have the same stationary points.

B. $f(x)$ and $g(x)$ have the same maximum value.

C. $f(x)$ and $g(x)$ have the same minimum value.

D. $f(x)$ and $g(x)$ differ by a constant.

E. the graphs of f and g have the same x-intercepts.

Question 16

Given that $\int_1^3 g(x)dx = 2,$ then $\int_1^3 (2g(x)+1)dx$ will equal

A. 3

B. 4

C. 5

D. 6

E. $4 + 2x$

Question 17

The graph of $f: [0, 3\pi] \to R, f(x) = b\sin(x)$ is shown.

Given that the area of the shaded region is equal to 2, then $b =$

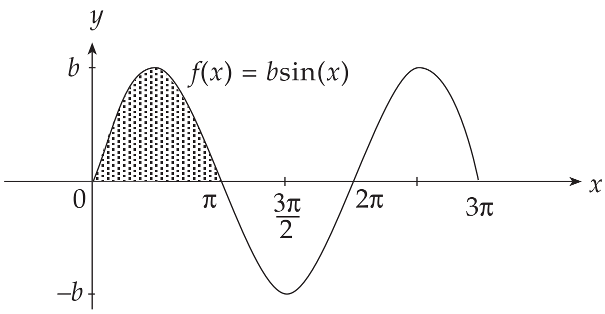
A. 1

B. 2

C. 4

D. $\frac{1}{2}$

E. $\frac{1}{4}$



Question 18

The area of the shaded region in the diagram shown is equal to

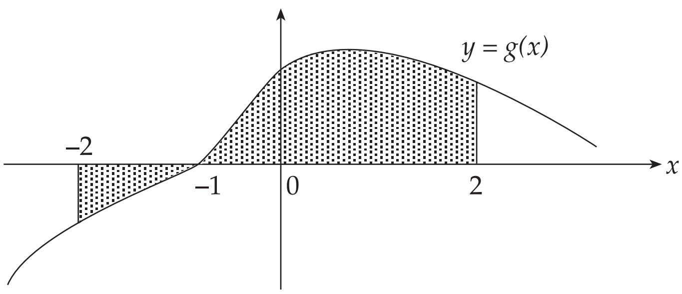
A. $\int_2^{-2} g(x)dx$

B. $\int_{-2}^{2} g(x)dx$

C. $\int_{-1}^{2} g(x)dx + \int_{-2}^{-1} g(x)dx$

D. $-\int_{-1}^{2} g(x)dx + \int_{-2}^{-1} g(x)dx$

E. $\int_{-1}^{2} g(x)dx - \int_{-2}^{-1} g(x)dx$



Question 19

An antiderivative of $e^{-2x} - \dfrac{1}{x}$ is

A. $-2e^{-2x} + \dfrac{1}{x^2}$

B. $-\dfrac{1}{2}e^{-2x} - \log_e x$

C. $-2e^{-2x} - \log_e x$

D. $-\dfrac{1}{2}e^{-2x} + \log_e x$

E. $-\dfrac{1}{2}e^{-2x} + \dfrac{1}{x^2}$

Question 20

An antiderivative of $\dfrac{1}{\sqrt{(5x+4)^5}}$ is equal to

A. $\dfrac{-2}{15\sqrt{(5x+4)^3}}$

B. $\dfrac{-5}{2\sqrt{(5x+4)^3}}$

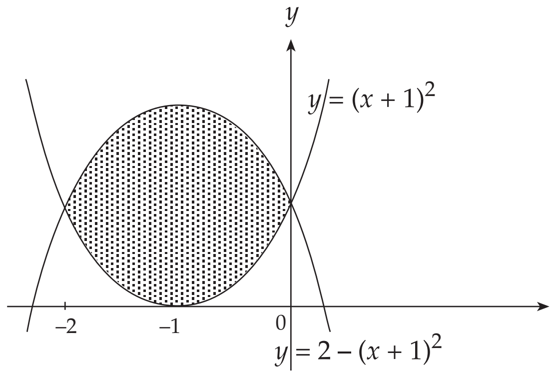
C. $\dfrac{-15}{2\sqrt{(5x+4)^3}}$

D. $\dfrac{15}{2\sqrt{(5x+4)^3}}$

E. $\dfrac{2}{15\sqrt{(5x+4)^3}}$

Question 21

The area of the shaded region shown is given by



A. $\int_{-1}^{1}\left(2-2(x+1)^2\right)dx$

B. $\int_{-1}^{1}\left(2(x+1)^2-2\right)dx$

C. $\int_{-2}^{0}\left(2(x+1)^2-2\right)dx$

D. $\int_{-2}^{0}\left(2-2(x+1)^2\right)dx$

E. $\int_{-2}^{0}(2)dx$

Question 22

Given that $g'(x)=\dfrac{1}{x+1}$ and $g(0)=2,$ then $g(x)=$

A. $\log_e\left(x+1\right)+2$

B. $-\dfrac{1}{\left(x+1\right)^2}+3$

C. $-\log_e\left(x+1\right)+3$

D. $\log_e\left(x+1\right)$

E. $\dfrac{1}{\left(x+1\right)^2}+1$

Question 23

When $(3a-2b)^k$ is expanded, there will be

A. $k-1$ terms

B. k terms

C. $k+1$ terms

D. 3^k-2^k terms

E. $6(ab)^k$ terms

Question 24

In a VCE Physics test the class mean was found to be 56 and the variance 16. It is expected that approximately 95% of the students' marks in this Physics class will lie in the interval

A. 40 to 72

B. 52 to 69

C. 24 to 88

D. 48 to 64

E. 50 to 62

Question 25

The random variable X has the following probability distribution:

x	1	2	3	4	5
$\Pr(X = x)$	0.1	0.2	a	0.4	0.1

The probability that X is greater than or equal to 3 is

A. 0.2

B. 0.3

C. 0.5

D. 0.7

E. unable to be determined

Question 26

A random variable X has a probability distribution with mean 6 and $E(X^2) = 52$. The standard deviation of X is

A. 4

B. 16

C. 36

D. $\sqrt{46}$

E. $\sqrt{58}$

Question 27

If the random variable X has a normal probability distribution with variance 4 and is such that $\Pr(X > 8) = 0.3$, then $E(X) =$

A. 5.902

B. 6.484

C. 6.589

D. 6.952

E. 9.049

The following information relates to Questions 28 and 29.

The length of chocolate coated licorice ‘sticks’ produced as being 100 mm long, has been found to be normally distributed with a mean of 105 mm and variance 16.

Question 28

The probability that such a stick is between 100 mm and 107 mm long is closest to

A. 0.4599

B. 0.5858

C. 0.2029

D. 0.4141

E. 0.3968

Question 29

The number of sticks that measure under 100 mm in a sample of 1000 would be closest to

A. 20

B. 106

C. 212

D. 788

E. 894

Question 30

Washing machines produced at KIP–m–KLEAN Pty Ltd, are known to have a *defective rate* of 12%. Ten machines from the plant are selected at random from a large production run. The probability that at most 8 of these machines are defective is given by

A. $^{10}C_8(0.12)^8(0.88)^2$

B. $1 - {}^{10}C_8(0.12)^8(0.88)^2$

C. $1 - {}^{10}C_8(0.12)^8(0.88)^2 - {}^{10}C_9(0.12)^9(0.88)^1 - {}^{10}C_{10}(0.12)^{10}(0.88)^0$

D. $1 - {}^{10}C_9(0.12)^9(0.88)^1 - {}^{10}C_{10}(0.12)^{10}(0.88)^0$

E. $1 - {}^{10}C_9(0.88)^9(0.12)^1 - {}^{10}C_{10}(0.88)^{10}(0.12)^0$

Part II (Short-answer questions)

Question 1

a. Find the linear factors of $x^4 - (ax)^2$. [1 mark]

b. Given that $a > 4$, sketch the graph of $f: R \to R$, where $f(x) = x^4 - a^2x^2$. [2 marks]

Do not *find the coordinates of any stationary points.*

Question 2

a. Find $\frac{d}{dx}(e^{x^2+1})$ [2 marks]

b. Hence, find the exact value of $\int_0^1 xe^{x^2+1}dx$. [2 marks]

Question 3

For the function $f:[0.2] \to R$, where $f(x) = 2 - 3\sin\left(\frac{\pi}{2}x\right)$, state

a. **i.** its amplitude

ii. its range [2 marks]

b. Find $\left\{x : f(x) = \frac{1}{2}\right\} \cap \{x : 0 < x < 2\}$, giving exact values for x. [3 marks]

Question 4

Given that the solution of the equation $\log_{10}x - \log_{10}(x - 2) = 1$ is $x = \frac{a}{b}$, find the values of a and b, where a and b are both integers. [2 marks]

Question 5

Given that the random variable X has a normal distribution with mean a and variance b^2, show that if $\Pr(X < 2) = 0.60$, then $a + 0.253b \simeq 2$. [2 marks]

Question 6

A random sample of 6 batteries is selected without replacement from a box containing 5 defective batteries and 4 ‘good’ batteries. Let X be the number of defective batteries in the sample.

i. List the possible values of X.

ii. Find the mean of X.

iii. Find the probability that X is greater than its mean. [1 + 1 + 2 = 4 marks]

Part II Total: 20 marks

Mathematical Methods
Practice papers based on MAV 1997 trial CATs
Examination 1 (Facts, skills and applications)
Answers and Solutions

Answers are given for multiple-choice questions. For short-answer questions and analysis questions, solutions (or solution outlines) are given to show the steps of *possible* methods and to provide answers. They are NOT intended to be full model solutions.

[A] represents a mark for a correct answer

[M] represents a mark for a correct method.

Part I (Answers to multiple-choice questions)

1. **C**	2. **D**	3. **E**	4. **D**	5. **D**
6. **E**	7. **C**	8. **E**	9. **B**	10. **A**
11. **A**	12. **C**	13. **E**	14. **C**	15. **D**
16. **D**	17. **A**	18. **E**	19. **B**	20. **A**
21. **D**	22. **A**	23. **C**	24. **D**	25. **D**
26. **A**	27. **D**	28. **B**	29. **B**	30. **D**

Part II (Solutions to short-answer questions)

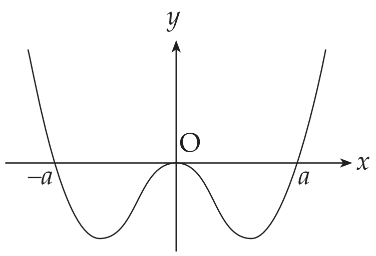
Question 1

a.

$$x^2(x^2 - a^2)$$
$$= x^2(x-a)(x+a)$$

[A]

b.



Shape: **[A]**

Intercepts: **[A]**

Question 2

a. Using chain rule: $2x.e^{x^2+1}$ **[M][A]**

b. Hence $\int_0^1 2x.e^{x^2+1}dx = [e^{x^2+1}]_0^1$ **[M]**

$\therefore \int_0^1 x.e^{x^2+1}dx = \frac{1}{2}\left(e^2 - e^1\right)$ **[A]**

Question 3

a. **i.** 3 **[A]**

ii. [–1, 2] **[A]**

b. $2 - 3\sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$

$\therefore \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$ **[M]**

$\frac{\pi}{2}x = \frac{\pi}{6}, \frac{5\pi}{6}$ **[M]**

$x = \frac{1}{3}, \frac{5}{3}$ **[A]**

Question 4

$\log_{10}\left(\frac{x}{x-2}\right) = 1$

$\Leftrightarrow \frac{x}{x-2} = 10$ **[M]**

$\Leftrightarrow \quad x = 10x - 20$

$\therefore x = \frac{20}{9} \therefore a = 20, b = 9$ **[A]**

(NB: multiples of 20 and 9 are acceptable)

Question 5

$$\Pr(X<2)=0.6 \Leftrightarrow \Pr\left(Z<\frac{2-a}{b}\right)=0.6$$

$\therefore \dfrac{2-a}{b} \simeq 0.253$ **[M]**

$\therefore 2 \simeq a+0.253$ **[A]**

Question 6

i. $X=2,3,4$ or 5 **[A]**

ii. $X \sim \text{Hg}(N=9, D=5, n=6)$

$E(X)=n\dfrac{D}{N}=6.\dfrac{5}{9}=3\dfrac{1}{3}$ **[A]**

iii. $\Pr\left(X>3\dfrac{1}{3}\right)=\Pr(X=4 \text{ or } 5)$

$$=\frac{{}^5C_4 \times {}^4C_2}{{}^9C_6}+\frac{{}^5C_5 \times {}^4C_1}{{}^9C_6}$$ **[M]**

$$=\frac{5\times 6}{\frac{9\times 8\times 7}{3\times 2\times 1}}+\frac{1\times 4}{84}$$

$$=\frac{30+4}{84}=\frac{17}{42}\left[\approx 0.405\right]$$ **[A]**