SECTION A

Specific Instructions for Section A

Section A consists of 33 questions.
Answer **all** questions in this Section on the multiple-choice answer sheet provided.
A correct answer scores 1, an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers. You should attempt every question. No credit will be given for a question if two or more letters are marked for that question.

Questions 1 and 2 refer to the points P and Q, with respective coordinates (–1,–1) and (3,7).

Question 1

The **length** of the line segment $\overline{PQ}$ is equal to

A 4 units   B $2\sqrt{10}$ units   C 8 units   D $4\sqrt{5}$ units   E 12 units
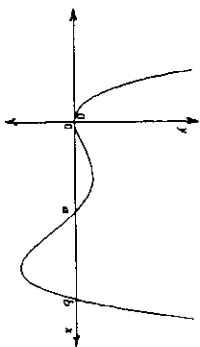
Question 2

Which one of the following equations defines a line **parallel** to the line segment $\overline{PQ}$?

A $2x - y = 0$ B $x + y = 0$ C $x - 2y = 0$
D $2x + y = 0$ E $x + 2y = 0$

Question 3

Which one of the following equations could define the graph sketched at right?

A $y = x^2(x + a)(x + b)$
B $y = x(x + a)^2(x + b)^2$
C $y = x^2(x - a)(x - b)$
D $y = x(x - a)^2(x - b)^2$
E $y = x(x - a)(x - b)$

Question 4

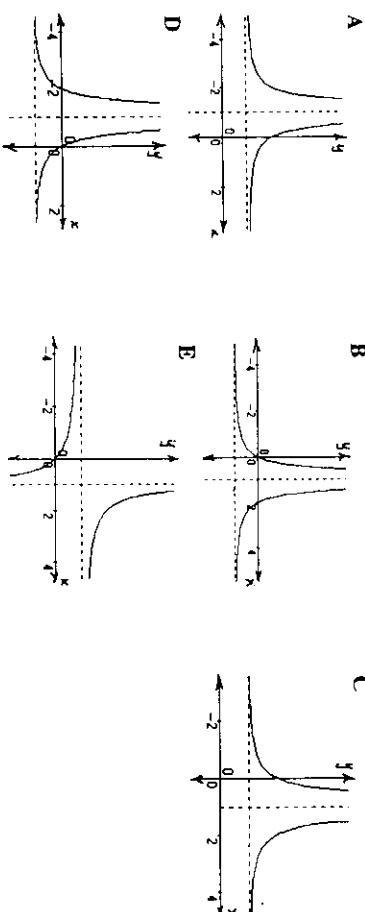For the function with rule $f(x) = 2 - \sqrt{x + 1}$, which one of the following statements is **NOT** true?

A The function has largest domain $\{x : x \geq 1\}$.
B The graph of f has an x-intercept at $(3,0)$.
C The function has range $\{y : y \leq 2\}$.
D f is a one-to-one function.
E The graph of f has a y-intercept at $(0,1)$.

Question 5

Which one of the following sketch graphs could represent the function

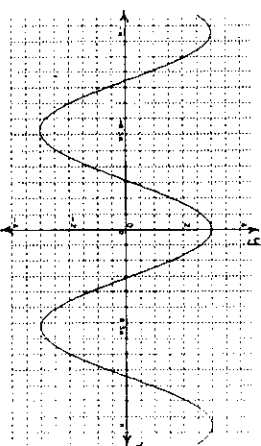$$g : R\backslash\{1\} \to R, \; g(x) = 10 + \frac{10}{(x - 1)^2} \; ?$$

Question 6

The graph at right represents the function defined by

$$y = a \cos (bx) \text{ where}$$

A $a = 3$ and $b = \frac{1}{2}$
B $a = 3$ and $b = 2$
C $a = 2$ and $b = 3$
D $a = 2$ and $b = \frac{1}{3}$
E $a = -3$ and $b = 2$

Question 7

The solution of the equation $2 \cos (2x) = \sqrt{3}$ between π and $\frac{3\pi}{2}$ is

A $\frac{\pi}{12}$ B $\frac{11\pi}{12}$ C $\frac{13\pi}{12}$ D $\frac{14\pi}{12}$ E $\frac{23\pi}{12}$
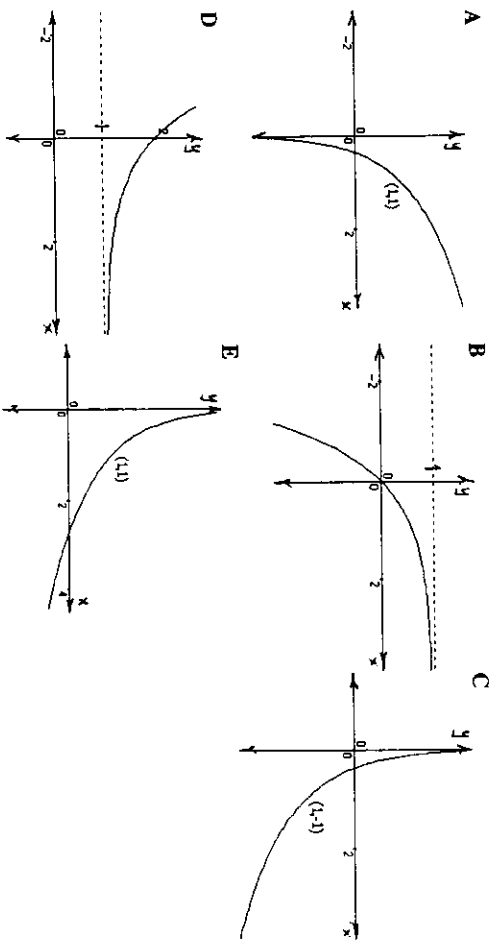
Question 8

If $0 \leq x \leq \pi$, then the equation $\sin (3x) = \cos (3x)$ has

A one solution   B two solutions   C three solutions
D four solutions   E five solutions

Question 9

Which one of the following graphs could represent the function $f: R^+ \to R$, $f(x) = 1 - \log_2 x$?

A

B

C

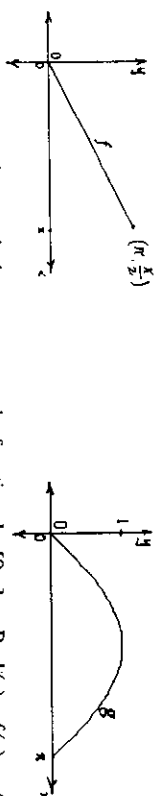**D**

E

Question 10

Data for two variables x and y can be modelled by the graph drawn at right.
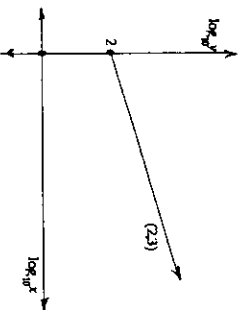
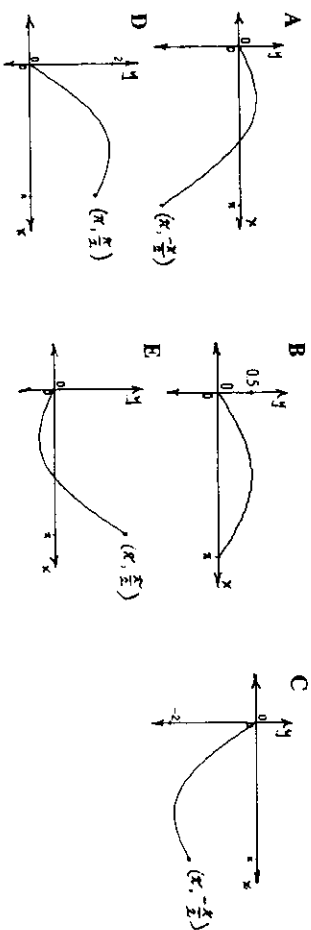A **rule** to model the data could be

A $y = 0.5x + 2$

B $y = 100 \times 10^{0.5x}$

C $y = \dfrac{100}{10^{0.5x}}$

D $y = 100\sqrt{x}$

E $y = \dfrac{100}{\sqrt{x}}$

Question 11

Consider the following graphs of two functions f and g, both with domains $[0, \pi]$:

Which one of the following graphs best represents the function $h: [0, \pi] \to R$, $h(x) = f(x) - g(x)$?
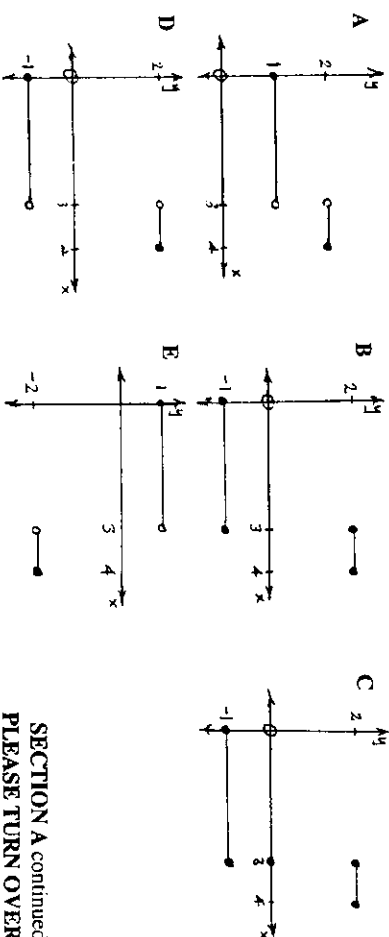
A

B

C

D

E

Question 12

The graph at right represents a function f with domain $[0, 4]$
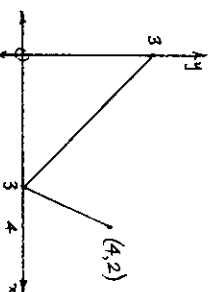
Which one of the following graphs best represents the **derivative function** f' ?

A

B

C

D

E

Question 13

The graph at right represents the **derivative function** f' for a function f defined on domain $[0,2]$.
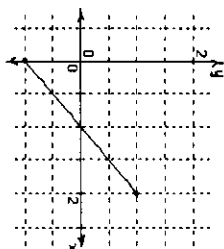
The graph of the function f could be

A

B

C

D

E

Question 14

If $f(x) = \sqrt{x}$, then $f'(4)$ is equal to

A ¼   **B** ½   **C** 1

D 2   **E** 4

Question 15

The derivative of $x \sin x$ is

A $x \cos x - \sin x$ **B** $\sin x - x \cos x$ **C** $x \sin x + \cos x$

D $\sin x + x \cos x$ **E** $\cos x - x \sin x$

Question 16

If $y = e^{\sin x}$, then, when $x = \dfrac{\pi}{2}$, $\dfrac{dy}{dx}$ is equal to

A $-e$ **B** -1 **C** 0

D 1 **E** e

Question 17

A normal (perpendicular) is drawn at the point $(1,2)$ on the curve defined by $y = x^2 + 1$. The equation which defines this normal is

A $x + 2y = 5$ **B** $2x - y = 0$ **C** $2y - x = 3$

D $2x + y = 4$ **E** $x + 3y = 7$

Question 18

The function $f: \mathbf{R} \to \mathbf{R}$, $f(x) = x^4 - 4x^2$ has a stationary point when $x = \sqrt{2}$. This stationary point is

A a local maximum

B a local minimum

C a stationary point of inflexion, with gradient positive on either side

D a stationary point of inflexion, with gradient negative on either side

E none of the above

Question 19

The amount, A gm, of radioactive material present at time t years is given by $A = 50 \, e^{-0t}$.
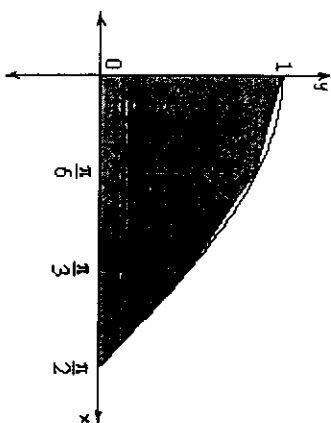After exactly five years, the amount of material is

A decaying at a rate of approximately 3 gm per year

B growing at a rate of approximately 3 gm per year

C neither growing nor decaying

D decaying at a rate of approximately 30 gm per year

E growing at a rate of approximately 30 gm per year

Question 20

The area of the region formed by the graph of the cosine graph, $y = \cos x$, and the coordinate axes, may be approximated by the shaded area (two trapeziums and one triangle) on the diagram at right.
The value of this approximation is

A 1 square unit

B $\dfrac{\pi}{6} (\sqrt{3} + 1)$ square units

C $\dfrac{\pi(\sqrt{3} + 1)}{12}$ square units

D $\dfrac{\pi(\sqrt{3} + 2)}{12}$ square units

E $\dfrac{\pi(\sqrt{3} + 3)}{12}$ square units

Question 21

An antiderivative of $3(x + 2)^2$ is

A $6x + 12$ B $3x^2 + 12x + 12$ C $(x + 2)^3$

D $x^3 + 6x^2$ E $x^3 + 6x^2 + 12$

Question 22

Correct to two decimal places, the value of $\int_0^1 e^{2x} \, dx$ is equal to

A 1.60 B 3.19 C 3.69

D 6.39 E 12.78

Question 23

The graph of $y = x^4 - 2x^3 + x^2$

A   touches the x-axis at 0, and cuts across the x-axis at 1

B   touches the x-axis at 1, and cuts across the x-axis at 0

C   touches the x-axis at 0, and cuts across the x-axis at 0

D   cuts across the x-axis at 0 and 1

E   touches the x-axis at -1 and 1

F   touches the x-axis at 0 and 1

Question 24

In the expansion of $(2x - 3)^4$, the coefficient of x^2 is equal to

A -216 B -96 C 36

D 96 E 216

Question 25

A set of seven integers has a mode of 2, a median of 3 and a mean of 4.
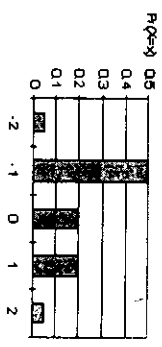If five of the integers are 1, 2, 3, 5 and 8, then the other two are

A 1 and 2   B 2 and 4   C 3 and 6

D 4 and 5   E 2 and 7

SECTION A - continued

Question 26

The column graph at right represents the probability
distribution for discrete random variable X.

The expected value of X is

A -1 D -0.3

B -0.5 E 0

C -0.4

**Questions 27, 28 and 29 refer to the random variable X, where X is the number of successes in a
binomial experiment, and the probability distribution is defined by the formula:**

$$\Pr(X = x) = {}^4C_x \, (0.1)^x \, (0.9)^{4-x}$$

Question 27

The expected value of X is

A 0.1 B 0.36 C 0.4

D 0.9 E 3.6

Question 28

The standard deviation for X is

A 0.3 B 0.36 C $\sqrt{0.4}$

D 0.6 E $\sqrt{1.2}$

Question 29

On 95% of repetitions of the binomial experiment, we would expect the number of successes to be

A none   B at most one   C at most two

D at most three   E any of 0, 1, 2, 3 or 4

Question 30

The amount of breakfast cereal in a box is labelled as 500 gm. It is found that the amount, packaged
by a particular machine, is **normally distributed**, with a mean of 505 gm and a standard deviation of
5 gm. The percentage of boxes packaged by this machine which are underweight (that is, contain less
than 500 gm of cereal) is approximately

A 30.9%   B 15.9%   C 6.7%

D 2.3%   E 0.6%

SECTION A - continued
PLEASE TURN OVER

Questions 31, 32 and 33 refer to a telephone poll, some years ago, taken of 1000 randomly selected Australian voters regarding the performance of the Prime Minister. At that time 49% of the sample approved of the Prime Minister's performance.

Question 31

49% is a

A	statistic	**B**	sample	**C**	parameter
D	population	**E**	sample parameter		

Question 32

The **standard error** of the proportion of the sample who approved of the Prime Minister's performance was approximately,

A	0·09	**B**	0·016	**C**	0·008
D	0·0005	**E**	0·00025		

Question 33

If the company conducting the telephone poll wished to be 95% confident that the population proportion approving the Prime Minister's performance was within 1% of the sample proportion, then

A a sample exceeding 20,000 would be required

B a sample exceeding 10,000 would be required

C a sample exceeding 5000 would be required

D a sample exceeding 2000 would be required

E the sample of 1000 is adequate
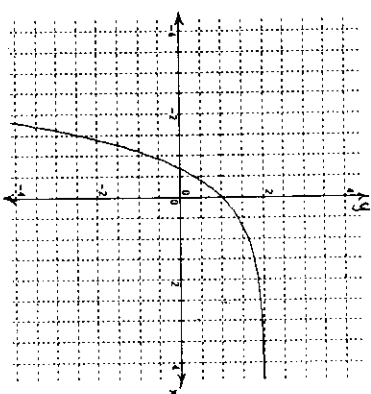
SECTION B

Specific Instructions for Section B

Section B consists of 6 questions. There are a total of 17 marks available.

Answer all six questions neatly on the lined paper provided, excepting question 1 (b), which should be answered on the back of the Section A Answer Sheet.

Please number each question clearly.

You need not give numerical answers as decimals unless instructed to do so.

Alternative forms may involve, for example, π, e, surds or fractions.

Full marks may not be given for answers which do not show appropriate working, or do not state answers clearly.

Question 1

The graph below represents the graph of the function f, where $f(x) = 2 - e^{-x}$.

It will be noted that the line defined by $y = 2$ is an horizontal asymptote to the graph.

(a) Algebraically locate the **precise** value of the x-intercept of the graph.

(b) On the graph grid on the back of your Section A Answer Sheet, sketch the graph of the **inverse function** f^{-1}.

Label the asymptote and the intercepts with the coordinate axes.

[1 + 2 + 2 = **5 marks**]

(c) State the **domain** and **rule** for the inverse function f^{-1}.

Question 2

(a) Find the **derivative** of $x \log_e x$.

(b) Use the result of part (a) to find an **antiderivative** of $\log_e x$.

[1 + 1 = **2 marks**]

Question 3

A rectangular dog run is to be built on flat ground from a length of 20 metres of wire fencing which forms three sides, the fourth side being part of a straight wooden fence.

If the width of the dog run is x m, then the area, A m², is given by the rule

$$A = x(20 - 2x).$$

(a) State the domain for this area function, that is, the set of allowable values for x.

(b) Find the **maximum area** of the dog run, carefully justifying your answer.
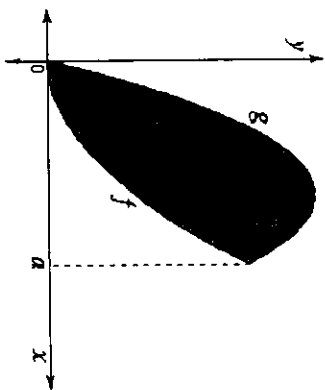
[1 + 2 = **3 marks**]

Question 4

The diagram at right shows the graphs of functions f and g where

$$f : [0,a] \rightarrow R, \ f(x) = \frac{x^2}{3} \ \text{and}$$

$$g : [0,a] \rightarrow R, \ g(x) = 4x - x^2 .$$

(a) Find the value of a where $f(a) = g(a)$, $a > 0$.

(b) Find the shaded area bounded by the graphs of f and g.

[1 + 2 = 3 marks]

Question 5

A four letter "word" is formed by selecting four letters at random from the alphabet (26 letters). Each letter may be chosen more than once. Find the probability that the "word" contains **at least one** of the five vowels (A, E, I, O or U).

[2 marks]

Question 6

A packager of breakfast cereal wishes to be 98% confident that each box will contain 500 gm of cereal as advertised on each packet. If the amount in each box packaged by a particular machine is **normally distributed** with a standard deviation of 5 gm, find the **mean weight** of cereal so that only 2% of the boxes filled by this machine are underweight (contain less than 500 gm).

[2 marks]

END OF QUESTION BOOKLET