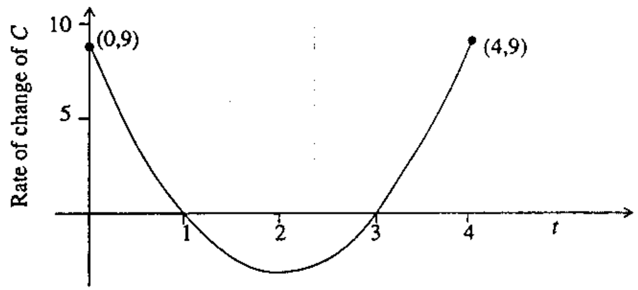
Specific Instuctions to Students
Answer **all** questions in the spaces provided

Question 1

The graph shows the approximate rate of change in the concentration of an enzyme over the first four hours of a digestion process. The concentration (C) is measured in milligrams per litre (mg/L), the time (t) is measured in hours (hr) and the rate of change in mg/L/hr.



a. i. Find the approximate rate of change of concentration when $t = 0.5$ hours.

[1 mark]

ii. Explain why the concentration has a local maximum when $t = 1$ and a local minimum when $t = 3$.

[2 marks]

b. Sketch the graph of the concentration of the enzyme (C) against the time (t), given that the initial concentration is 2 mg/L.

[3 marks]

c. Given that the concentration of enzyme is modelled by the function:

$$C(t) = at^3 + bt^2 + ct + d\ , \quad 0 \le t \le 4$$

and that all the previous information from parts **a.**, & **b.**, still hold, find a, b, c and d.

[6 marks]

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d. Find the maximum concentration and the time(s) when this occurs.

[2 marks]

e. Find the time(s) at which the concentration is 4 mg/L.

[5 marks]

Total marks 19

Question 2

A breed of small marsupial has a variety of distinctive features:

60% of the animals have brown fur and
40% have grey fur.

75% of the animals have short tails and
25% have long tails.

10% of the animals have long tails and grey fur.

a. State, with reasons, whether or not the two traits (fur colour and tail length) are independent.

[2 marks]

A sample of 10 of the marsupials is taken at random. Let X = the number of animals in the sample that have grey fur.

b. **i.** Find the values of E(X) and the standard deviation of X (correct to 2 decimal places).

[2 marks]

ii. Using the values in **i.**, give a rough sketch of the distribution of X.

[3 marks]

iii. What is the probability that exactly half the animals in the sample have grey fur? (Giving your answer to 3 d.p.)

[2 marks]

A random sample of 200 of a similar species was taken and in this sample, 95 animals had grey fur and the remainder had brown fur.

c. **i.** Find the approximate 95% confidence limits for the population proportion of this second species that have grey fur.

[3 marks]

ii. Does the second species have the same distribution of fur colours as the first? Explain your answer.

[2 marks]

The second species have weights that are normally distributed with a mean of 12.3kg and a standard deviation of 1.9kg.

d. **i.** Find the proportion of the animals that weigh more than 13.5kg.

[2 marks]

ii. Find the percentage of the animals that weigh between 12.0 and 14.0kg.

[2 marks]

iii. The lightest 10% of the animals are classified as being malnourished. Below what weight are the animals considered to be malnourished? (*Give your answer to 1 decimal place.*)

[2 marks]

Total marks 20

Question 3

a. For the function $y = xe^{-0.1x}, x \geq 0$,

i. Find $\frac{dy}{dx}$

[2 marks]

ii. Find the value of x for which $\frac{dy}{dx} = 0$.

[2 marks]

iii. What will happen to the value of $xe^{-0.1x}$ for very large values of x ?

[1 mark]

We define the demand function to represent the number of units, x, that consumers are willing to buy at a given price of p dollars per unit. The demand function for a particular product is modelled by

$$p(x) = 60e^{-kx}, x \geq 0 .$$

A demand of 100,000 units means that the price per unit will be $20.00.

b. **i.** Show that $k = 0.00001 \times \log_e 3$.

[2 marks]

ii. Find (to the nearest cent) the price per unit if 50,000 units are required.

[1 mark]

iii. On the set of axes shown below, sketch the graph of p versus x.

[2 marks]

iv. In one sentence, clearly describe what is happening in the graph you have sketched.

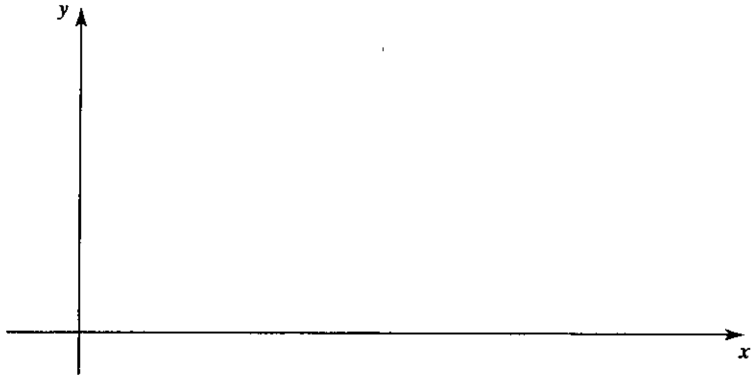
[1 mark]

The revenue after selling x units is modelled by the equation $R(x) = xp(x)$, $x \geq 0$.

c. Find the price per unit that will yield the maximum revenue. (Give your answer to the nearest cent.)

[4 marks]

d. Using part **a.**, and **c.**, to help you, sketch the graph of $y = R(x)$, $x \geq 0$ on the set of axes provided.

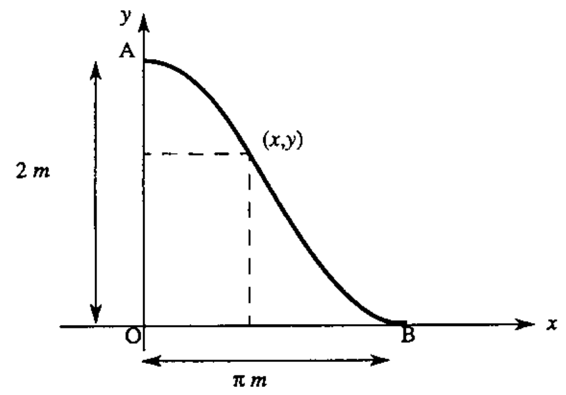


[3 marks]
Total 18 marks

Question 4

A slide at the local primary school was felt to be unsafe because of the increase in the speed that students would experience as they would slide down. Costing factors forced the school to alter the slide rather than replace it. The slide was altered by placing a straight ramp at the point on the slide where it was felt that the increase in speed was too great.
The original slide (shown below) had a cross–sectional shape that could be modelled by the equation:

$$y = a + b\cos x,\ 0 \le x \le \pi.$$



x and y are measured in metres, and represent the height and horizontal distance of any point on the slide relative to the origin O.

a. i. Write down the coordinates of the points A and B.

[1 mark]

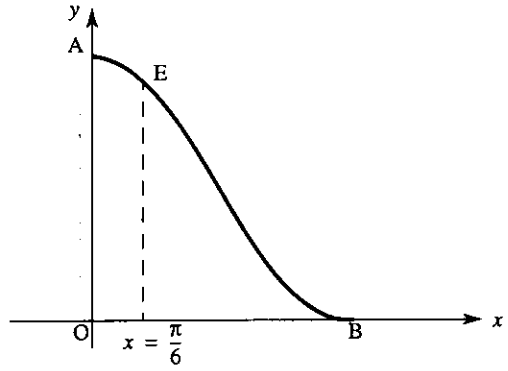
ii. Show that $a + b = 2$ and $a - b = 0$

[2 marks]

iii. Hence, determine the equation which models the cross section of the slide.

[2 marks]

The position on the slide, where it is felt that students will still enjoy the ride (even after the ramp is put into place) is at point E (as shown on the diagram), and is such that $x = \frac{\pi}{6}$.



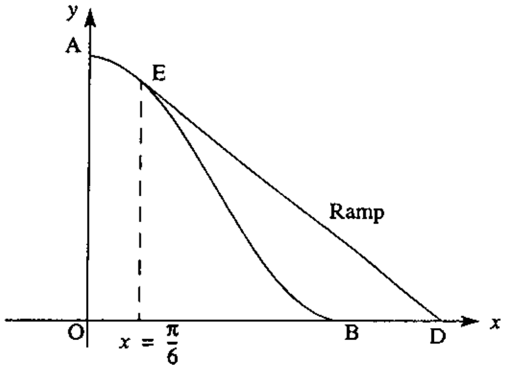
b. Find the slope of the slide at this point.

[2 marks]

A straight ramp will be placed at the point E so that children using the slide will have a smooth ride as they move from the original slide and onto the ramp.

c. The ramp is modelled by a straight line with equation $y = mx + c$, find the value of m and c (giving your value of c to 2 decimal places).

[3 marks]



d. In fact, the ramp is modelled by the equation

$$y = mx + c, \quad \frac{\pi}{6} \le x \le k.$$

Find the value of k (to 2 d.p.) and hence, state the coordinates of the point D, where the ramp meets the ground.

[3 marks]

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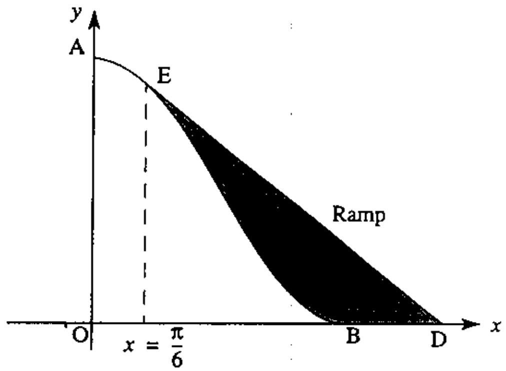

To add strength to the slide, a lightweight, yet very strong material is fitted between the slide and the ramp (as shown above).

e. Find the cross sectional area covered by this material.

[5 marks]

Total marks 18