M indicates a method mark awarded for use of a correct method
A indicates an answer mark for correct answer in the correct format.

Question 1

a. **i.** Read from graph (i.e. no tangent to be drawn). Allow between 2 & 5. [A]

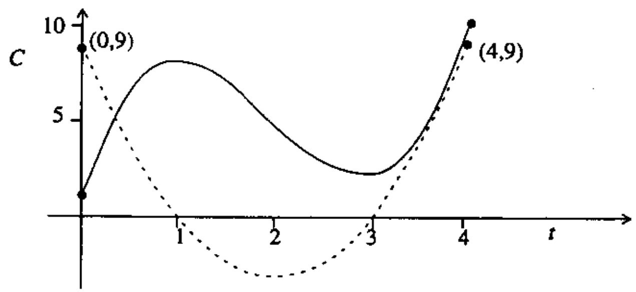
ii. Around $t = 1$ the rate goes from positive ($t < 1$) ↗ to zero at $t = 1$ ⟶ to negative ($t > 1$) ↘ .

The stationary point is a maximum. [A]

Around $t = 3$ the rate goes from negative ($t < 3$) ↘ to zero at $t = 3$ ⟶ to positive ($t > 3$) ↗ .

The stationary point is a minimum. [A]

b.



Maximum at 1 and minimum at 3. [A]
General shape. [A]
Correct domain, [0,4] [A]

c. The initial concentration is 2 so $d = 2$. [A]
There are three other pieces of information:
The initial rate of change is 9 (from the graph)

$C'(t) = 3at^2 + 2bt + c$

$C'(0) = c$

$= 9$

$\Rightarrow c = 9$ [A]

Also, there are stationary points at 1 & 3 so that:

$t = 1 \Rightarrow 3a + 2b + 9 = 0, [1]$ [M]

$t = 3 \Rightarrow 27a + 6b + 9 = 0, [2]$ [M]

Attempt to solve such as [2] - 2[1] [M]

$18a - 18 = 0 \Rightarrow a = 1$ [A]

$3 + 2b + 9 = 0$

$2b = -12$

$b = -6$ [A]

$\therefore C(t) = t^3 - 6t^2 + 9t + 2 \qquad 0 \le t \le 4$

d. We already know that t = 1 is a local maximum so no calculus is necessary:
$C(1) = 6$ [A]
It is also necessary to check the end-points of the domain to find overall maxima and minima:
$C(0) = 2, C(4) = 6$ so the maximum also occurs at $t = 4$. [A]

e. Equating their equation for C to 4. [M]

$C(t) = t^3 - 6t^2 + 9t + 2 = 4$

$P(t) = t^3 - 6t^2 + 9t - 2 = 0$

$P(2) = 0 \Rightarrow t - 2$ is a factor [A]

$$\begin{array}{r|l} & t^2 - 4t + 1 \\ t-2 & t^3 - 6t^2 + 9t - 2 \\ & t^3 - 2t^2 \\ & -4t^2 + 9t \\ & -4t^2 + 8t \\ & t - 2 \end{array}$$ [M]

Solving their quadratic quotient by an appropriate method. [M]

$t^2 - 4t + 1 = 0$

$t = \frac{4 \pm \sqrt{16 - 4}}{2}$

$t = 2 \pm \sqrt{3}$

The required times are 2, 0.27 and 3.7 hours. [A]

Question 2

a. Use of the definition of independence: $Pr(A \cap B) = Pr(A) \times Pr(B)$ [M]

$Pr(\text{grey fur \& long tail}) = 0.1$

$Pr(\text{grey fur}) \times Pr(\text{long tail}) = 0.4 \times 0.25$

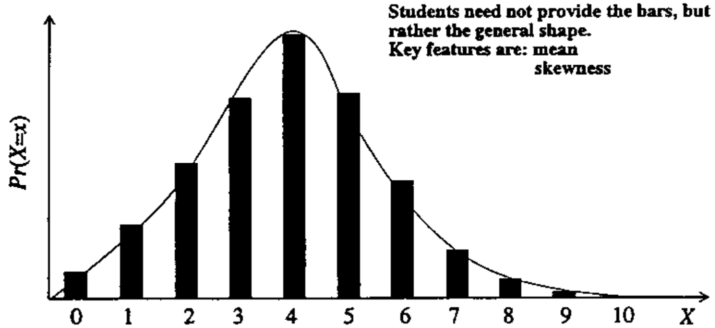
$= 0.1$

The traits are independent. [A]

b. i. $E(X) = np = 10 \times 0.4 = 4$ [A]

$SD(X) = \sqrt{npq} = \sqrt{10 \times 0.4 \times 0.6} \approx 1.55$ [A]

ii.



Tallest bar at 4. [A]
Positive skew [A]
Indication that 9 & 10 are very rare. [A]

iii. Use of the binomial distribution. [M]

$Pr(X=5) = {}^{10}C_5 \times 0.6^5 \times 0.4^5$

$= 0.2007$ (0.201) [A]

c. i. $se(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$ [M]

$= \sqrt{\frac{0.475 \times 0.525}{200}}$

$= 0.0353$ [A]

The ~95% confidence interval is $0.475 \pm 2 \times 0.0353 \rightarrow 0.404$ to 0.546 [A]

ii. No, at the 95% confidence level. [A]

 VMM3S

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This is because the first species has a population proportion of animals with grey fur of 0.4 and this does not lie in the 95% confidence interval for the second sepecies. [A]

d. i.

Convert to a standard problem: $z = \frac{x-\mu}{\sigma} = \frac{13.5 - 12.3}{1.9} = 0.632$ (to 3 dec. pl.). [M]

$Pr(X > 13.5) = Pr(Z > 0.632) = 1 - Pr(Z < 0.632) = 1 - 0.7363 = 0.2637$ [A]

ii.

$Pr(12.0 < X < 14.0) = Pr(X < 14.0) - Pr(X < 12.0)$

$= Pr\left(Z < \frac{14 - 12.3}{1.9}\right) - Pr\left(Z < \frac{12 - 12.3}{1.9}\right)$

$= Pr(Z < 0.895) - Pr(Z < -0.158)$ [M]

$= Pr(Z < 0.895) - Pr(Z > 0.158)$

$= Pr(Z < 0.895) - (1 - Pr(Z < 0.158))$

$= 0.8147 - (1 - 0.5628)$

$= 0.3775$

Approx 37.75% of the animals have weights in this range. [A]

iii. The lightest 10% have a Z value of -1.282 (look up 0.900 in the body of the table). [M]

$-1.282 = \frac{X - 12.3}{1.9}$

$X = 1.9 \times -1.282 + 12.3$

$= 9.86$ kg [A]

 VMM3S

Question 3

a. **i.** Using the product rule, we have:

$$\frac{dy}{dx} = 1 \times e^{-0.1x} + x \times (-0.1e^{-0.1x})$$ [M]

$$= e^{-0.1x} - 0.1xe^{-0.1x}$$

$$= e^{-0.1x}(1-0.1x)$$ [A]

ii. $e^{-0.1x}(1-0.1x) = 0$

$e^{-0.1x} = 0$ or $(1-0.1x) = 0$ [M]

No real solution to $e^{-0.1x} = 0$.

Therefore only real solution is $x = \frac{1}{0.1} = 10$ [A]

iii. As the values of x increase, the value of $xe^{-0.1x}$ will decrease. [A]

(In fact, we can say that $\lim_{x \to \infty} xe^{-0.1x} = 0$)

b. **i.** Solving for k we have:

$$20 = 60e^{-100000k}$$ [M]

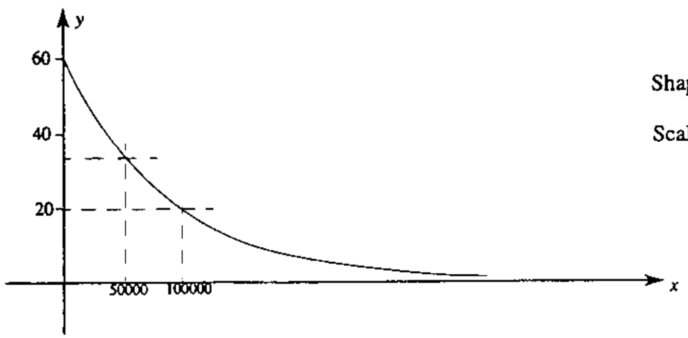
$$-100000k = \log_e\left(\frac{1}{3}\right) = [-\log_e 3]$$

$$k = 0.00001 \times \log_e 3$$ [M]

ii. For $x = 50000$, we have $p = 60e^{-50000 \times 0.00001 \log_e 3}$

≈ 34.64 [A]

iii.



Shape [A]

Scale [A]

iv. As the demand increases, the price per unit decreases. [A]

c. $R'(x) = xp'(x) + 1 \times p(x) = -x \times 60ke^{-kx} + 60e^{-kx}$ [M]

$= 60e^{-kx}(-kx+1)$

$R'(x) = 0 \Leftrightarrow 60e^{-kx}(-kx+1) = 0$ [M]

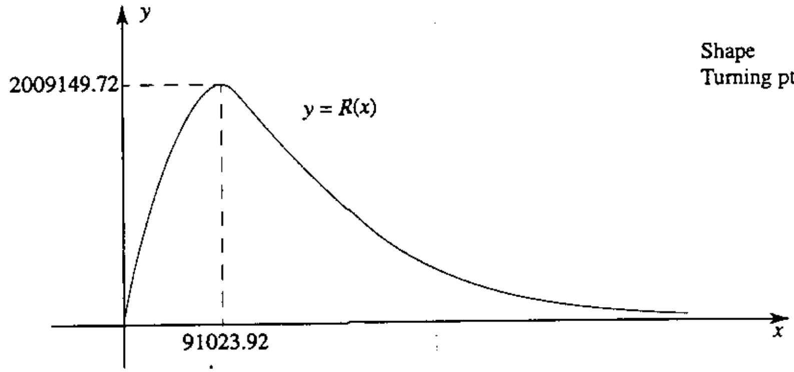
$\Leftrightarrow x = \frac{1}{k}$

That is, $x = \frac{1}{0.00001 \times \log_e 3} \approx 91023.92$ [A]

Therefore, price per unit is $p\left(\frac{1}{k}\right) = 60e^{-k \times \frac{1}{k}} = 60e^{-1}$

$\approx \$22.07$ [A]

d.



Shape [A]
Turning pt [A]
[A]

Question 4

a. **i.** A(0,2), B(π,0) [A]

ii. From A: $2 = a + b\cos(0)$, giving $2 = a + b$ [M]
From B: $0 = a + b\cos(\pi)$, giving $0 = a - b$ [M]

iii. Solving for a and b, we have: $2 = 2a$, so that $a = 1$ and $b = 1$ [M]
Therefore, we have that $y = 1 + \cos x \quad 0 \le x \le \pi$ [A]

b. $\left.\frac{dy}{dx}\right|_{\frac{\pi}{6}} = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$ [M] [A]

c. At E, $y = 1 + \cos\left(\frac{\pi}{6}\right) = 1 + \frac{\sqrt{3}}{2}$.
Substituting into $y = mx + c$, we have:

$$1 + \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{\pi}{6}\right) + c \Leftrightarrow c = 1 + \frac{\sqrt{3}}{2} + \frac{\pi}{12} \approx 2.13$$ [M] [A]

Therefore, we have:

$$y = -\frac{1}{2}x + 2.13$$ [A]

d. At the point D we have $y = 0$. Therefore we can solve for at $x = k$, $y = 0$. [M]

$$0 = -\frac{1}{2}k + 1 + \frac{\sqrt{3}}{2} + \frac{\pi}{12} \Leftrightarrow k = 2 + \sqrt{3} + \frac{\pi}{6} \approx 4.26$$ [M]

Therefore, the coordinate of D is (4.26, 0) [A]

e. Required area (in appropriate units) is given by:

$$\int_{\frac{\pi}{6}}^{4.26}\left(-\frac{1}{2}x + 2.13\right)dx - \int_{\frac{\pi}{6}}^{\pi}(1 + \cos x)dx$$ [M] [M]

$$= \left[-\frac{1}{4}x^2 + 2.13x\right]_{\frac{\pi}{6}}^{4.26} - [x + \sin x]_{\frac{\pi}{6}}^{\pi}$$ [A] [A]

≈ 1.36 (allow between 1.3 and 1.4) [A]