No. 25

QUESTION ONE (15 marks)

Consider the function:

$$f(x) = \frac{1}{4000}(x + 40)(x - 50)^2$$

(a) Find the y-intercept for the graph of $y = f(x)$.

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(1 mark)

(b) State at which value(s) of x , the graph of $y = f(x)$ crosses or touches the x - axis.

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(1 mark)

(c) Calculate $f'(x)$ ie. $\frac{dy}{dx}$

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(2 marks)

(d) Give the coordinates of all the turning points and briefly explain why your points are turning points.

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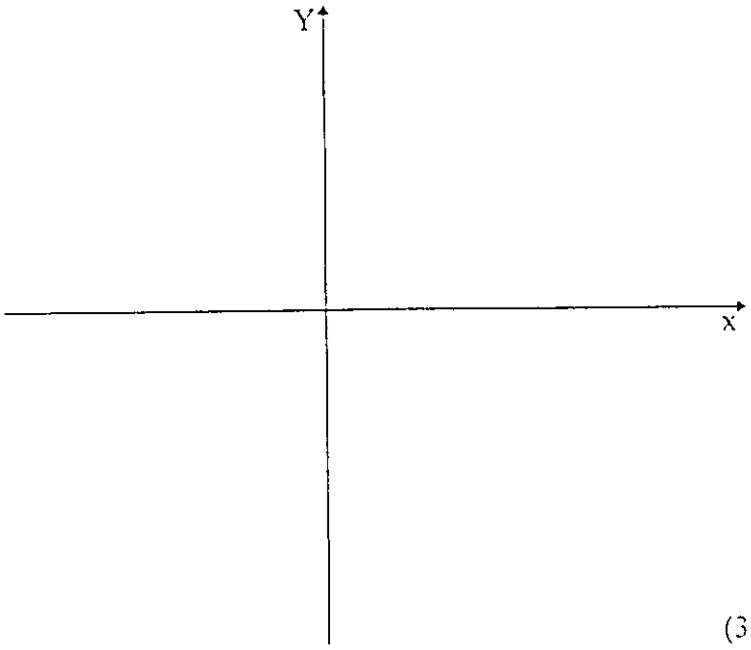
(2 marks)

(e) On the axis below, sketch the graph of $y = f(x)$ carefully labelling the important points. (You do not need to show the position of inflexion points.)



(3 marks)

A new waterslide is designed for Ocean World and the side view of the slide is given by

$$y = \frac{1}{4000}(x+40)(x-50)^2 \qquad \text{where} \quad 0 \le x \le 50$$

where y = the height of the slide in metres

x = the horizontal length of the slide in metres.

(f) State the maximum height of the slide.

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(1 mark)

(g) Find the gradient at the top of the slide.

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(1 mark)

(h) Calculate the average gradient of the slide.

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(1 mark)

(i) Calculate $\frac{d^2y}{dx^2}$ or $f''(x)$.

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(2 marks)

(j) Find where the slide is steepest.

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(1 mark)

QUESTION TWO (15 MARKS)

For this question, unless stated otherwise, give probabilities to an accuracy of three decimal places.

Regal Casino examined the statistics of its patrons. The amount lost by its customers followed a normal distribution with mean of $60 and standard deviation $30.

Assuming that these statistics are accurate,

(a) find the probability that a customer will lose at least $90.

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(1 mark)

(b)Find the probability that a customer will make a profit.

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(2 marks)

(c) Calculate the probability that a patron loses between $10 and $50.

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(2 marks)

A group of five friends visit the Regal Casino and they all gamble.

(d) What is the probability that only one of the group wins on this visit?(A winner here means a person who makes a profit on a visit.)

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(1 mark)

(e) Calculate the probability that the majority of the group will leave, having lost money.

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(2 marks)

The individuals lost amounts of $10, $20, $30, $50 and one gained $20.

(f) Calculate the average loss for the group.

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(1 mark)

(g) Using $s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$, calculate the standard deviation of the loss for the group.

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(2 marks)

(h) Give the probability that for a random group of five gamblers, all will lose less than $60.

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(2 marks)

(i) Is it reasonable to conclude that the five friends had a lucky night?

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(2 marks)

QUESTION THREE (15 MARKS)

The percentage of available power in a rechargeable battery changes with time according to the rule:

$P = A\log_e(3.5-t)$

where P = the percentage of available power

A = a constant value

t = the time in hours

Initially the battery has 100% power available.

(a) Find the value of A correct to two decimal places.

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(2 marks)

(b) If the percentage of power always remains between 0% and 100%,and includes these endpoints, what restrictions are there on the domain (t) ?

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(2 marks)

(c) Give the percentage of power available after two hours of operation. Your answer should be correct to one decimal place.

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(1 mark)

(d) State the rate of change of P .

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(2 marks)

(e) Find the rate of change of P after two hours of battery operation.

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(1 mark)

(f) What percentage of power is lost in the last hour of battery operation? (ie in the hour prior to a power level of 0%)

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(2 marks)

(g) How long does it take the battery to drop to 50% of its available power?

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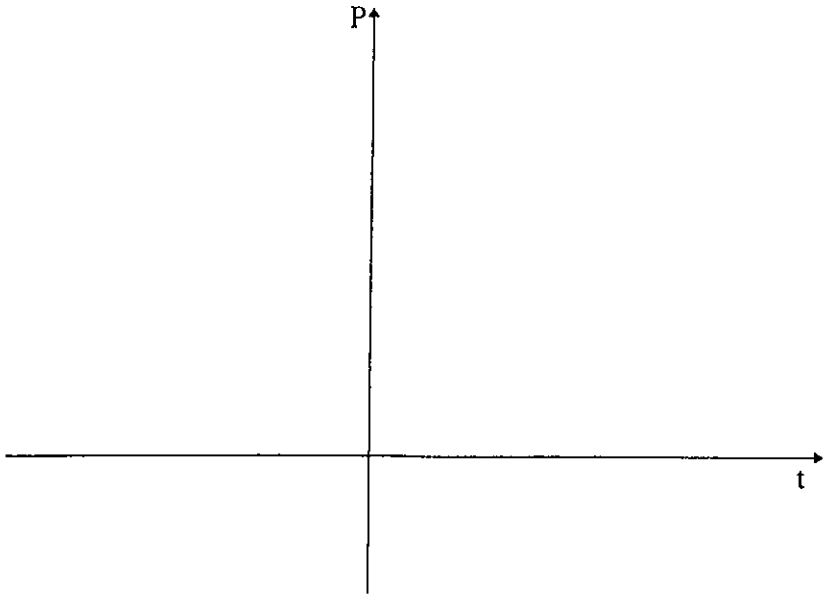
(2 marks)

A heavy duty battery has its available power changing according to the rule

$$P = 43.43\log_e(10 - t)$$

(h) On the axes below sketch the graph of this function.(The function should be restricted so that $0 \le P \le 100$.)



(3 marks)

QUESTION FOUR (15 MARKS)

An object is moving in a straight line. Its position, x metres , from an origin at time t seconds is given by:

$$x = e^{-t}\sin t, \quad \text{where} \quad 0 \le t \le 4\pi$$

(a) Show that the velocity, v, of the object can be written as

$$v = e^{-t}(\cos t - \sin t).$$

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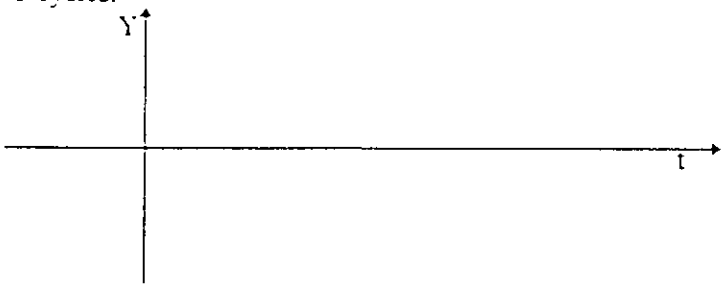
(1 mark)

(b) On the axes below sketch and label the graphs of y = cos t and y = sin t for two cycles.



(2 marks)

(c) Using parts (a) and (b) or otherwise, find the times when the object comes to rest.

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(1 mark)

(d) State the times when the object will be at the origin.

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(1 mark)

(e) Where is the object after 3 seconds and what direction is it travelling?

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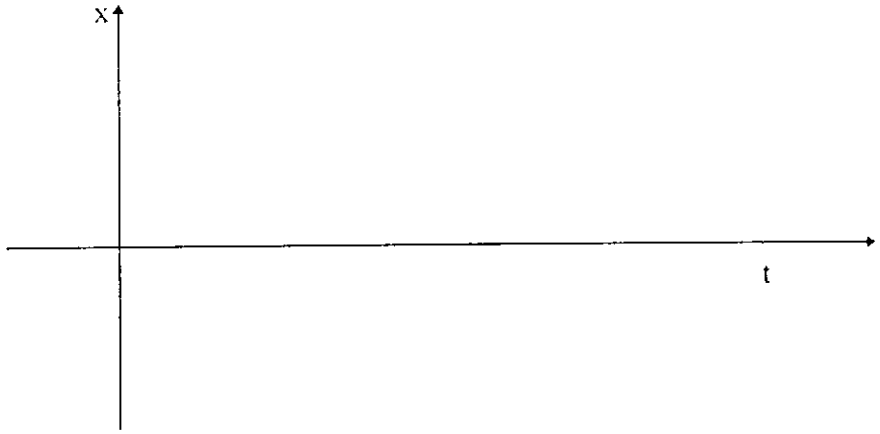
(2 marks)

(f) Sketch the graph of x(t) showing turning points and axes intercepts.



(3 marks)

(g) What is the total distance that the object has moved between the times $t = 0$ and $t = 2\pi$?

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(3 marks)

END OF BOOKLET.