YEAR 12
IARTV TEST — OCTOBER 1996
MATHEMATICAL METHODS CAT 3
ANSWERS & SOLUTIONS

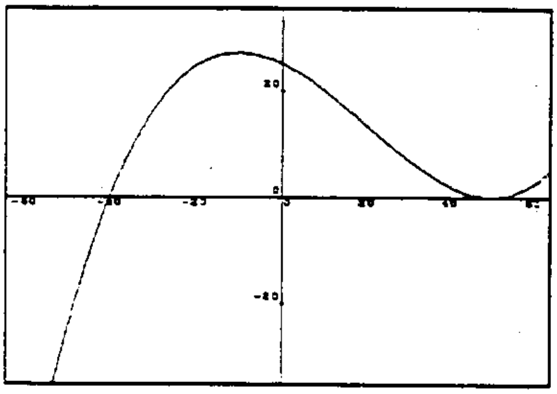
1(*a*) $x = 0, f(0) = 25$

(*b*) $x = -40,\ 50$

(*c*) $f'(x) = \frac{3}{4000}(x - 50)(x + 10)$

(*d*) (50, 0) and (-10, 27)

cubic functions have either one inflection point or two turning points, since there are two points which have zero gradient then the cubic function will have two turning points.



1(*f*) 25metres

(g) $f'(0) = -\frac{3}{8}$

(*h*) *average gradient* $= -0.5$

(*i*) $f''(x) = \frac{3(2x - 40)}{4000} = \frac{3(x - 20)}{2000}$

(*j*) The gradient has its largest value when $f''(x) = 0$ ie. when $x = 20$.

2. $X =$ *loss on one visit*

(*a*) $\Pr(X \geq 90) = \Pr(Z > 1)$

$= 1 - \Pr(Z < 1) = 0.159$

(*b*) $\Pr(X < 0) = \Pr(Z < -2)$

$= 1 - \Pr(Z < 2) = 0.023$

(*c*) $\Pr(10 < X < 50)$

$= \Pr(\frac{-5}{3} < Z < \frac{-1}{3})$

$= \Pr(Z < \frac{5}{3}) - \Pr(Z < \frac{1}{3})$

$= 0.322$

(*d*) $p = \Pr($*gambler makes profit*$)$

From 5 gamblers the

$\Pr($*just one profits*$)$

$= {}^5C_1 p(1-p)^4 = 0.105$

(*e*) $\Pr($*majority lose*$) = {}^5C_0(1-p)^5$

$+ {}^5C_1 p(1-p)^4 + {}^5C_2 p^2 (1-p)^3$

$= 0.998$

(*f*) *total loss* $= \$90$

average loss $= \$18$

(*g*) $s^2 = 670 \Rightarrow s = 25.9$

(*h*)(1) $\Pr($*loss* $< \$60) = 0.5$

$\Pr($*all* 5 *lose* $< \$60) = \frac{1}{32}$

(2) on this visit all 5 lost less than \$60. The chance of this happening is $1/32$ which is infrequent; thus the group could be considered lucky. However more information should be requested to confirm this.

$3(a)\ 100 = A\log_e 3.5 \Rightarrow A = 79.82$

(b) since $0 \le P \le 100 \Rightarrow 0 \le t \le 2.5$

$(c)\ P(2) = 79.82\log_e 1.5 = 32.37$

$(d)\ \dfrac{dP}{dt} = -\dfrac{A}{3.5-t}$

(e) *when* $t = 2,\ \dfrac{dP}{dt} = -53.22$

(f) *from* $t = 1.5$ *to* $t = 2.5$ *there is a loss of* $P = 79.82\log_e 2 = 55.32$

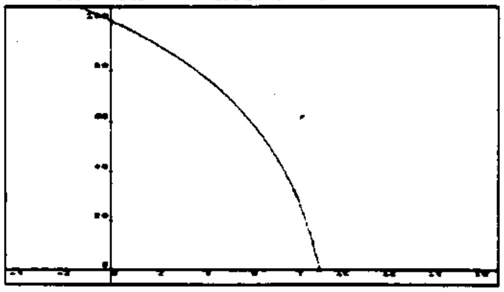
(g) *If* $P = 50,$

$$50 = \frac{100}{\log_e 3.5}\log_e(3.5-t)$$

$$0.5\log_e 3.5 = \log_e(3.5-t)$$

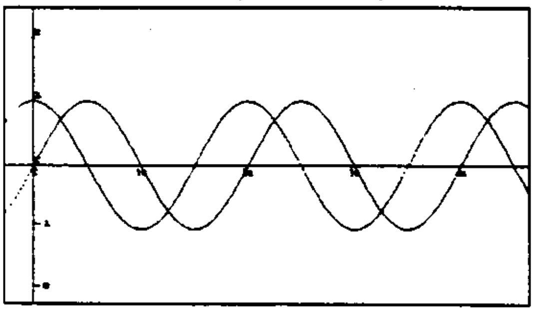
$$3.5 - t = 3.5^{0.5}$$

$$t = 3.5 - 3.5^{0.5} = 1.63 \textit{hours}$$



$$4(a)\ v = \frac{dx}{dt} = -e^{-t}\sin t + e^{-t}\cos t$$

$$= e^{-t}(\cos t - \sin t)$$



(c)The object comes to rest when $\cos t = \sin t$, and this is at the point of intersection of the 2 curves,

$$t = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

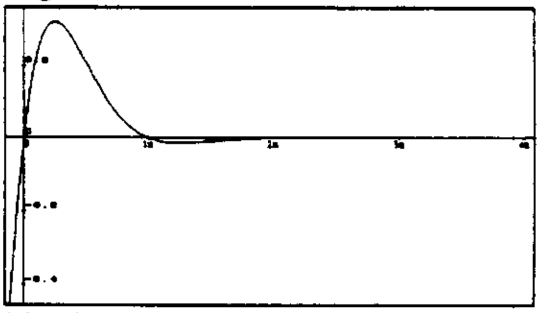
(d) The object will be at the origin when $x = 0$. ie. when $\sin t = 0$

$t = 0, \pi,\ 2\pi,\ 3\pi,\ 4\pi.$

$(e)\ x = e^{-3}\sin 3 = 0.007,$

$v = e^{-3}(\cos 3 - \sin 3) = -0.056$

so the particle is at 0.007cm from the origin and moving towards the negative direction.



The sketch graph would give the additional turning points and intercepts.

(f) Total distance travelled

$$= 2\left(\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}} + \frac{\sqrt{2}}{2}e^{-\frac{5\pi}{4}}\right)$$

$= 0.6727$