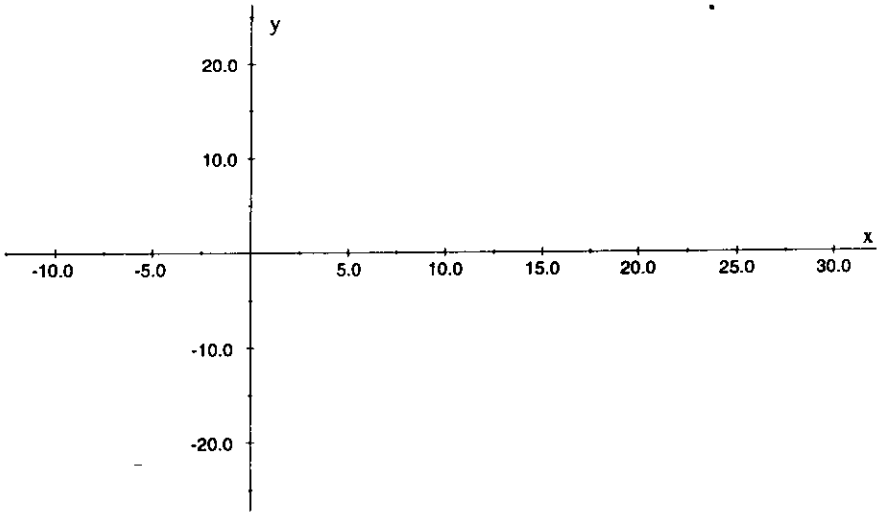
Question 1

a. On the set of axes provided below, sketch the curve with equation $f(x)=\frac{25}{10-x}, x \neq 10$.

Clearly showing all axial intercepts and labelling all asymptotes.



3 marks

b. If $g(x)=\frac{25x}{10-x}, x \neq 10$, show that

i. $g(x)=\frac{250}{10-x}-25, x \neq 10$

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ii. $g(x)=10f(x)-25$

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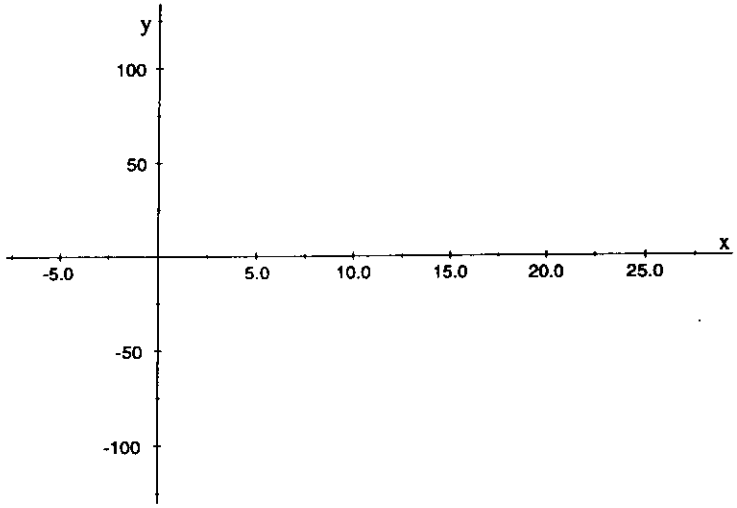
3 marks

15

c. On the set of axes provided below, sketch the graph of $g(x)=\frac{25x}{10-x}, x \neq 10$.



2 marks

It is found that the cost , $ C, in millions of dollars for a municipality to remove x% of industrial pollutants discharged into a river system is approximately modelled by

$$C(x)=\frac{25x}{100-x}, 0 \leq x < 100$$

d. Find the cost of removing 60% of industrial pollutants discharged into a river system.

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1 marks

The municipality can only allocate $ 10 million for the removal of industrial pollutants from their river system.

e. What percentage of pollutants remain in the river system after the clean up process ?

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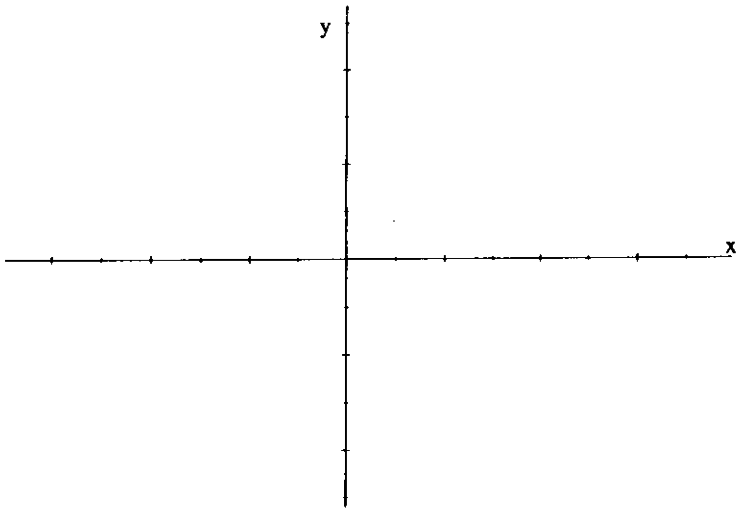
2 marks

f. Sketch the graph of $C(x) = \dfrac{25x}{100 - x}, 0 \le x < 100$ on the set of axes shown below.



2 marks

g. Find an expression for the inverse function $C^{-1}(x)$.

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2 marks

h. If $N(u) = C^{-1}(u)$, what physical property does $N(u)$ represent ?

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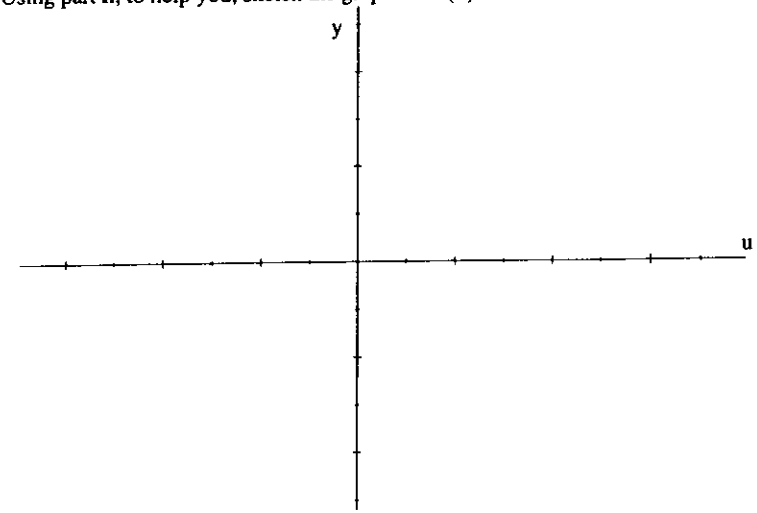
1 mark

i. Using part **f.**, to help you, sketch the graph of $N(u)$ on the set of axes shown below.



2 marks

j. If possible, how much would it cost to completely remove all pollutants from the river system ? Explain your answer.

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2 marks

Total 20 marks

Question 2

For the National Junior Skateboard Championship a ramp has been built so that the participants can display their many talents. The cross section of this ramp is made up of three parts, and is modelled by the equation

$$R(x)=\begin{cases}\frac{1}{9}(3-x)^3 & for \quad 0\le x<3\\ \frac{1}{3}(x-3)^3 & for \quad 3\le x<4\\ mx+c & for \quad 4\le x\le a\end{cases}$$

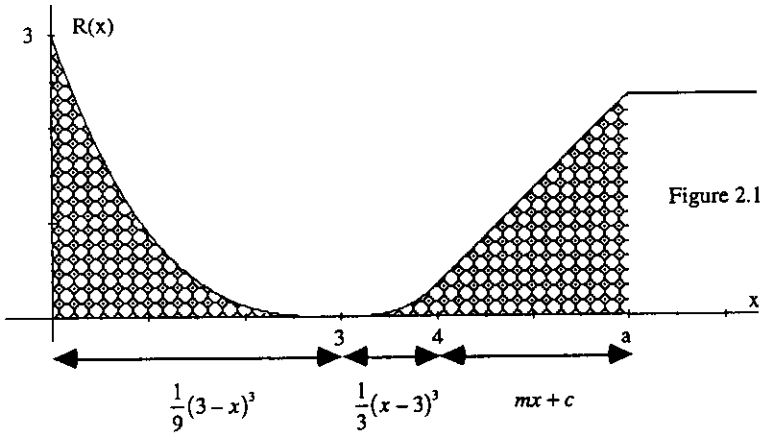

Figure 2.1

A hardened foam material is used to fill the space beneath the surface of the ramp over the region $0 \le x \le a$.

Let A_1 denote the area beneath the section of the curve $y=\frac{1}{9}(3-x)^3, 0\le x<3$ and $y=\frac{1}{3}(x-3)^3, 3\le x<4$ that is approximated by using the rectangles shown in Figure 2.2.

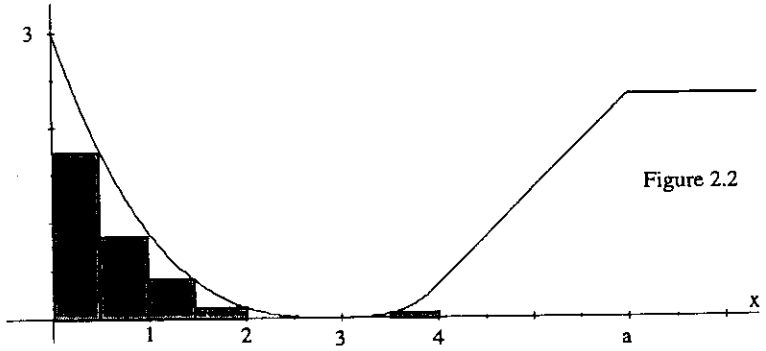

Figure 2.2

a. Find the value of A_1.

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2 marks

Let A_2 denote the area 'beneath' the section of the curve $y=\frac{1}{9}(3-x)^3, 0\le x<3$ and $y=\frac{1}{3}(x-3)^3, 3\le x<4$ that is approximated by using the rectangles shown in Figure 2.3.

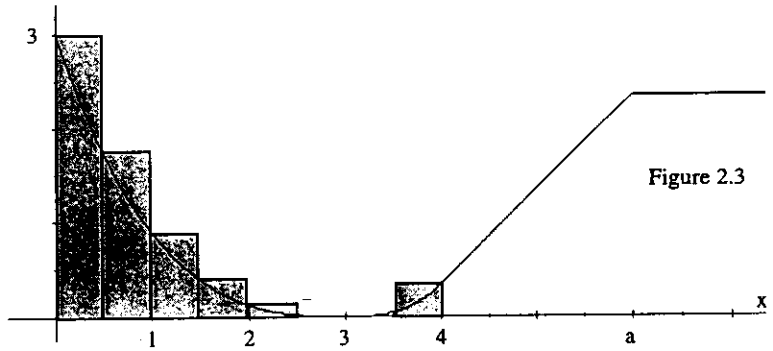

Figure 2.3

b. Find the value of A_2.

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2 marks

c. Show that the true cross sectional area, A , beneath the ramp, from x = 0 to x = 4 is such that $A_1 < A < A_2$.

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4 marks

d. Find the numerical value of the gradient of the curve at the point where x = 4.

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3 marks

At x = 4, a straight ramp is to be placed with equation $y = mx + c, 4 \le x \le a$, so that the participants have s *smooth* ride as they move from the curved section of the ramp onto that region which is straight.

e. **i.** Give a reason why the value of m is 1.

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ii. Hence, show that the equation of the striaght line is $y = x - \frac{11}{3}, 4 \le x \le a.$.

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3 marks

f. If the straight ramp reaches a maximum vertical height of 2.5 metres, show that $a = \frac{37}{6}$.

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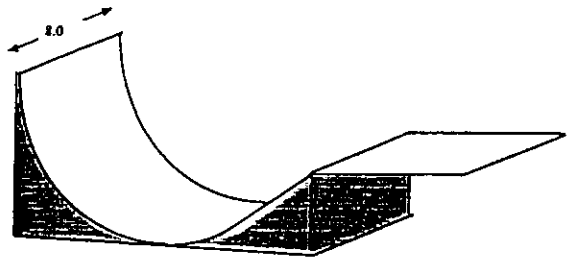
1 mark

Foam is only to be placed over the interval $0 \le x \le \frac{37}{6}$ (that is, until the platform becomes horizontal).



The width of the ramp is 8 units.

g. **i.** Determine the cross sectional area of the foam that is used.

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4 marks

ii. Hence determine the volume of foam that is reqired to complete this task.

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1 mark

Total 19 marks

Question 3

The blood pressure, P(t) millimeters of mercury, at time t seconds for a person at rest is appoximated by the equation

$$P(t) = 10\left(10 - 2\cos\left(\frac{5\pi}{3}t\right)\right)$$

a. Find

i. the period of the function

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ii. the amplitude of the function

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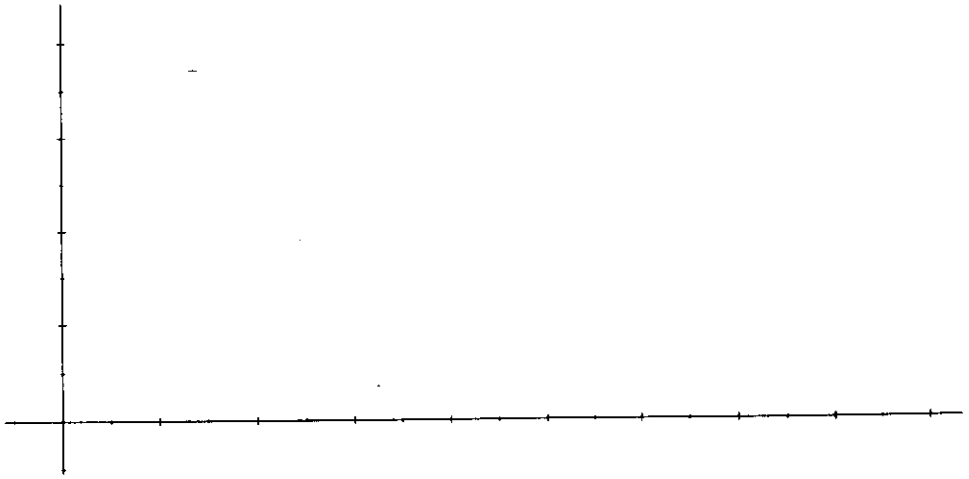
2 marks

b. On the set of axes provided, sketch the graph of the pressure function over a period of 3.6 seconds.



3 marks

c. For what percentage of the time can you expect the blood pressure to be at least 110 millimeters ?

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3 marks

Total 8 marks

Question 4

A study was carried out at Moorewood Secondary College to determine the amount of time spent on homework per subject per night by year 12 students. It was found that the times per subject were approximately normally distributed with mean 40 minutes and standard deviation 6 minutes.
Define the random variable X to measure the time spent by students on homework (per subject).

a. If $X \overset{d}{=} N(\mu, \sigma^2)$, what are the values of

i. μ

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ii. σ

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2 marks

b. Find the probability that the time taken to complete the homework set for one particular subject exceeds 50 minutes.

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3 marks

c. What is the probability that the student completes the homework for a particular subject within 45 minutes ?

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2 marks

On a particular night a student is given homework for only three subjects.

d. What is the probability that a student completes the homework for the three subjects, each one within 45 minutes ?

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2 marks

On the following night, the student is set homework in five subjects.

e. What is the probability that this student completes the homework for only three of the subjects each one within 45 minutes ?

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2 marks

f. A student starts to work on the homework for one subject at 7.00 p.m. When should dinner be served so that there is a 90% chance that the homework for that subject is completed ? Give your answer to the nearest minute.

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3 marks

g. What is the probability that it will take at most 2.5 hours to complete the homework that has been set for only three subjects ?

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4 marks

Total 18 marks