YEAR 12

IARTV TEST — OCTOBER 1995

MATHEMETICAL METHODS Units 3 and 4

CAT 2 — Facts and Skills Task

SECTION A — ANSWERS & SOLUTIONS

NAME: SOLUTIONS

Question 1	D	Question 12	D	Question 23	C
Question 2	C	Question 13	E	Question 24	B
Question 3	A	Question 14	A	Question 25	A
Question 4	D	Question 15	B	Question 26	B
Question 5	E	Question 16	D	Question 27	C
Question 6	A	Question 17	B	Question 28	C
Question 7	D	Question 18	E	Question 29	D
Question 8	E	Question 19	A	Question 30	C
Question 9	E	Question 20	A	Question 31	B
Question 10	C	Question 21	D	Question 32	D
Question 11	B	Question 22	B	Question 33	D

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MATHEMETICAL METHODS Units 3 and 4

CAT 2 — Facts and Skills Task — SECTION B — ANSWERS & SOLUTIONS

1 $f(x) = x^4 - 2x^3 = x^3(x - 2)$

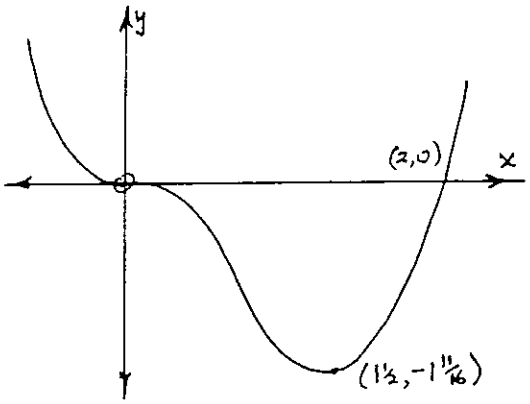
Hence • inflexion at (0,0)

• intercept at (2,0)

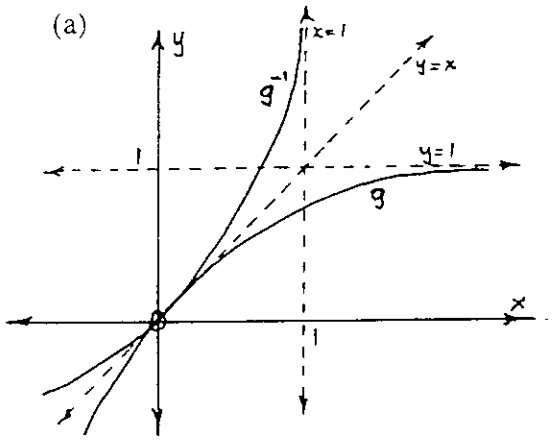
$f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$

Hence • stationary point of inflexion at (0,0)

• local minimum at $x = 1\frac{1}{2}$



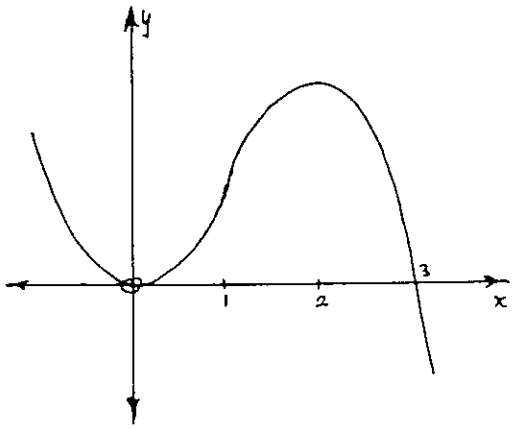
2 (a)



(b) $g^{-1} : (-\infty,1) \to \mathbb{R}$,

$g^{-1}(x) = -\log_e(1 - x)$

3



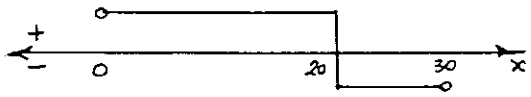
4 (a) $\{x : 0 < x < 30\} = (0,30)$

(b) $V(x) = 360x^2 - 12x^3$

$V'(x) = 720x - 36x^2$

$= -36x(x - 20)$

Sign diagram for $V'(x)$:



Hence maximum when $x = 20$.

Hence dimensions for maximum volume are: depth 20 cm, length 60 cm, width 40 cm.

5 Let X be number of pins landing "point up".

$X \sim \text{Bi}(5,\frac{2}{3})$

(a) $\Pr(X = 2) = {}^5C_2(\frac{2}{3})^2(\frac{1}{3})^3$

$= \frac{40}{243} \approx 0.165$

(b) $\Pr(X \geq 1) = 1 - \Pr(X = 0)$

$= 1 - (\frac{1}{3})^5$

$= \frac{242}{243} \approx 0.996$

6 Let X ml be amount of drink in cup

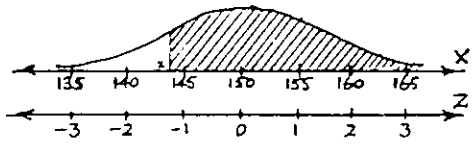
$X \sim \text{N}(150,5^2)$

(a) $\Pr(X > 165) = \Pr(Z > 3)$

$= 1 - 0.9987$

$= 0.0013$

(b) $\Pr(X > x) = 90\% = 0.9$



$\Pr(Z < 1.2816) = 0.9$

$\Rightarrow \Pr(Z > -1.2816) = 0.9$

Hence $x = 150 - 1.2816 \times 5$

$= 143.592$

That is, 90% of cups contain more than 143.592 ml.