SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

Victorian Certificate of Education 1994

MATHEMATICAL METHODS

Common Assessment Task 3: Analysis task

Wednesday 9 November 1994: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOKLET

Structure of booklet

Number of questions	*Number of questions to be answered*
4	4

Directions to students

Materials
Question and answer booklet of 12 pages, including 2 blank pages for rough working. There is a detachable sheet of miscellaneous formulas in the centrefold.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the centre of this booklet during reading time.
Ensure that you write your **student number** in the space provided on the cover of this booklet.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 60 marks available for the task.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task
Hand in this question and answer booklet.

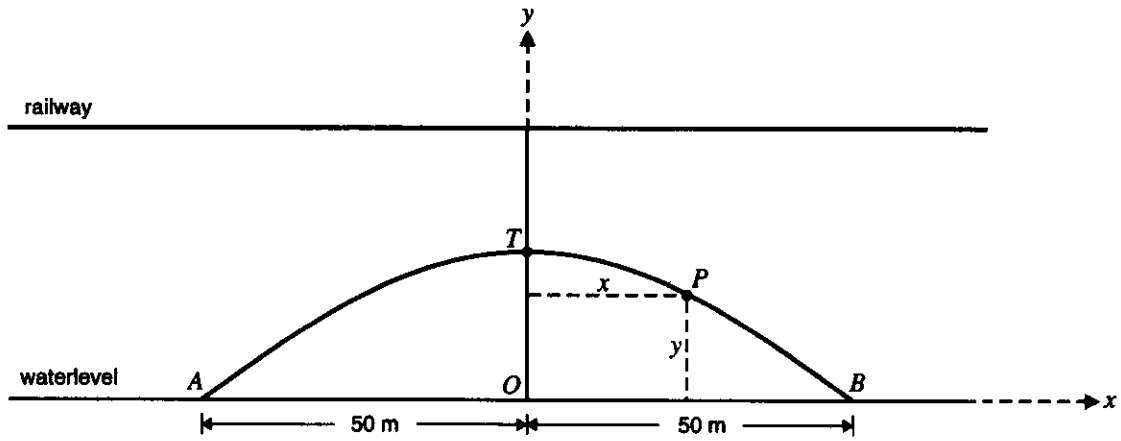
Question 1

A railway bridge over a river in countryside Victoria has a central arch. In the diagram below, the straight line AB indicates the water level under the bridge, and the curved line ATB represents the underside of the arch, and T is the top of the curve ATB.

The arch has shape given by the relation

$$y = -0.01x^2 + 25$$

where y metres is the distance from a point on the curve AB to the line AB, and x metres is the distance of the point P from the line OT.



a. What is the height of T above the water level?

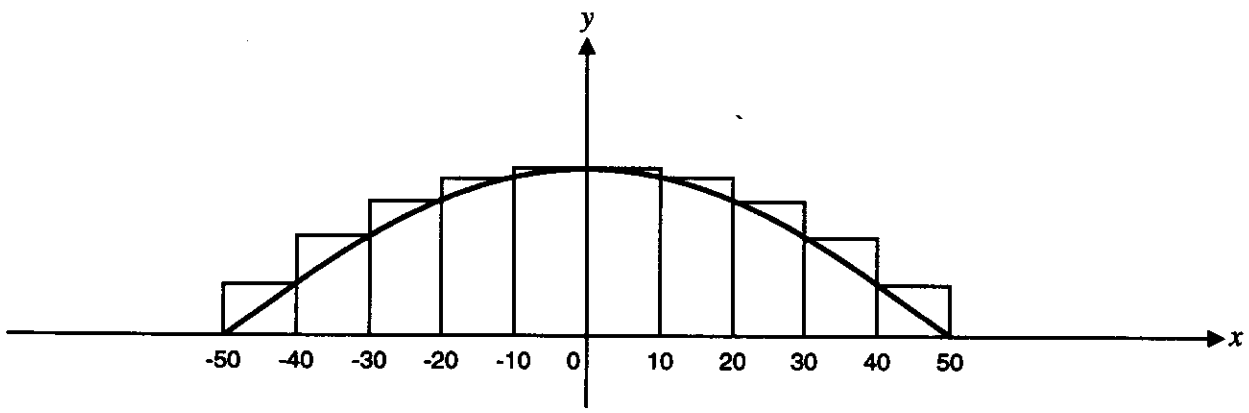
1 mark

b. The cross-sectional area enclosed by the underside of the arch and the water lies between the area of the triangle ATB and the area of the rectangle with base AB and with height OT.

Evaluate these lower and upper approximations for the cross-sectional area.

2 marks

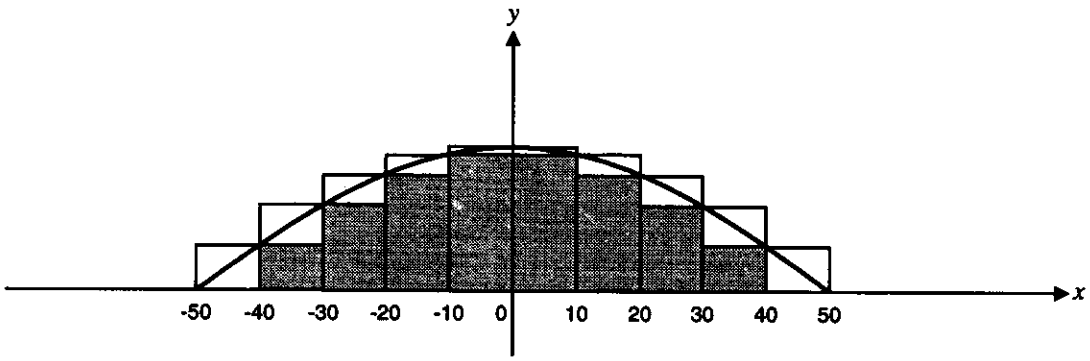
c. A more accurate approximation to the cross-sectional area is made by constructing ten equally spaced 'upper rectangles', as shown on the diagram below.



Find the value of this approximation.

2 marks

d. Find the value of the approximation to the area obtained if the areas of the 'lower rectangles' (shaded below) are summed.



2 marks

e. Using the information from parts **c.** and **d.**, between what two values must the actual cross-sectional area lie?

1 mark

f. Use calculus to find the actual cross-sectional area bounded by the underside of the arch and the water level, to the nearest square metre.

2 marks

Total 10 marks

Question 2

The amount of a radioactive substance decreases spontaneously over time because of the process of radioactive decay. The decrease is known to be exponential, so that the amount y (gram) of a particular radioactive substance present after time t (years) of decay can be modelled by the equation

$$y = A\,e^{-kt} \qquad \text{(equation } \alpha\text{)}$$

where A and k are positive constants.

a. The amount y (gram) present at time t (years) in a sample of the radioactive element radium (${}_{88}Ra^{226}$) is given approximately by

$$y = 1000\,e^{-0.0004\,t}$$

i. Calculate the amount of radium in the sample initially (that is, at time $t = 0$).

1 mark

ii. Calculate the amount of radium in the sample after 20 years.

1 mark

iii. What is the average rate of decay (in grams per year) of the radium sample over the first 1000 years?

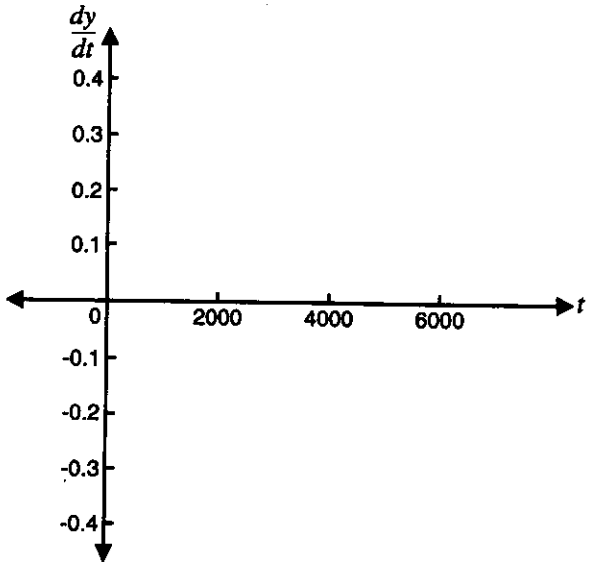
2 marks

iv. Write down an equation for the instantaneous rate of change at any time t of the amount of the given sample of radium.

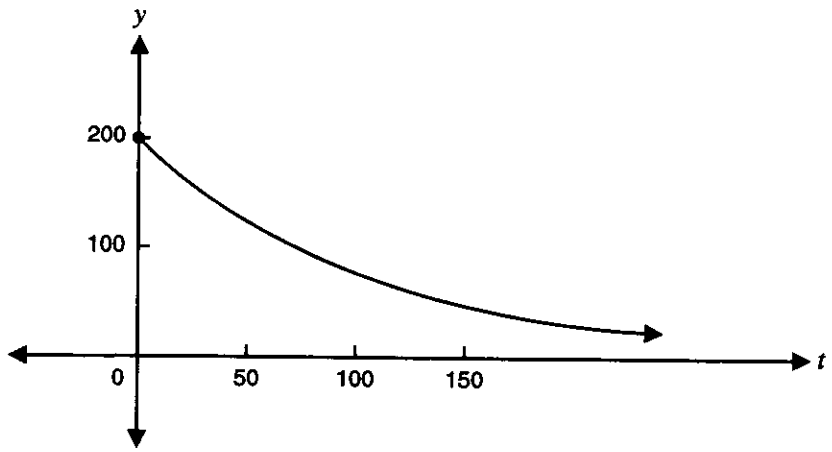
2 marks

v. On the set of axes provided below, sketch the graph of the rate of change of the amount of radium versus time in the sample as it decays radioactively.

2 marks



b. A sketch graph of the equation for the amount of a sample of the radioactive element uranium ($_{92}U^{232}$) is shown below.



It is known that the half-life of this element is 74 years. That is, it takes 74 years for a given sample of uranium to decay to half its initial amount.

The equation which models the amount of uranium (y gram) present in a given sample at any time (t years) is given by the same equation α on page 4.

From the information given above, find A and k in equation α for the element uranium. (Find k to four decimal places.)

4 marks

Total 12 marks

Question 3

The annual rainfall of Melbourne is known to be approximately a normally distributed random variable with a mean of 660 mm and a standard deviation of 125 mm. A year is regarded as 'very wet for Melbourne' if the rainfall exceeds 820 mm. The following table gives descriptions for other rainfalls.

Rainfall for the year	Description of the year
more than 820 mm	very wet
between 595 mm and 820 mm	moderate
between 500 mm and 595 mm	fairly dry
between 485 mm and 500 mm	very dry
less than 485 mm	drought year

a. State the probability that the rainfall in 1995 will exceed 660 mm.

1 mark

b. Calculate the probability that 1995 will have a rainfall that is 'very wet for Melbourne'. Give your answer correct to three decimal places.

2 marks

c. Calculate the probability that a particular year will have a rainfall that is 'fairly dry for Melbourne'. Give your answer correct to three decimal places.

3 marks

d. Assuming that the weather for any year is independent of the weather for any other year, find

i. the probability that in a given three-year period, all three years will be 'fairly dry for Melbourne'. Give your answer correct to three decimal places.

2 marks

ii. the probability that in a given seven-year period, exactly three years will be 'fairly dry for Melbourne'. Give your answer correct to three decimal places.

2 marks

e. How many millimetres of rainfall annually are exceeded about 95 per cent of the time? Give your answer correct to the nearest millimetre.

4 marks

f. Show that the probability of a 'drought year' is 0.0808. Assuming the independence of the weather for one year from the weather for any other year, how often would you expect a 'drought year' to occur? Explain very briefly how you arrived at your answer.

4 marks

Total 18 marks

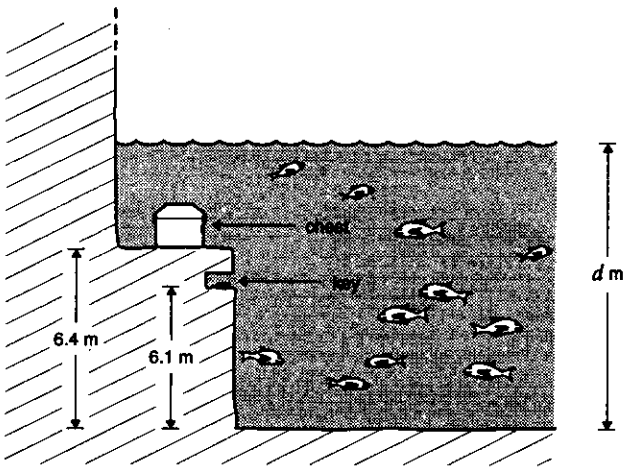
Question 4

In the depths of a jungle in Brazil, an Indian tribe keeps its treasure in a stone chest which is on a rock ledge on the banks of the piranha-infested Amazon River.

The chest cannot be moved and can only be opened by using a metal key which is kept in a hole in the rock just near the chest.

Because the piranha fish eat human flesh, the key can only be safely taken from the hole when the water level in the river falls below the level of the key.

The ledge that the chest is on is 6.4 metres above the river bed, and the hole that the key is in is 6.1 metres above the river bed.



Tasmania Jones, the intrepid adventurer, is keen to find the treasure. He is aware that the depth of water in the river could be modelled by the relation

$$d(t) = 10.0 + 4.0 \cos \frac{\pi t}{14}$$

where d metres is the depth at time t hours after 12 noon on a given Monday.

a. Write down the minimum and maximum depths of the river.

2 marks

b. Show that the depth of water is the same at 12 noon every Monday.

2 marks

c. Find the day and time when the water first reaches its minimum level.

2 marks

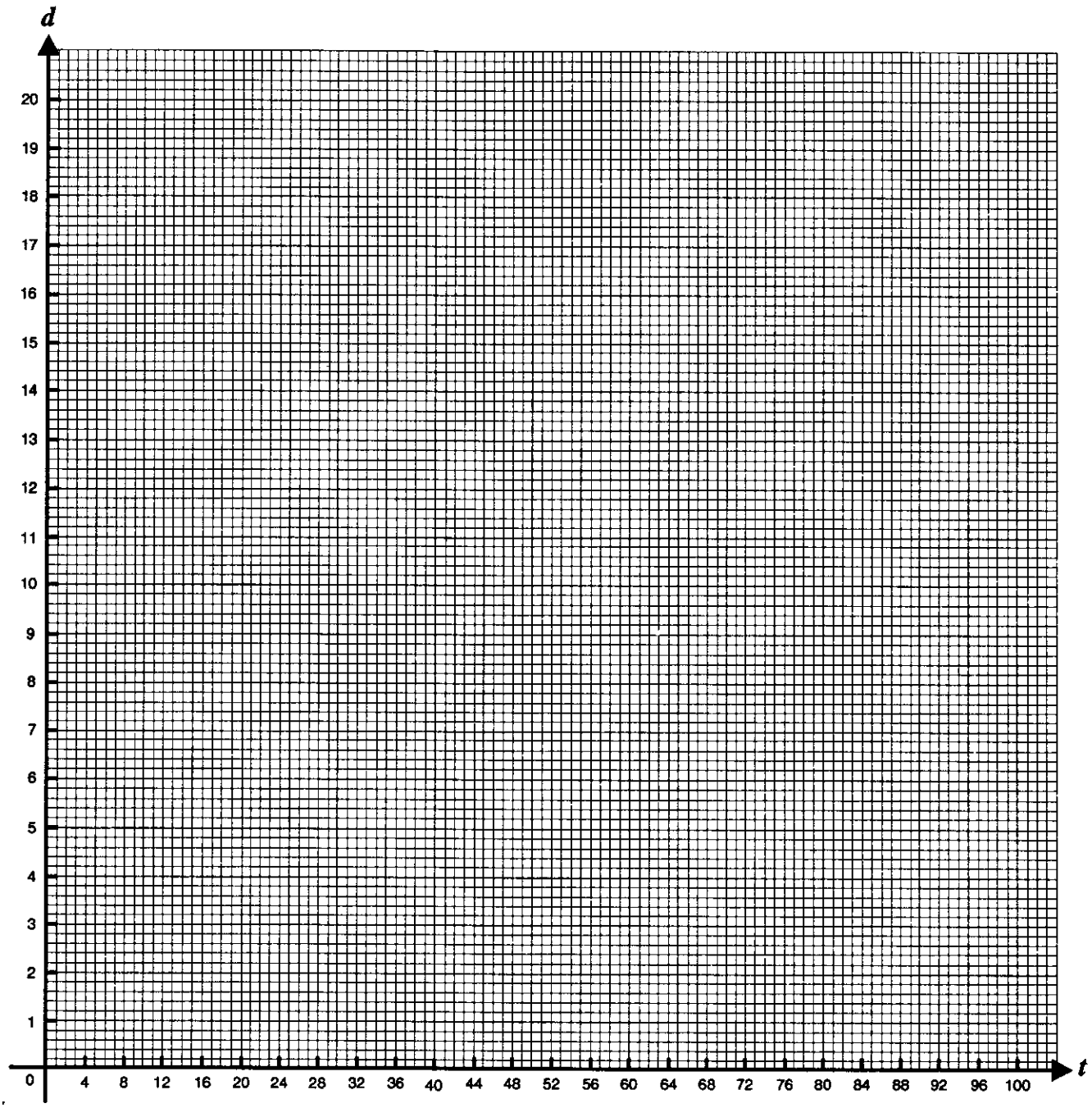
d. Find the depth of the river at

i. 12 noon on Monday ($t = 0$).

ii. 2 am on Tuesday ($t = 14$).

2 marks

e. On the set of axes provided below, sketch the graph of the depth of the river versus time showing **three** complete cycles of the graph.



3 marks

f. Determine the first time after 12 noon on a Monday when

i. the chest is completely uncovered.

ii. the key is able to be taken from the rock.

4 marks

g. Find out the length of time that Tasmania Jones will have to try to remove the key from the rock, take the treasure and return the key, stating your answer in minutes.

2 marks

Unfortunately for Tasmania, the Indians capture him before he is able to obtain the treasure. They tie him by a vine 20 metres above the bottom of the river at 4 pm on a Tuesday and slowly lower him towards the water at a rate of one metre per hour.

h. Draw a graph, on the same set of axes as part **e.** on page 11, showing Tasmania's height above the bottom of the river versus time.

2 marks

i. Using your graph, determine the day and time at which Tasmania Jones will first touch the water of the Amazon River. Give your answer below.

1 mark

Total 20 marks

END OF QUESTION AND ANSWER BOOKLET