1994
VCE
MATHEMATICAL METHODS
CAT 2

DETAILED SUGGESTED SOLUTIONS

CHEMISTRY ASSOCIATES
P.O. BOX 2227
KEW, VIC., 3101
AUSTRALIA
TEL:(03) 9817 5374 FAX: (03) 9817 4334

email: chemas@vicnet.net.au

Internet: http://www.vicnet.net.au/~chemas/education.htm

Victorian Certificate of Education
1994

MATHEMATICAL METHODS

Common Assessment Task 2:
Facts, skills and applications task

Tuesday 8 November 1994: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOKLET

This task has two parts: part I (multiple-choice questions) and part II (short-answer questions).
Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this booklet.

At the end of the task
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

MATHEMATICS

MATHEMATICAL METHODS

Common Assessment Tasks 2 and 3

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\frac{d}{dx}(\sin ax) = a\cos ax$

$\frac{d}{dx}(\cos ax) = -a\sin ax$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}\,dx = \log_e x + c$, for $x > 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x) = \Sigma x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x(1-p)^{n-x}$	np	$np(1-p)$
Continuous distributions			
normal	If X is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $N(0,1)$.		

sample mean: $\bar{x} = \frac{\Sigma x}{n}$

sample variance: $s^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2 = \frac{1}{n-1}(\Sigma x^2 - n\bar{x}^2)$

sample proportion	mean	variance	standard error
$\hat{p}$	$E(\hat{p}) = p$	$\text{var}(\hat{p}) = \frac{p(1-p)}{n}$	$\text{se}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

This table is provided for use with Part I Question 31 and Part II Question 3

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
33	33	33

Directions to students

Materials
Question booklet of 18 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.
An approved calculator may be used.

The task
Detach the formula sheet from the centre of this booklet during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

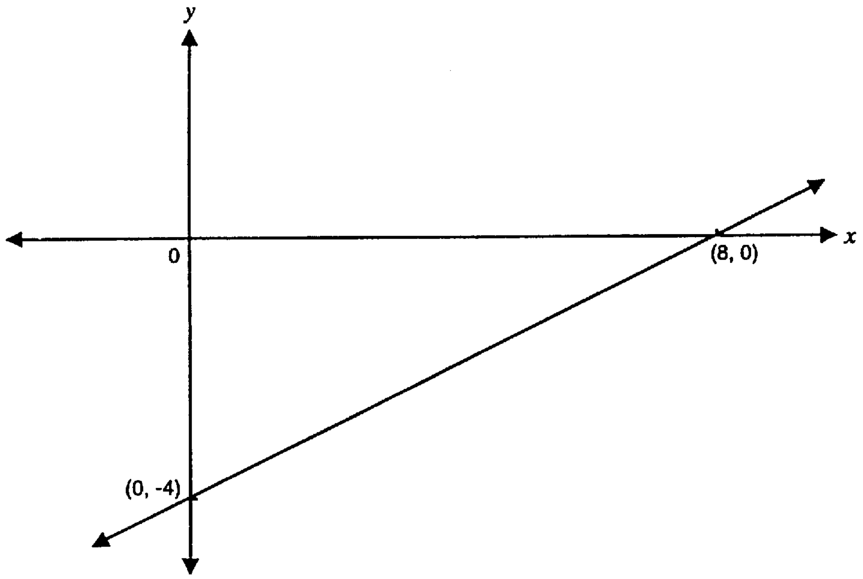
At the end of the task
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

Specific instructions to students

This part consists of 33 questions.
Answer **all** questions in this part on the answer sheet provided for multiple-choice questions.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers. You should attempt every question.
No credit will be given for a question if two or more letters are marked for that question.

Question 1

The equation of the line which contains the points with coordinates (0, – 4) and (8,0) is



A. $y = \frac{x}{2} - 4$

B. $y = -\frac{x}{2} - 4$

C. $y = 2x - 4$

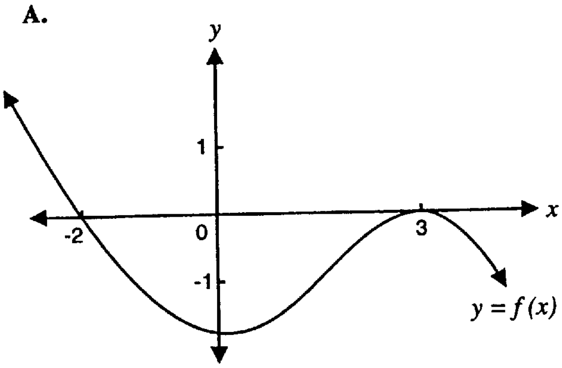
D. $y = -2x - 4$

E. $y = \frac{x}{2} + 4$

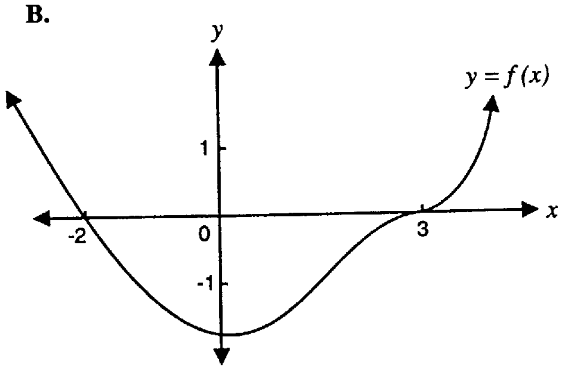
Question 2

Which one of the following could be the graph of $f: R \to R$, $f(x) = k(x-3)^2(x+2)$, k is a constant and $k < 0$?

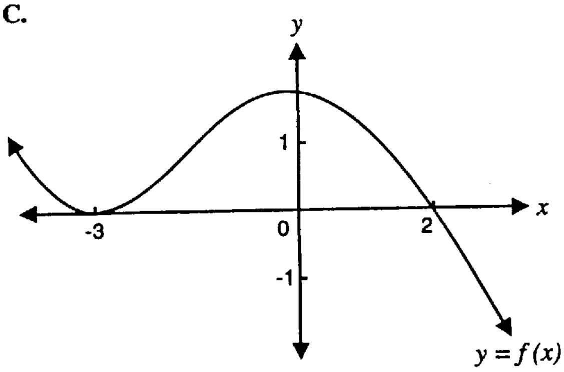
A.



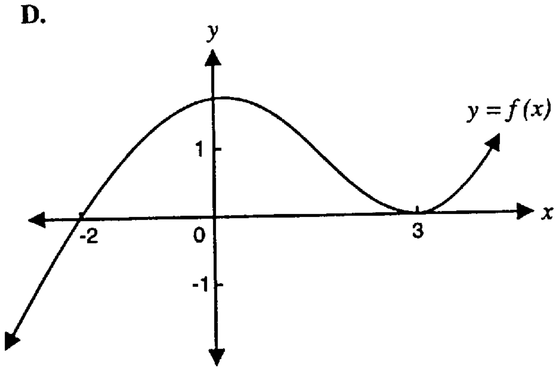
B.



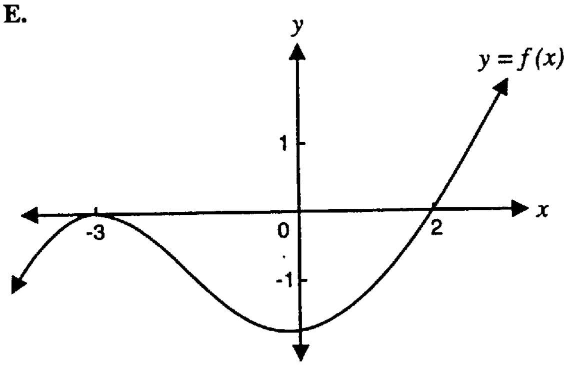
C.



D.



E.



Question 3

The parabola with equation $y = x^2$ is translated so that its image has its vertex at $(-4, 3)$. The equation of the image is

A. $y = (x-4)^2 + 3$

B. $y = (x-3)^2 + 4$

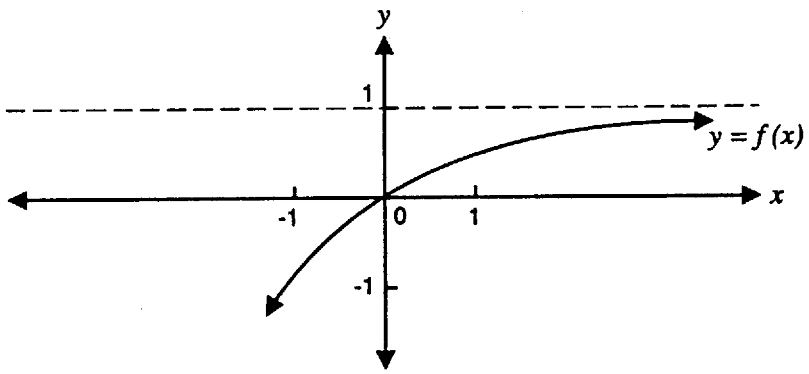
C. $y = (x+4)^2 + 3$

D. $y = (x+3)^2 - 4$

E. $y = -4x^2 + 3$

Question 4

The graph of the function f is shown below.



The rule for f is most likely to be

A. $f(x) = 1 - e^{-x}$

B. $f(x) = 1 - e^{x}$

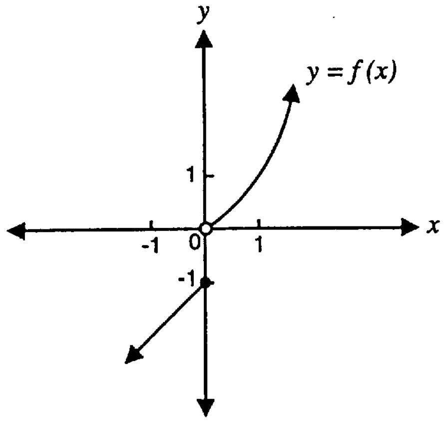
C. $f(x) = e^{x} - 1$

D. $f(x) = \log_e x + 1$

E. $f(x) = \log_e (x+1)$

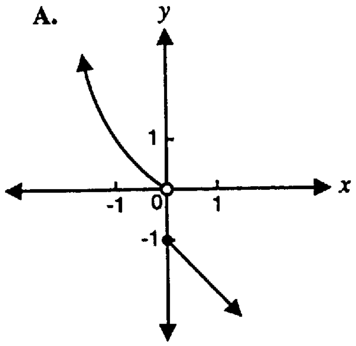
Question 5

The graph of the function f is shown below.

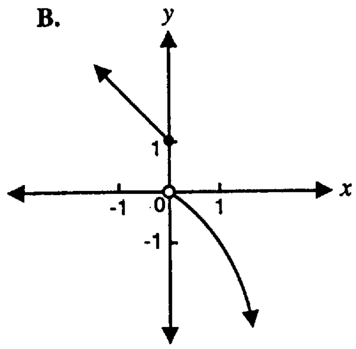


The graph of the inverse function f^{-1} is most likely to be

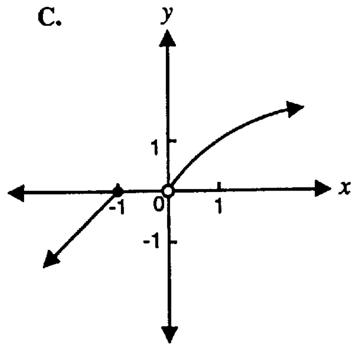
A.



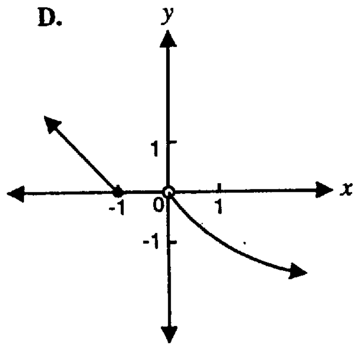
B.



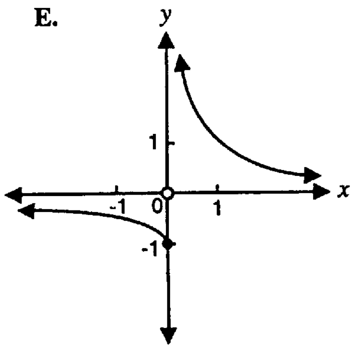
C.



D.



E.



Question 6

The function $f: R \rightarrow R,\ f(x) = 2(\sin x - 1)$ has range

A. $[0, 2]$

B. $[-2, 0]$

C. $[-2, 2]$

D. $[-4, 0]$

E. R

Question 7

The period of the function f whose rule is $f(x) = 3\cos\left(2x + \frac{\pi}{4}\right)$ is

A. 2π

B. 3

C. π

D. $2\pi + \frac{\pi}{4}$

E. $\pi + \frac{\pi}{4}$

Question 8

The possible equation for the graph shown is

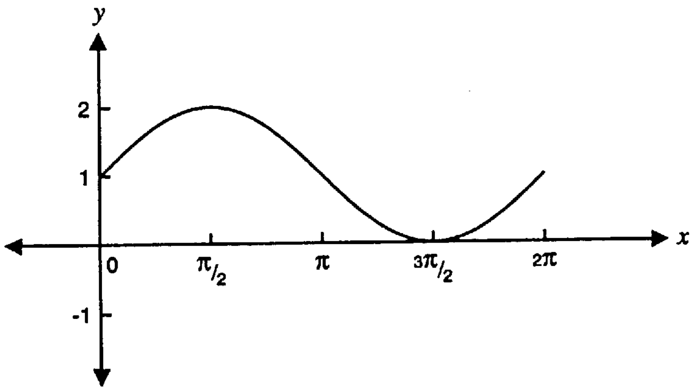
A. $y = 1 + \cos x$

B. $y = 1 + \sin x$

C. $y = 1 + \cos 2x$

D. $y = 1 + \sin 2x$

E. $y = 1 + \sin\left(x + \frac{\pi}{2}\right)$



Question 9

The solution of the equation $\cos x - \frac{1}{2} = 0$ on the domain $[0, \pi]$ is

A. 0

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. π

Question 10

$3\log_{10} 5 + 2\log_{10} 2 - \log_{10} 20$ is equal to

A. $\log_{10}\left(\frac{19}{20}\right)$

B. $\log_{10} 109$

C. $\log_{10} 480$

D. $2\log_{10} 5$

E. $6\log_{10}\left(\frac{1}{2}\right)$

Question 11

The fifth and sixth rows of Pascal's triangle are shown below.

	1		4		6		4		1	
1		5		10		10		5		1

The coefficient of x^3 in the expansion of $(x+2)^5$ is

A. 2^2

B. $6(2)^2$

C. $6(2)^3$

D. $10(2)^2$

E. $10(2)^3$

Question 12

The derivative of $\frac{1}{x^6}$ is equal to

A. $-\frac{1}{6x^5}$

B. $-\frac{1}{5x^5}$

C. $\frac{1}{5x^5}$

D. $-\frac{6}{x^7}$

E. $-\frac{1}{6x^7}$

Question 13

The derivative of $\sin(e^x)$ is equal to

A. $\sin(e^x)$

B. $e^x\cos(e^x)$

C. $e^x\sin(e^x)$

D. $\cos(e^x)$

E. $-e^x\cos x$

Question 14

If $f(x) = \log_e 2x$, then $f'(3)$ is equal to

A. $3\log_e 2$

B. 1

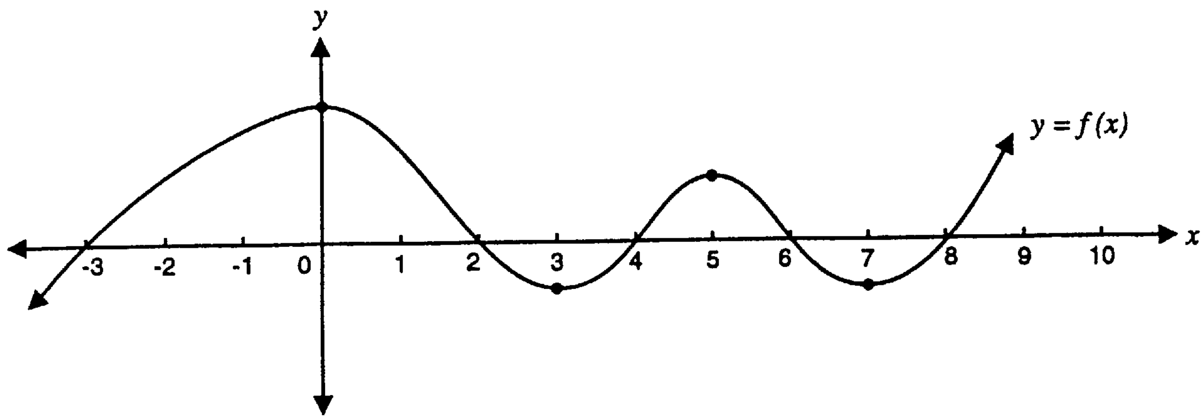
C. $\frac{2}{3}$

D. $\frac{1}{6}$

E. $\frac{1}{3}$

Question 15

The graph of the function f is shown below.



$f(x)$ and $f'(x)$ are both positive over the intervals

A. $(-\infty, 0) \cup (3, 5) \cup (7, \infty)$

B. $(-3, 2) \cup (4, 6) \cup (8, \infty)$

C. $(-\infty, -3) \cup (2, 4) \cup (6, 8)$

D. $(-3, 0) \cup (4, 5) \cup (8, \infty)$

E. $(4, 5) \cup (8, \infty)$

Question 16

The x coordinate of the turning point of the graph of the relation $y = e^{2x} - 2x$ is

A. $\log_e 2$

B. $\log_e 4$

C. 0

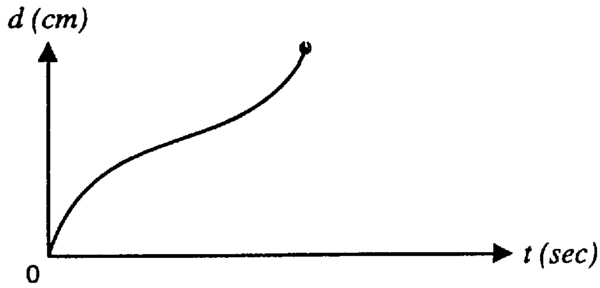
D. $\frac{1}{2}\log_e 2$

E. e^0

Question 17

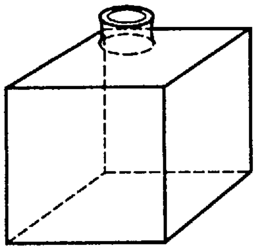
Liquid is poured into a container at a constant rate.

The graph of the depth of the liquid versus time is shown below.

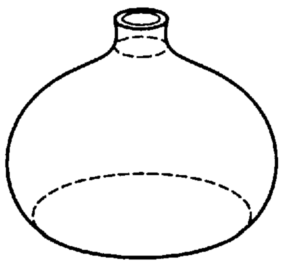


Which one of the following is most likely to be the container used?

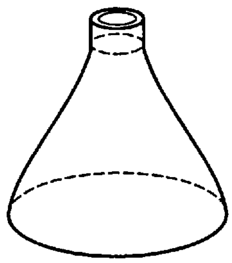
A.



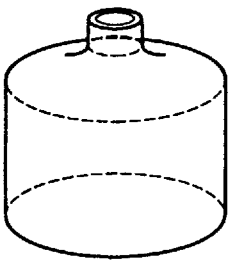
B.



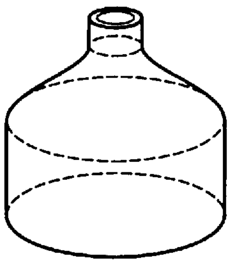
C.



D.

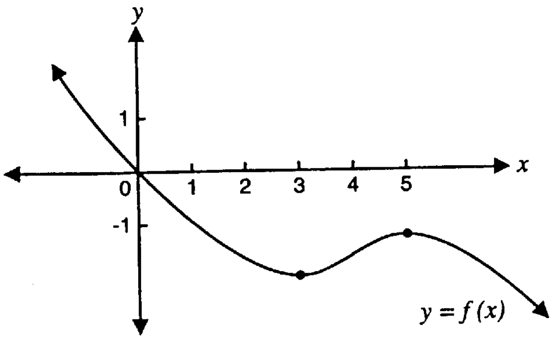


E.



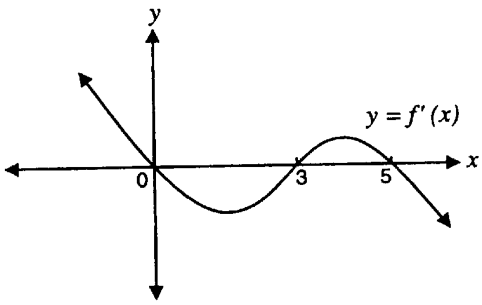
Question 18

The graph of f is shown below.

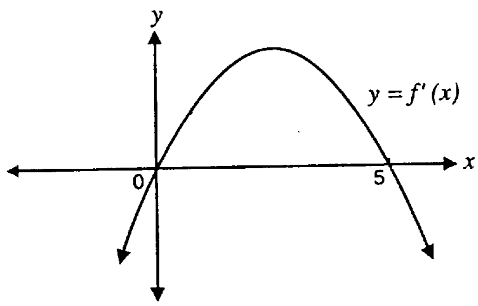


Which one of the following could be the graph of the derivative of f?

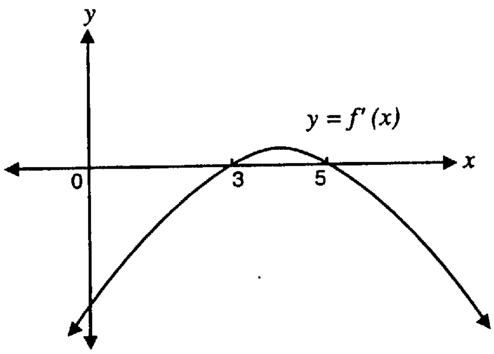
A.



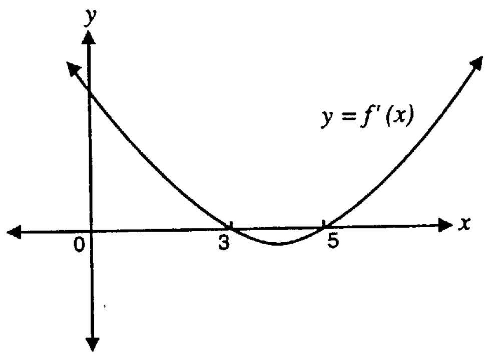
B.



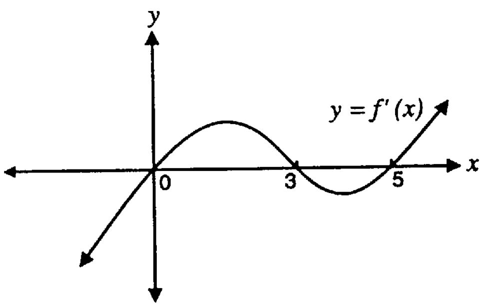
C.



D.



E.



Question 19

$\int (3x+5)^4\,dx$ is equal to

A. $\dfrac{(3x+5)^5}{15}+c$

B. $\dfrac{(3x+5)^5}{5}+c$

C. $\dfrac{(3x+5)^5}{3}+c$

D. $\dfrac{\left(\frac{3x^2}{2}+5x+c\right)^5}{15}$

E. $12\,(3x+5)^3+c$

(c is an arbitrary constant)

Question 20

If $g'(x)=6e^{2x}$, then $g(x)$ is equal to

A. $3e^{2x}+c$

B. $6e^{2x}+c$

C. $12e^{2x}+c$

D. $3xe^{2x}+c$

E. $6e^{x^2}+c$

(c is an arbitrary constant)

Question 21

To find an approximation to the area between the graph with equation $y=x^2$ and the x-axis between the lines with equations $x=1$ and $x=4$, the partitioning shown, using rectangles, can be used.

The area of the shaded rectangles is equal to

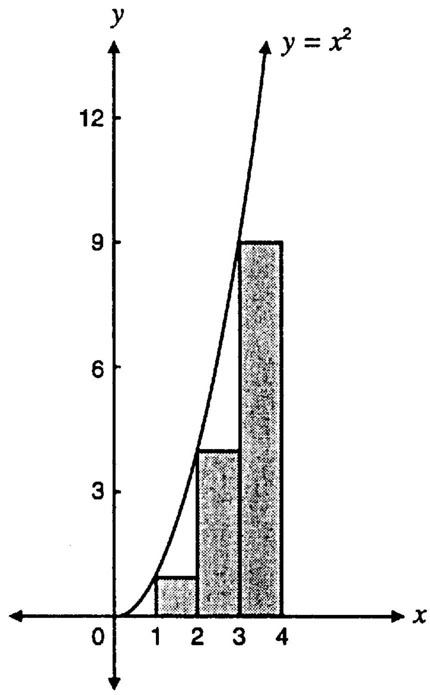
A. 14

B. 15

C. 21

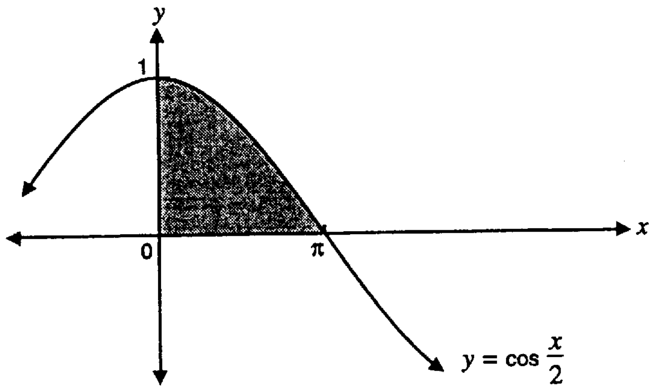
D. 30

E. 54



Question 22

The area between the curve with equation $y = \cos\frac{x}{2}$ and the x-axis between the lines with equations $x = 0$ and $x = \pi$ is equal to



A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. 4

Question 23

If $\int_0^a (3x - 6)\, dx = 0$, $a \neq 0$, then a is equal to

A. -4

B. -2

C. 2

D. 4

E. 6

Question 24

Which one of the following random variables is **not** discrete?

A. the price of petrol at a local petrol station in cents per litre

B. the number of goals kicked by a full forward in each game of the season

C. the number of cartons of milk purchased by a family each week for a year

D. the number of Mazda cars sold each day for a year

E. the height of a person as she grows over a period of one year

The following information refers to Questions 25 and 26

Rambo, the poodle, has fleas. His owner, Angela, counts the number of fleas on Rambo each day for twenty days. The results are given in the table below.

Number of fleas (x)	0	1	2	3	4	5	6
Number of days Rambo had this number of fleas (f)	1	2	1	4	7	4	1

Question 25

During the 20-day period, the proportion of days on which Angela observed more than three fleas is

A. 0.2

B. 0.4

C. 0.5

D. 0.6

E. 0.8

Question 26

The mean number of fleas per day that Rambo had was equal to

A. 0.95

B. 3

C. 3.5

D. 19

E. 70

Question 27

Andrew throws a basketball towards a goal ring. If the ball passes through the ring, Andrew scores a goal. Andrew knows that on average he scores a goal 8 times out of every 10 throws.

If Andrew throws the ball 20 times, then the mean and variance, respectively, of the number of goals that he scores are

A. 16 and 1.6

B. 20 and 1.6

C. 8 and 3.2

D. 16 and 3.2

E. 8 and 1.6

Question 28

A ticket collector at Flinders Street Railway Station has observed that, in the long run, 60 per cent of all tickets collected are full-fare and the remaining 40 per cent are concession. A ticket inspector has taken a random sample of 20 tickets from a day's takings. The probability that this sample contains exactly 12 full-fare tickets is equal to

A. 1

B. ${}^{20}C_{12}\,(0.4)^{8}\,(0.6)^{12}$

C. ${}^{20}C_{12}\,(0.4)^{12}\,(0.6)^{8}$

D. $(0.4)^{12}\,(0.6)^{8}$

E. $(0.4)^{8}\,(0.6)^{12}$

Question 29

A random sample of 100 people were asked their opinions about the Australian flag and, of those asked, 32 believed that the flag should be changed. The standard error for the proportion of the population who would like the flag changed is

A. 0.0022

B. 0.0466

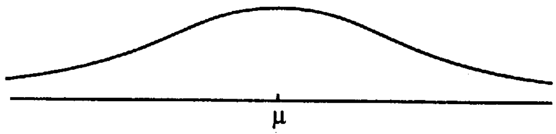
C. 0.2176

D. 0.3200

E. 0.4665

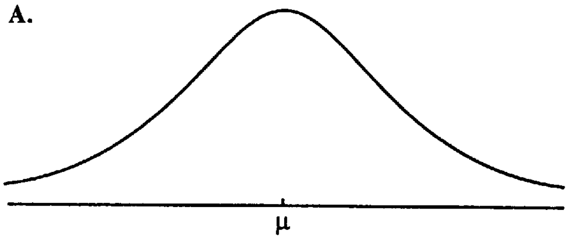
Question 30

A normal distribution where the mean is μ and the standard deviation is σ is shown below.

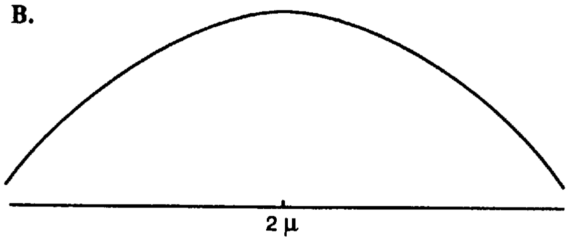


Using the **same** scale, a normal distribution with mean 2μ and standard deviation $\frac{\sigma}{2}$ would look most like

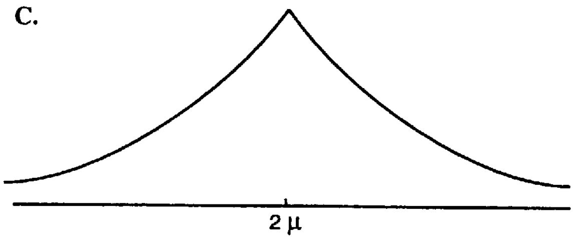
A.



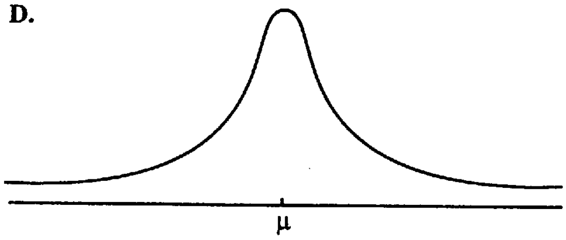
B.



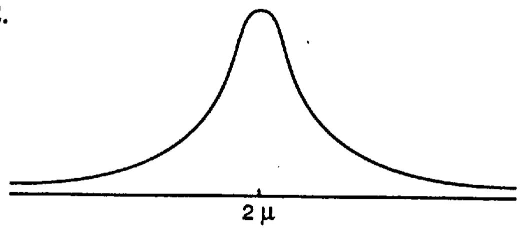
C.



D.



E.



The following information refers to Questions 31 and 32

See detachable formula sheet – Table 1 Normal distribution – cdf

Wizzi Cherry Drink is sold in 500 mL bottles. The company determines that the volume of drink in each bottle is normally distributed with mean 498 mL and standard deviation 2.5 mL.

Question 31

The probability that a bottle selected at random will contain more than 500 mL is equal to

A. 0.1056

B. 0.2119

C. 0.5000

D. 0.7881

E. 0.8944

Question 32

The 95 per cent confidence interval within which the volumes of the drink in Wizzi Cherry Drink bottles will lie is

A. 498 to 503 mL

B. 495.5 to 500.5 mL

C. 493 to 498 mL

D. 493 to 503 mL

E. 490.5 to 505.5 mL

Question 33

When she fires an arrow at a target, the probability that Julie hits the target is 0.4. When she fires N arrows at the same target, the probability that Julie hits the target at least once is 0.92224.

The value of N is equal to

A. 3

B. 4

C. 5

D. 6

E. 7

V.C.E. Mathematical Methods 1994 Solutions to CAT 2 ①

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

Question 1

Let $(x_1, y_1) = (0, -4)$ and $(x_2, y_2) = (3, 0)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad c = -4$

$= \frac{0 - -4}{3 - 0}$

$= \frac{4}{8}$

$= \frac{1}{2}$

∴ Equation of the line is $y = \frac{x}{2} - 4$ **A.**

Question 2

For $f(x) = k(x-3)^2(x+2), \quad k < 0$

A negative cubic graph has the general shape

The x-intercepts are $(-2, 0)$ and $(3, 0)$

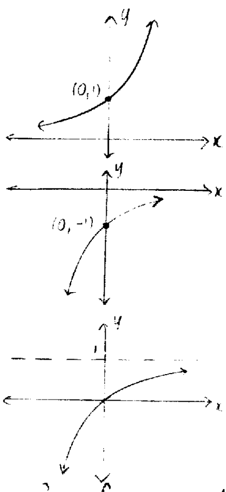
∴ Graph **A** could be the graph of $f(x)$

Question 3

The parabola $y = (x+4)^2 + 3$ has its vertex at $(-4, 3)$.

∴ The equation of the image is **C.**

Question 4



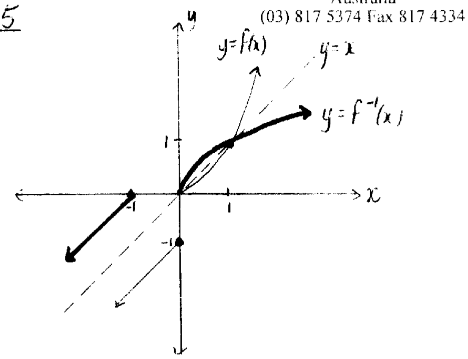
$f(x) = e^x$

$f(x) = -e^{-x}$

$f(x) = 1 - e^{-x}$

∴ The rule for $\check{f}$ is most likely to be **A**.

Question 5

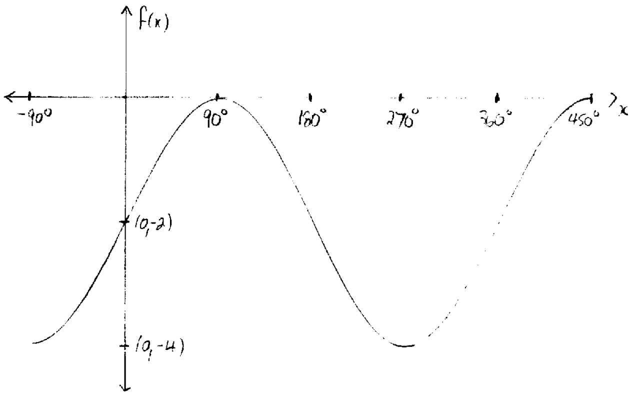


∴ The graph of the inverse is most likely to be **C.**

Question 6

$f(x) = 2(\sin x - 1)$

$\quad = 2\sin x - 2$



The function f has range $[-4, 0]$. **D**

Question 7

$f(x) = 3\cos(2x + \frac{\pi}{4})$

$\quad = 3\cos 2(x + \frac{\pi}{8})$

period $= \frac{2\pi}{2}$

$\quad = \pi$ **C.**

Question 8

The graph shown is the general sine curve with a period of 2π, amplitude of 1 unit and translated 1 unit vertically.

The possible equation is $y = 1 + \sin x$ **B.**

Question 9

$\cos x - \frac{1}{2} = 0 \qquad 0 \le x \le \pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

The solution of the equation is **C.**

Question 10

$3\log_{10} 5 + 2\log_{10} 2 - \log_{10} 20$

$= \log_{10} 5^3 + \log_{10} 2^2 - \log_{10} 20$

$= \log_{10} 125 + \log_{10} 4 - \log_{10} 20$

$= \log_{10}\left(\frac{125 \times 4}{20}\right)$

$= \log_{10} 25$

$= \log_{10} 5^2$

$= 2\log_{10} 5$ **D.**

Question 11

$(x+2)^5 = x^5 + 5x^4(2) + 10x^3(2)^2 + 10x^2(2)^3 + 5x(2)^4 + 2^5$

$\therefore$ coefficient of x^3 is $10(2)^2$ **D.**

Question 12

$f(x) = \frac{1}{x^6}$

$= x^{-6}$

$f'(x) = -6x^{-7}$

$= -\frac{6}{x^7}$ **D.**

Question 13

$y = \sin(e^x)$

Let $u = e^x$ so that $y = \sin u$

$\therefore \frac{du}{dx} = e^x$ and $\frac{dy}{du} = \cos u$

$= \cos(e^x)$

By the chain rule:

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= e^x \cdot \cos(e^x)$ **B.**

Question 14

$y = \log_e 2x$

Let $u = 2x$ so that $y = \log_e u$

$\frac{du}{dx} = 2$ and $\frac{dy}{du} = \frac{1}{u}$

$= \frac{1}{2x}$

By the chain rule:

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{2x} \times 2$

$= \frac{1}{x}$

$\therefore$ if $f(x) = \log_e 2x$

then $f'(x) = \frac{1}{x}$

and $f'(3) = \frac{1}{3}$ **E.**

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

Question 15

$f(x)$ is positive over the intervals

$(-3,2) \cup (4,6) \cup (8,\infty)$

$f'(x)$ is positive over the intervals

$(-\infty,0) \cup (3,5) \cup (7,\infty)$

$f(x)$ and $f'(x)$ are both positive over the intervals

$(-3,0) \cup (4,5) \cup (8,\infty)$ **D.**

Question 16

$y = e^{2x} - 2x$

$\frac{dy}{dx} = 2e^{2x} - 2$

For turning point, let $\frac{dy}{dx} = 0$

let $2e^{2x} - 2 = 0$

$2(e^{2x} - 1) = 0$

$e^{2x} - 1 = 0$

$e^{2x} = 1$

$2x = \log_e 1$

$2x = 0$

$x = 0$ **C.**

Question 17

For container B, the depth of liquid initially increases quickly but as the container becomes wider the depth of liquid increases at a slower rate. The container begins to narrow again so the depth of liquid increases at a greater rate. **B.**

Question 18

x	<3	3	$3<x<5$	5	>5
$f'(x)$	$-$	0	$+$	0	$-$

The derivative of f could be **C.**

Question 19

$\int (3x+5)^4 \, dx$

$= \frac{(3x+5)^5}{3 \times 5} + c$

$= \frac{(3x+5)^5}{15} + c$ **A.**

Question 20

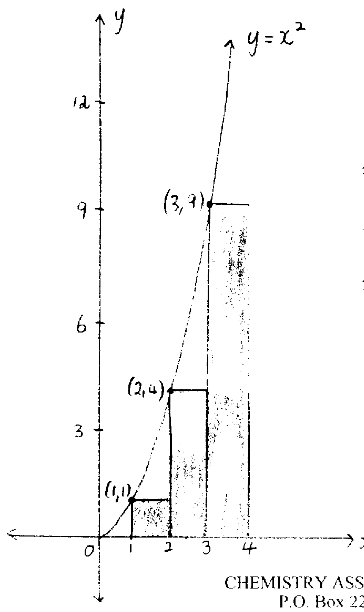
$g'(x) = 6e^{2x}$

$g(x) = \int 6e^{2x} \, dx$

$= \frac{6}{2} e^{2x} + c$

$= 3e^{2x} + c$ **A.**

Question 21



Area of shaded rectangles

$= 1 \times 1 + 1 \times 4 + 1 \times 9$

$= 1 + 4 + 9$

$= 14$ square units **A.**

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

Question 22

$$\text{Area} = \int_0^{\pi} \cos\frac{x}{2}\, dx$$
$$= \left[2\sin\frac{x}{2}\right]_0^{\pi}$$
$$= 2\sin\frac{\pi}{2} - 2\sin 0$$
$$= 2 \text{ square units}$$

D.

Question 23

$$\int_0^a (3x-6)\,dx = 0 \qquad a \neq 0$$
$$\therefore \left[\frac{3}{2}x^2 - 6x\right]_0^a = 0$$
$$\therefore \frac{3}{2}a^2 - 6a = 0$$
$$\therefore 3a^2 - 12a = 0$$
$$\therefore 3a(a-4) = 0$$

Since $a \neq 0$, $a = 4$

D.

Question 24

The height of a person as she grows over a period of one year is NOT discrete

E.

Question 25

Proportion of days on which Angela observed more than three fleas $= \frac{12}{20}$
$= 0.6$

D.

Question 26

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}$$
$$= \frac{0+2+2+12+28+20+6}{20}$$
$$= \frac{70}{20}$$
$$= 3.5 \text{ fleas per day}$$

C.

Question 27

Let X = number of goals Andrew scored

X is Binomial

$n = 20 \qquad p = \frac{8}{10} = 0.8$

$$\text{mean} = np$$
$$= 20 \times 0.8$$
$$= 16$$

$$\text{variance} = np(1-p)$$
$$= 20 \times 0.8 \times 0.2$$
$$= 3.2$$

D.

Question 28

Let X = number of full-fare tickets

X is Binomial

$n = 20 \qquad p = \frac{60}{100} = 0.6$

$$Pr(X=12) = {}^{20}C_{12}\,(0.6)^{12}\,(0.4)^{8}$$

B.

Question 29

$n = 100 \qquad \hat{p} = \frac{32}{100} = 0.32$

$$\text{standard error} = \sqrt{\frac{0.32(1-0.32)}{100}}$$
$$\approx 0.0466$$

B.

Question 30

The normal distribution must be symmetrical about the mean 2μ and the smaller standard deviation $\frac{\sigma}{2}$ makes the graph steeper.

E.

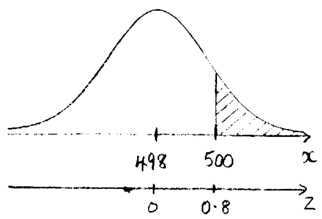
CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

Question 31

Let X = volume of drink in each bottle (mL)

X is Normal

$\mu = 498 \qquad \sigma = 2.5$



When $x = 500$

$z = \dfrac{500-498}{2.5}$

$= 0.8$

$\Pr(X > 500)$

$= \Pr(Z > 0.8)$

$= 1 - \Pr(Z < 0.8)$

$= 1 - 0.7881$

$= 0.2119$

B.

Question 32

95% confidence limits $= \mu \pm 2\sigma$

$\mu - 2\sigma = 498 - 2(2.5)$

$= 493$ mL

$\mu + 2\sigma = 498 + 2(2.5)$

$= 503$ mL

∴ 95% confidence interval is 493 to 503 mL.

D.

Question 33

Let X = number of times Julie hits the target

X is Binomial

$n = N \qquad p = 0.4$

$\Pr(X \geq 1) = 0.92224$

∴ $\Pr(X = 0) = 0.07776$

∴ ${}^{N}C_0 (0.4)^0 (0.6)^N = 0.07776$

$(0.6)^N = 0.07776$

$\log_e (0.6)^N = \log_e 0.07776$

$N \log_e 0.6 = \log_e 0.07776$

$N = \dfrac{\log_e 0.07776}{\log_e 0.6}$

$= 5$

C.

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

(6)

Specific instructions to students

Answer **all** questions in this part in the spaces provided.

Question 1

On the set of axes below, sketch the graph with equation

$$y = \frac{4}{(x-2)} + 1,$$

marking clearly the **coordinates** of its intersection with the axes and the **equations** of the **asymptotes**.

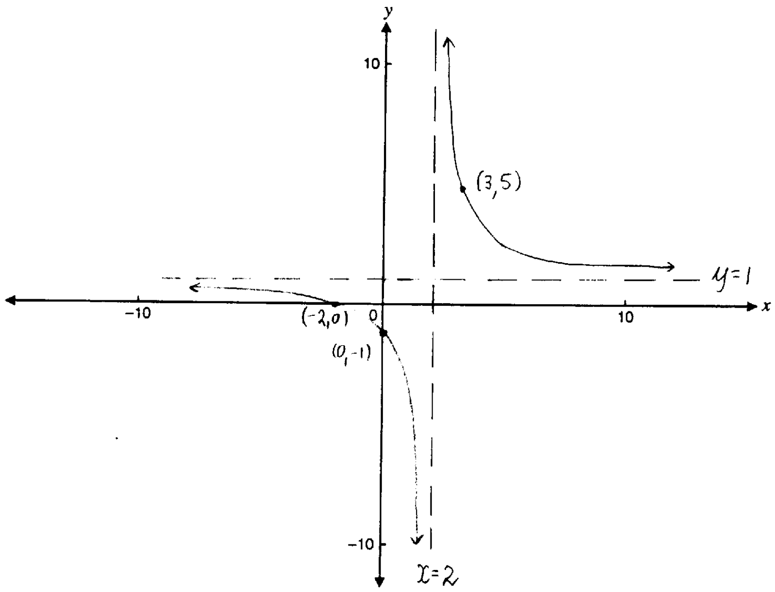
y intercept: let $x = 0$ — x intercept: let $y = 0$

$y = \frac{4}{-2} + 1$ — $0 = \frac{4}{x-2} + 1$

$= -2 + 1$ — $-(x-2) = 4$

$= -1$ — $-x + 2 = 4$

$x = -2$



3 marks

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

CHEMISTRY ASSOCIATES
P.O. Box 2227
Kew, Vic., 3101
Australia
(03) 817 5374 Fax 817 4334

7

Question 2

By finding the coordinates of any intercepts **and** turning points, sketch on the set of axes below the graph with equation

$$y = x^3 - 12x$$

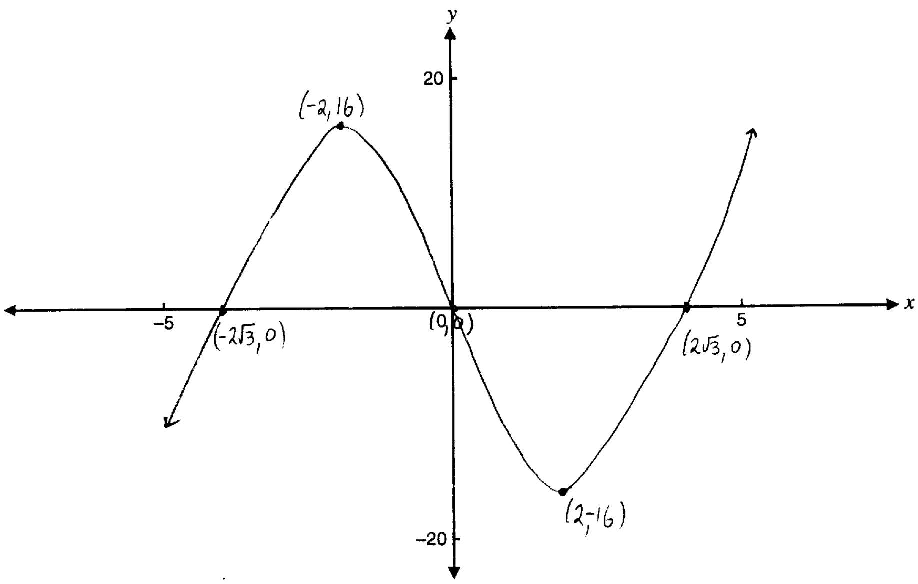
Clearly label the **coordinates** of all intercepts and turning points on the sketch.

y intercept let $x=0$, $y=0$

x intercept let $y=0$
$0 = x^3 - 12x$
$0 = x(x-\sqrt{12})(x+\sqrt{12})$
$0 = x(x-2\sqrt{3})(x+2\sqrt{3})$
$x = 0, 2\sqrt{3}, -2\sqrt{3}$

turning points: let $\frac{dy}{dx} = 0$
$3x^2 - 12 = 0$
$3(x-2)(x+2) = 0$
$x = 2, -2$
If $x = 2$, $y = 2^3 - 12(2) = -16$
If $x = -2$, $y = (-2)^3 - 12(-2) = 16$

3 marks



Question 3

See page 7 Part II – Table 1 Normal distribution – cdf

Tessa has measured her heartrate, upon waking up in the morning, over several months. She has found that her heartrate in beats per minute is normally distributed with a mean value of 70 beats per minute and a standard deviation of 5 beats per minute.

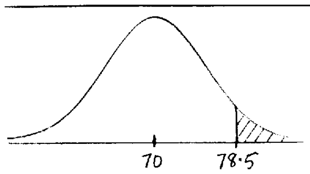
On a particular morning, what is the probability that her waking heartrate will exceed 78.6 beats per minute?

Let X = Tessa's heartrate (beats per minute) $\mu = 70$, $\sigma = 5$

When x 78.6
$z = \frac{78.6-70}{5}$
$= 1.72$

$Pr(X > 78.6) = Pr(Z > 1.72)$
$= 1 - Pr(X < 1.72)$
$= 1 - 0.9573$
$= 0.0427$

2 marks



Question 4

A new drug has been developed to treat distemper in dogs. Over a period of time, 40 per cent of dogs with distemper were cured. Ten dogs with distemper are to be selected randomly and treated with this drug.

i. How many dogs would be expected to be cured?

Let X = number of dogs cured. X is Binomial, $n = 10$ $p = 0.4$

$E(X) = np = 10 \times 0.4 = 4$ Four dogs would be expected to be cured.

ii. Calculate the probability, to four decimal places, of more than eight of the dogs being cured.

$Pr(X > 8) = Pr(X = 9) + Pr(X = 10)$

$= {}^{10}C_9 (0.4)^9 (0.6)^1 + {}^{10}C_{10} (0.4)^{10} (0.6)^0$

$= 0.0017$

1 + 2 = 3 marks

Question 5

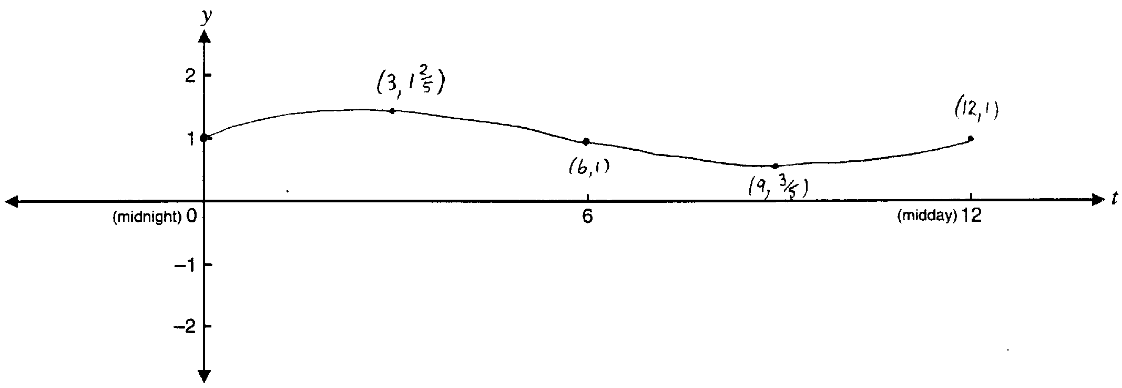
The height of the sea level (above a fixed point) on a given day varies over time according to the effect of the tides.

The height, y cm, is given by the equation

$$y = \frac{2}{5}\sin\left(\frac{\pi t}{6}\right) + 1,$$

where t represents the number of hours after midnight on 21 September 1994.

a. Sketch, on the set of axes below, a graph representing the height of the sea level on 21 September 94 from midnight to midday.



b. State

i. the period of the function

period = 12

ii. the amplitude of the function

amplitude = $\frac{2}{5}$

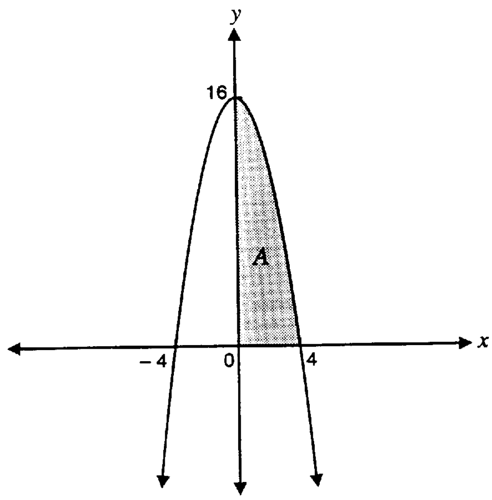
2 + 2 = 4 marks

TURN OVER

9

Question 6

Let A denote the area in the first quadrant bounded by the lines with equations $x = 0$ and $y = 0$ and the parabola $y = 16 - x^2$.



i. Express the area A as an integral.

$A = \int_0^4 (16 - x^2)\, dx$

ii. Evaluate the area A.

$A = \left[16x - \frac{1}{3}x^3\right]_0^4$

$= \left(64 - \frac{64}{3}\right) - (0 - 0)$

$= 42\frac{2}{3}$ square units

1 + 1 = 2 marks

Total 17 marks

END OF SUGGESTED SOLUTIONS

1994 VCE MATHEMATICAL METHODS CAT 2

CHEMISTRY ASSOCIATES
P.O. BOX 2227
KEW, VIC., 3101
AUSTRALIA
TEL:(03) 9817 5374 FAX: (03) 9817 4334