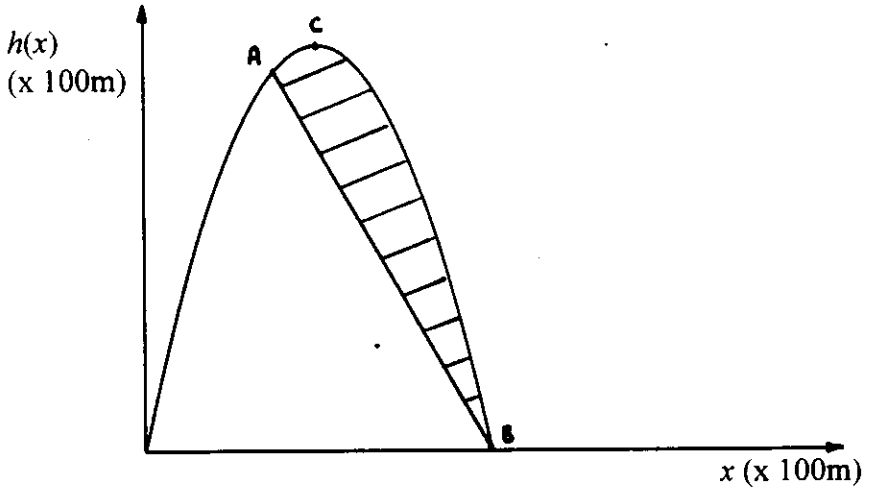
IARTV '94
CAT 3

QUESTION ONE (15 marks)

An East-West cross section of Mt. Sterling is shown below. The height, h metres, at any point x units east of the western edge is given by $h(x) = 6x - x^2$ [Note; x is measured in units of 100m].

The Grape and Paelli development company wants to build a Ski Resort on the mountain at point A and to improve the skiing the company wants to remove that part of the mountain shaded in the diagram.

Point A is 200m east of the western edge at a height of 800m.



(a) Find the height of the lookout at the top of Mt. Sterling, located at C.

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[3 marks]

(b) Find the coordinates of position B.

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[3 marks]

(c) Calculate the area of the cross section that is to be removed.

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[6 marks]

(d) If the section of the mountain removed runs for a length of 500m in the North-South direction, calculate the volume of earth removed.

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[3 marks]

QUESTION TWO (15 marks)

The diameters of apples grown on a certain orchard in Maffra are normally distributed, with mean 84 mm, and standard deviation 5 mm. The apples are classified into grades according to their size.

GRADE A: apples with diameters 88mm or more.
GRADE B: apples with diameters between 78mm and 88mm.
GRADE C: apples with diameters 78mm or less.

(a) Calculate the proportion of apples in each grade.

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[3 marks]

Apples in grades A and B are sold at 40 cents each and 30 cents each respectively. Apples in grade C are crushed and the juice is extracted. The price obtained for the juice averages 10 cents per apple.

(b) Of the apples that are not crushed, what proportion are grade A?

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[3 marks]

(c) The orchardist is considering combining grades A and B into a single grade. What price should be charged for apples in this single grade, if the overall return is to remain the same?

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[3 marks]

(d) On one particular day 50,000 apples are picked. Calculate the expected value of the apples picked that day.

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[2 marks]

(e) Suppose that the orchardist wanted to introduce a new grade for the largest 15% of the apples. What should the lower size limit for the new grade be?

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[4 marks]

QUESTION THREE (15 marks)

The concentration, C *(t)* units, of a drug in a patients bloodstream *t* hours after injection is given by

$$C(t) = 15\,t^2\,e^{-4t},\ t \geq 0$$

(a) Complete the table below for the given values of *t*

t	0	0.25	0.5	0.75	1	1.5	2
C*(t)*		0.345		0.42		0.084	0.02

[3 marks]

(b) Find the maximum concentration and the number of hours after injection at which it occurs.

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[6 marks]

(c) What happens to the concentration after a long period of time? Justify your answer briefly.

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[2 marks]

(d) Using the axes drawn on the next page, sketch the graph of the C(*t*) function for $0 \leq t \leq 5$.

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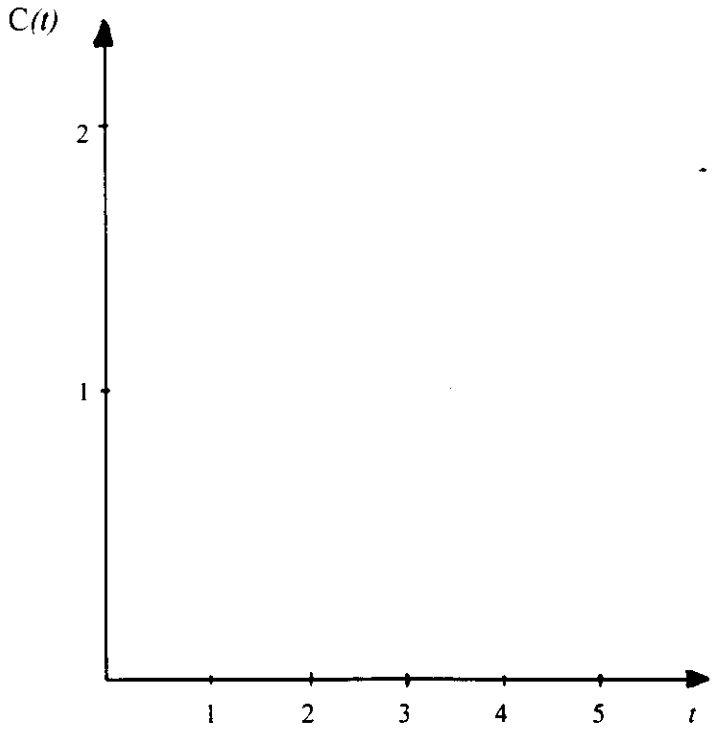

[4 marks]

QUESTION FOUR (15 marks)

(a) A study commissioned by the TAC into the driving habits of men aged 50 and over has shown that the pressure exerted on the brake pedal for a standard family sedan can be approximated by the formula.

$$P(x) = 4000 + 4x + \frac{10000}{x}$$

where x is the force exerted on the brake pedal, measured in Leg Newtons, by the driver

Find the value of Leg Newtons that minimizes the pressure on the brake pedal for this group of drivers.

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[5 marks]

(b) The driving characteristics of a car, and a car towing a caravan, are slightly different. For example, the car towing a caravan is a heavier 'unit', so that greater pressure on the brake pedal is needed to stop it. The probability P that an average driver driving a car towing a caravan will forget that he has to press harder on the brake pedal t driving hours after hitching on the caravan can be approximated by the equation

$$\frac{dP}{dt} = -6P$$

It can be shown that a possible solution to this equation is:

$$\log_e P = -6t + \log_e A$$

(i) Find the value of A and hence make P the subject of the equation given that P = 0.5 when $t = 0$.

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(ii) What is the probability that an average driver will forget to press harder on the brake pedal 10 driving minutes after hitching on a caravan?

Express your answer correct to 3 decimal places.

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[3 marks]

END OF BOOKLET