SECTION A

Specific Instructions for Section A

Section A consists of 33 questions.
Answer all questions in this part on the multiple-choice answer sheet provided.
A correct answer scores 1, an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers and you are urged to attempt every question.

Question 1

$(2x - 1)^4$ may be expressed as

A $16x^4 + 1$
B $16x^4 - 8x^3 + 4x^2 - 2x + 1$
C $16x^4 + 8x^3 + 4x^2 + 2x + 1$
D $2x^4 - 8x^3 + 12x^2 - 8x + 1$
E $16x^4 - 32x^3 + 24x^2 - 8x + 1$

Question 2

The graph of the function $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^4 - 4x^2$, has

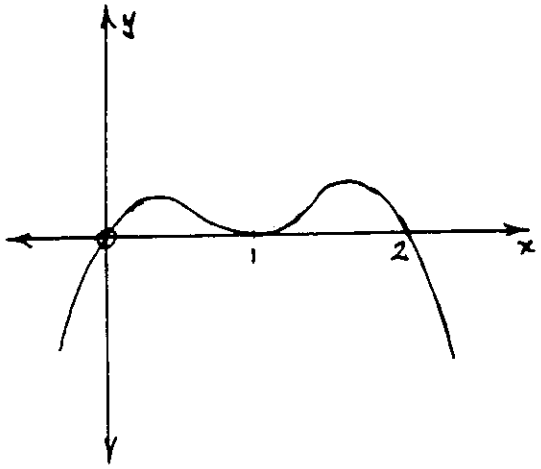
A no x-intercepts.
B one x-intercept.
C two x-intercepts.
D three x-intercepts.
E four x-intercepts.

Question 3

The graph sketched at right represents a polynomial function of degree 4.

The rule could be

A $y = x(x - 1)^2(2 - x)$
B $y = -x(x - 1)(x - 2)^2$
C $y = x(x - 1)^2(x - 2)$
D $y = x^2(x - 1)(x - 2)$
E $y = -x(x + 1)^2(x + 2)$



Question 4

The function $g : \mathbb{R}\backslash\{2\} \to \mathbb{R}$, defined by $g(x) = \frac{1}{(x-2)^2} + 3$, has a **range** of

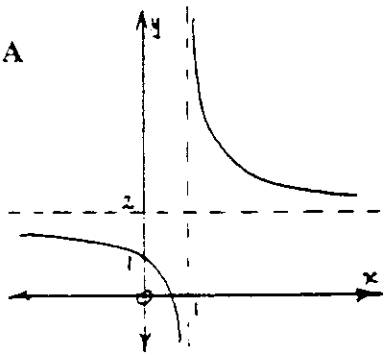
A	$\mathbb{R}$	**D**	$\{y : y > 3\}$
B	$\mathbb{R}\backslash\{3\}$	**E**	$\{y : y > 2\}$
C	$\mathbb{R}\backslash\{0\}$		

Question 5

Which one of the following graphs best represents the function

$$h : \mathbb{R}\backslash\{-1\} \to \mathbb{R}, \quad \text{where } h(x) = 2 - \frac{1}{x+1}$$

A



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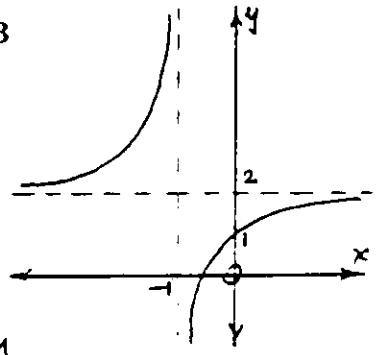

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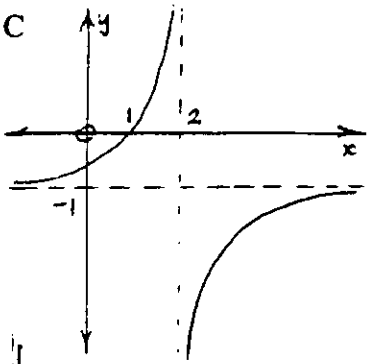

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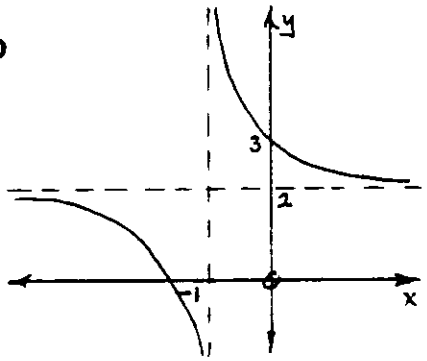
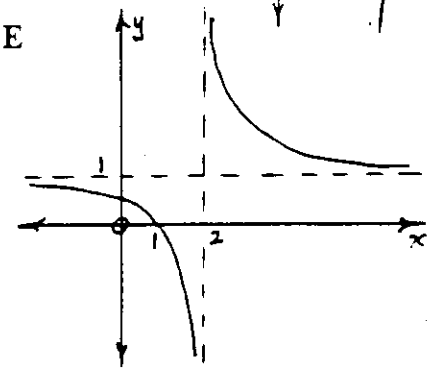


E

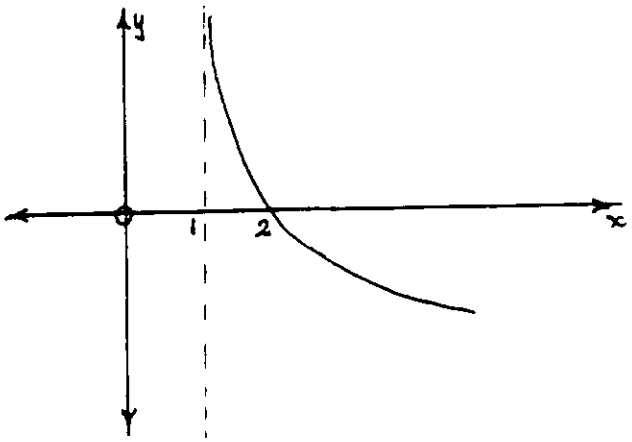


Question 6

Which one of the following functions does **not** have an **inverse function**?

A	$\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6)\}$
B	$\{(x,y) : y = \log_2 x\}$
C	$f : \mathbb{R} \to \mathbb{R}, f(x) = x^2$
D	$\{(x,y) : y = 2^x\}$
E	$g : \mathbb{R} \to \mathbb{R}, g(x) = 2x$

Questions 7 and 8 refer to the sketch graph below which represents a function f with domain $\{x : x > 1\}$.



Question 7

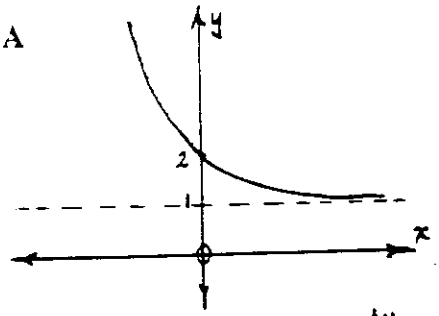
The rule of the function f could be

A $y = \log_e(x - 1)$
B $y = e^{x-1}$
C $y = -\log_e(x - 1)$
D $y = e^{x-2} - 1$
E $y = -\log_e(x + 1)$

Question 8

The graph of the **inverse function** f^{-1} could be

A



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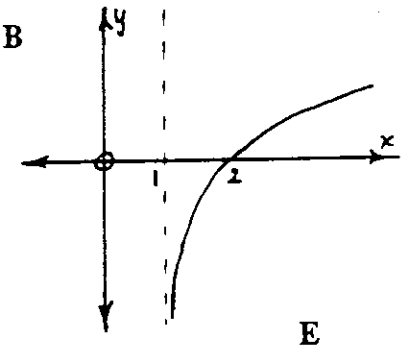

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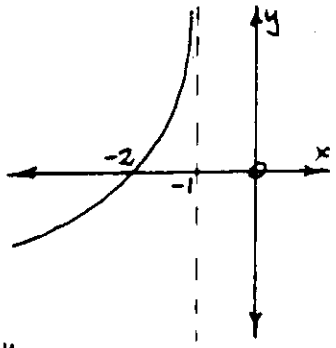

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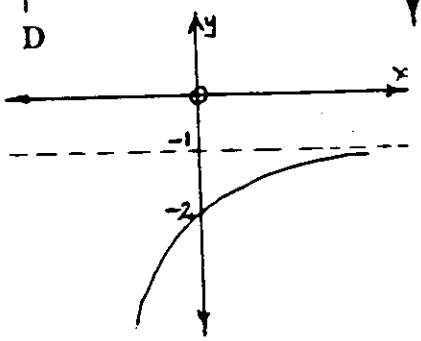
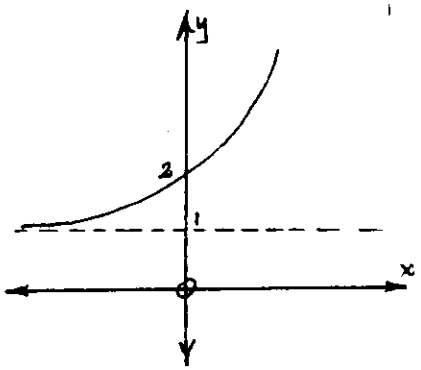


E



Question 9

If $x = \log_a 2$ and $y = \log_a 5$, then $\log_a 0{\cdot}8$ is equal to

A $x + y$
B $2x - y$
C $x - 2y$
D $3x - 2y$
E $4x - 2y$

Question 10

If $2^{2x-1} = 0{\cdot}125$, then x is equal to

A -3
B -2
C -1
D 0
E $1{\cdot}5$

Question 11

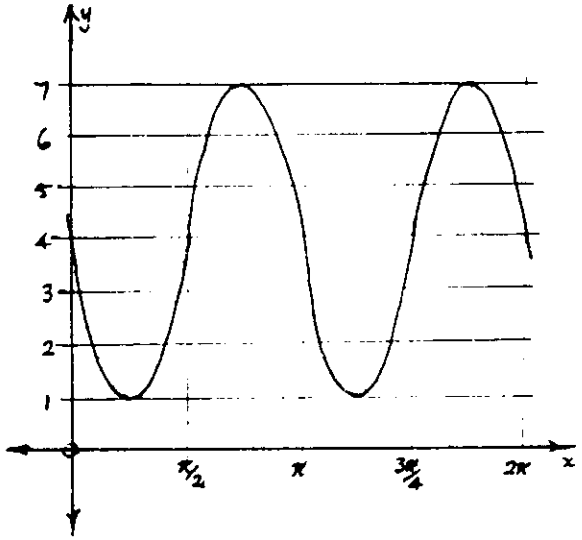
If $\log_{10} y = -\frac{1}{4}x + 2$, then y is equal to

A $y = 2 \times 10^{-x/4}$
B $y = 100 \times 10^{x/4}$
C $y = 10^{-0.25} \times 10^{2x}$
D $y = 10^2 \times 10^{-4x}$
E $y = 100 \times 10^{-x/4}$

Question 12

The graph sketched at right represents part of a function of the form

$f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 4 + a \sin kx$.



The values of a and k are, respectively,

A -3 and 2
B $-\frac{1}{3}$ and 2
C $\frac{1}{3}$ and $\frac{1}{2}$
D -3 and $\frac{1}{2}$
E 3 and 2

Question 13

Within the restricted domain $0 \le x \le 2\pi$, the equation $2 \cos 3x = \sqrt{3}$ has

A two solutions
B three solutions
C four solutions
D five solutions
E six solutions

Question 14

If $y = 2x \sin x$, then $\frac{dy}{dx}$ is equal to

A $2 \cos x$
B $2 \cos x + 2x \sin x$
C $2 \sin x - 2x \cos x$
D $2 \sin x + 2x \cos x$
E $2 \cos x - 2x \sin x$

Question 15

If $f(x) = \log_e(x^2 + 1)$, then $f'(x)$ is equal to

A $2\log_e(x^2 + 1)$

B $\dfrac{2x}{x^2 + 1}$

C 2

D $\dfrac{1}{x}(x^2 + 1) + 2x\log_e x$

E $\dfrac{x^2 + 1}{\log_e(2x)}$

Question 16

If $g(x) = \dfrac{x^3 + 2}{e^x}$, then $g'(0)$

A is equal to -2
B is equal to -1
C is equal to 0
D is equal to 2
E is not defined

Question 17

For a particular cubic function f, when $x < -1$, $f'(x) < 0$;
$f'(-1) = 0$;
when $-1 < x < 1$, $f'(x) > 0$;
$f'(1) = 0$; and
when $x > 1$, $f'(x) < 0$.

From this information, the graph of f has

A a local maximum when $x = -1$
B a stationary point of inflection when $x = -1$
C a local minimum when $x = 1$
D a stationary point of inflection when $x = 1$
E a local maximum when $x = 1$

Question 18

The volume of a rectangular prism, V cm^3, is given by the formula $V = x(x - 3)^2$, where x cm is one of the side lengths, and $0 < x < 3$. The **maximum** volume of the prism occurs when x is equal to

A ½
B 1
C 1½
D 2
E 3

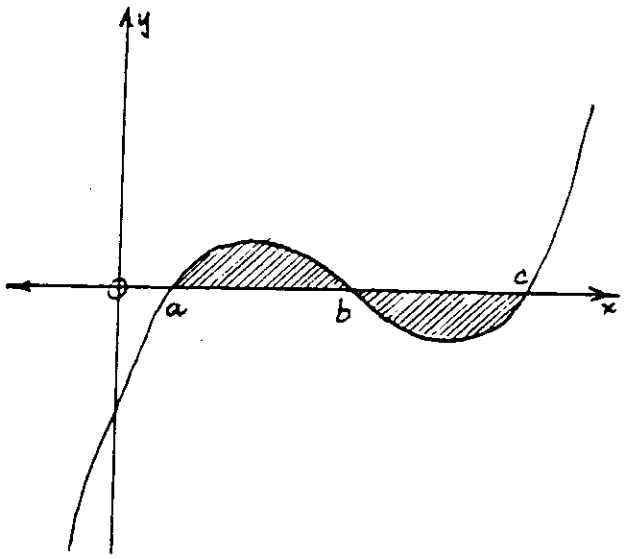
Question 19

An antiderivative of $3x^2 + \sin x$ is

A $x^3 - \cos x$
B $6x - \cos x$
C $x^3 + \cos x$
D $6x + \cos x$
E $x^3 + \sin x$

Question 20

The graph of a particular cubic function f, sketched at right, has three x-intercepts: a, b and c.



The area of the shaded region is given by

A $\int_a^c f(x)\,dx$

B $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

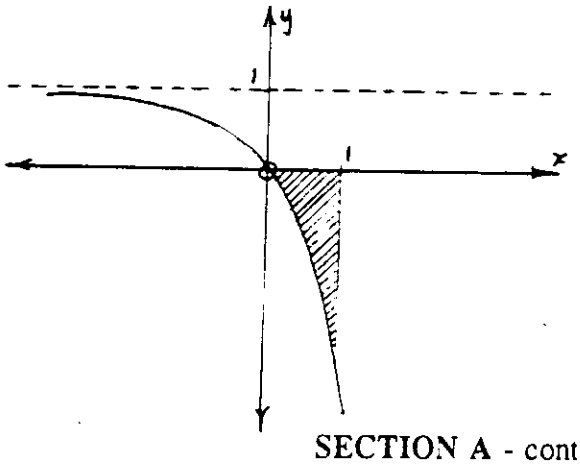
C $\int_a^b f(x)\,dx - \int_b^c f(x)\,dx$

D $\int_a^c f(x)\,dx - \int_b^c f(x)\,dx$

E $\int_b^c f(x)\,dx - \int_a^b f(x)\,dx$

Question 21

The graph at right represents part of the function defined by $f(x) = 1 - e^x$.



The area of the shaded region is

A (e - 2) square units
B 1 square unit
C (e - 1) square units
D 2 square units
E e square units

Question 22

$\int_{-1}^{2} \frac{1}{2x+4}\,dx = \log_e k$, where k is equal to

A ½
B 2
C 4
D 8
E 16

Questions 23 and 24 refer to the following probability distribution for random variable *X*:

x	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.3	0.3	0.1

Question 23

The **expected value** of X is

A 1.0
B 2.0
C 2.1
D 2.2
E 2.5

Question 24

The **variance** of X is

A 1.14
B 1.29
C 2.10
D 3.04
E 5.70

Question 25

A fair die is rolled 180 times. The number of sixes has a **standard deviation** of

A $\sqrt{5}$
B $2\sqrt{5}$
C 5
D 10
E 25

Question 26

A married couple have four children. Assuming that the probability of a male child remains at a constant value of ½, the probability that the couple will have precisely two boys and two girls is

A 1/16
B 1/5
C ¼
D ⅜
E ½

Question 27

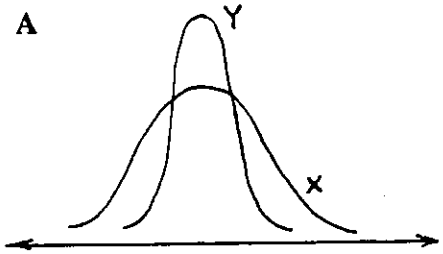
A fair coin is tossed 400 times. The expected number of heads is 200. If this experiment were repeated many times, we would expect the number of heads to be close to 200 on most occasions. On 95% of occasions we would expect the number of heads to be between

A 195 and 205
B 190 and 210
C 185 and 215
D 180 and 220
E 170 and 230

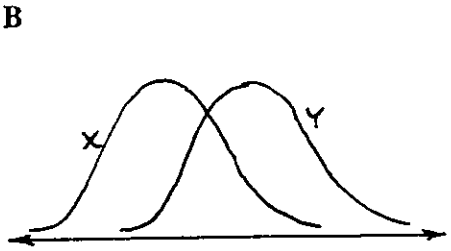
Question 28

The following diagrams represent the probability distributions for two **normally distributed random variables**, X and Y. In which one of these diagrams does Y have a **larger mean** and a **smaller standard deviation** than X?

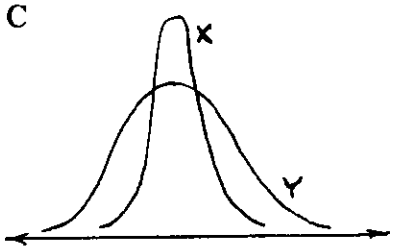
A



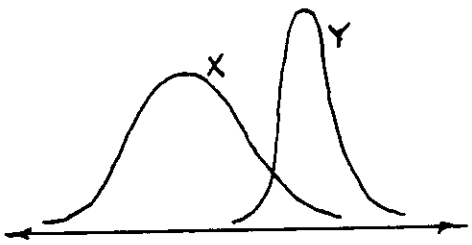
B



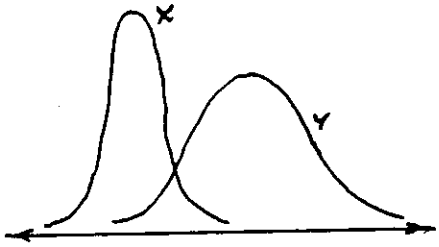
C



D



E



Question 29

If Z has a standard normal distribution, $\Pr(Z < 2) = 0.9772$ and $\Pr(Z < 1) = 0.8413$, then $\Pr(-1 < Z < 2)$ is equal to

A 0.1359
B 0.1815
C 0.3185
D 0.8185
E 0.8664

No. 25

Question 30

Which one of the following random variables, X, with Binomial distribution is **best** approximated by the normal distribution?

A $X \sim \text{Bi}(n=20, p=\frac{1}{2})$
B $X \sim \text{Bi}(n=100, p=\frac{1}{3})$
C $X \sim \text{Bi}(n=100, p=\frac{1}{8})$
D $X \sim \text{Bi}(n=50, p=\frac{3}{4})$
E $X \sim \text{Bi}(n=100, p=\frac{1}{2})$

Question 31

Which one of the following symbols is used to represent a **population parameter**?

A $\bar{x}$
B s_x
C μ
D $\hat{p}$
E none of these - all are symbols for **sample statistics**

Question 32

From a random sample of 400 students, 80 agreed with the statement that "Corporal punishment is sometimes justified in a school." If p denotes the proportion of the student population that agrees with this statement, then the standard error of the estimate of p is

A 0.0004
B 0.002
C 0.02
D 0.04
E 0.12

Question 33

A telephone poll in late June 1994 asked 1000 voters, selected at random, who they thought would make the better Prime Minister: Mr Keating or Mr Downer. 54% stated a preference for Mr Downer. On the basis of this study, we could be 95% confident that (at that time) the proportion of Australian voters who would prefer Mr Downer is approximately

A 53% to 55%
B 51% to 57%
C 49% to 59%
D 44% to 64%
E 40% to 68%

END OF SECTION A

SECTION B

Specific Instructions for Section B

Section B consists of 5 questions. There are a total of 17 marks available.

Answer all five questions neatly on the lined paper provided.

Please number each question clearly.

Full marks may not be given for answers which do not show appropriate working, or do not state answers clearly.

Question 1

Express each of the following polynomials as the product of linear factors:

(a) $2x^3 + x^2 - 8x - 4$
(b) $6x^4 + 5x^3 - 6x^2$

[2 + 2 = **4 marks**]

Question 2

These questions refer to the function $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = \frac{1}{3}x^3 - x^2 + x$

(a) Sketch a graph of the **derivative function** f'.
(b) State the feature of (i) the graph of f' at $x = 1$
(ii) the graph of f at $x = 1$

[2 + 1 + 1 = **4 marks**]

Question 3

On the same set of Cartesian axes, and using the same scale on both axes, sketch graphs of

(a) $f : [2,\infty) \to \mathbb{R}$, where $f(x) = \sqrt{x - 2} + 1$, and
(b) the **inverse function** f^{-1}.

Label the graphs carefully.

[1 + 2 = **3 marks**]

Question 4

An object is thrown vertically upwards so that its height above the ground, h m, at time t seconds ($0 \le t \le 10$) is given by $h = 100t - 10t^2$.

Calculate

(a) the **average** rate at which the height is increasing over the first 5 seconds, and

(b) the **initial** rate at which the height is increasing, at the **instant** the object is thrown.

[1 + 2 = 3 marks]

Question 5

Suppose that the height of an adult male in a population is a normal random variable with mean 175 cm and standard deviation 12 cm. Find the **interquartile range** of adult male heights in this population, correct to the nearest centimetre. [The interquartile range is the difference between the first and third quartiles.]

You may use a calculator or the following inverse cumulative normal distribution function (inverse *cdf*) table:

Pr(Z<z)	z
0.50	0.0000
0.55	0.1257
0.60	0.2533
0.65	0.3853
0.70	0.5244
0.75	0.6745
0.80	0.8416
0.85	1.0364
0.90	1.2816
0.95	1.6449

[3 marks]

END OF BOOKLET